WORLDDYN AS THE TOOL FOR STUDY OF WORLD DYNAMICS WITH FORRESTER’S MODEL: THEORY, ALGORITHMS, EXPERIMENTS

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Abstract: *Forrester´s model (FM) is a system of nonlineal differential equations constructed at far 70-s. Since that time FM remains a classical example of applications of system dynamics principles. This paper shortly describes the theory, algorithms and program WorldDyn for end-users being interested to meet themselves with FM. Moreover the program allows to model undefiniteness presented in the form of additive or multiplicative noise. By the moment there is no any free-share software having these possibilities.*

Keywords: *Forrester model, numerical analysis, noise-immunity, applied software.*

1. Classic Forrester’s model

1.1. Principles of system dynamics

System dynamics is based on two main principles. First of all, equations of the same type are written for all variables:

$$\frac{\mathrm{dy}}{\mathrm{dt}} = \mathrm{y}^{+} - \mathrm{y}^{-}. \quad (1)$$

Here y+ is the positive rate of variable change (it includes all factors calling the rise of the variable y), y– is the negative rate of variable change (it includes all factors calling decrease of the variable y).

Thereafter it is supposed that all rates (the positive and negative ones) could be presented in the form of function compositions, which depend on one factor (combination of main variables):

$$y^{\pm} = g(y_1, y_2, \dots, y_n) = f(F_1, F_2, \dots, F_k) = f_1(F_1) f_2(F_2) \dots f_k(F_k). \quad (2)$$

1.2. Forrester’s model

Roman Club is nongovernmental organization, which joins political und scientific personalities and is working on modeling of World Crisis. At far 1970 the elite Roman club asked prof. J. Forrester from MIT to develop a model of world dynamics. Speaking world dynamics we mean the dynamic interactivity of the main macroeconomics variables. The first version of the model named “World-1” was presented in 4 weeks and next year the corrected version “World-2” [2] was accepted as the classical J. Forrester model. In spite of its long history the J. Forrester model retains its actuality being the basis for modern models, so we can say, that this model is actual even today, in spite of elderly age. Such models, based on J. Forrester’s model, can predict crisis and sometimes avoid it. So, such models are very important.

As far classical model, J. Forrester in his work [2] saw five main problems, because of which the World Crisis can appear. It is overpopulation of our planet, lack of basis resources, the critical level of pollution, food shortages and industrialization and the related industrial growth. He tied a single variable with each of these issues. So, we have a five-level system, on which is built the structure of the system:

- Population (P);
- Pollution (Z);
- Natural resources (R);
- Fixed capital (K);
- Capital investment in agriculture fraction (X).

For system level J. Forrester brought the following differential equation:

$$\frac{dP}{dt} = P(c_B B_C B_P B_F B_Z - c_D D_C D_P D_F D_Z) \quad (3)$$

$$\frac{dK}{dt} = c_K P K_C - \frac{K}{T_K} \quad (4)$$

$$\frac{dX}{dt} = \frac{X_F X_Q - X}{T_X} \quad (5)$$

$$\frac{dZ}{dt} = P Z_K - \frac{Z}{T_Z} \quad (6)$$

$$\frac{dR}{dt} = -P R_C \quad (7)$$

Here he used tabulated functions (with linear interpolation) $(B_C, B_P, B_F, B_Z, D_C, D_P, D_F, D_Z, K_C, X_F, X_Q, Z_K, R_C)$ and constants:

c_B=0,04 (normal fertility rate), c_D=0,028 (normal death rate), c_K=0,05 (normal rate of capital), T_K=40 (time of depreciation main funds), T_X=15 (time of depreciation agricultural funds), t_N=1970 (initial year), P_N=3,6·10^9 (population in initial year), X_N=0,3 (capital investment ratio in agriculture in initial year), Z_N=3,6·10^9 (pollution in initial year).

Initial data are:

$$t_0 = 1900, P_0 = 1{,}65 \cdot 10^9, K_0 = 0{,}4 \cdot 10^9, X_0 = 0{,}2, Z_0 = 0{,}2 \cdot 10^9, R_0 = 900 \cdot 10^9.$$

Standard pollution Z_N is numerically equal to the population, and R_0 was taken on the assumption that resource at a constant rate of consumption (equal to the rate of consumption in 1970) should be sufficient for 250 years.

In addition Forrester imputed such variables, as consumption (F) and material level of living (C):

$$F = F_X F_P F_Z \quad (8)$$

$$C = K_p \frac{1 - X}{1 - X_0} E_R \quad (9)$$

Here F_X, F_P, F_Z, K_p, E_R are tabulated functions.

1.3. Experiment 1: instability

If we try to solve equations (3)-(7), we will get following results. The behavior of the model parameters is shown in Figure1:

One can see (Figure 1), that after a period of growth, the population P begins to decline since 2020. Non-renewable natural resources in 2100 are less than ⅓ of the original stock. Pollution reaches its maximum in 2050, about 6 (more precisely, 5.8) times exceeding the standard level, then drops due to the general decline of industry and population decline. Material standard of living reaches its maximum about 2000, and then decreases.

The reason of it is resource depletion. Reducing the supply of resources R causes lower material standard of living C. This causes increase mortality and reduce investment. And, finally, we have a sharp population decline and fall of industrial production (of funds). J. Forrester tried to change the original settings in order to avoid the crisis, but every time the crisis arose.

1.4. Experiment 2: Global equilibrium

So Forrester suggested changing some constants in the model that means political reforms in 1970 (the year, when he built his model). More specially, he offered [2]:

1. To decrease natural resources usage rate in 4 times (as compared with 1970 year);
2. To decrease pollution generation in 2 times;
3. To decrease capital investment on 40%;
4. To decrease birth rate on 30%.

In the Figure 2 we can see the behavior of main variables under these assumptions:

In this case we come to so-called “global equilibrium”. Behavior of model is improved, but we don’t solve the problem. Nevertheless we can adapt to situation.

In this paper we will discuss “stable” model.

1.5. Forrester’s followers

Forrester’s model was developed more than 40 years ago, so many researches worked with this model.

D. Meadows [5] developed World-3. He suggested including more than one variable in each “problem” sector. But unfortunately he had too little data for his model, so he tried to retrieve data according to Forrester’s model.

In USSR and Russia there were Forrester’s followers too. Matrosov [4] suggested including new factors, such as biomass of the Earths, scientific-and-technological advance, political tension. He proved, that there are stationary solutions and they are stable. Egorov [1] influenced on material standard of living, pollution ratio, food coefficient. And he supposed that there is technology of utilization and recovery resources, artificial cleaning pollution and investment in agriculture can be changed. S. Makhov [3] removed from the model such variables, as pollution and capital investment in agriculture fraction, but includes energy resources and education level.

By the moment there is no an accessible end-user program based on this model and our goal was the development of such a program with a comfortable graphical interface. And no one has investigated an influence of white noise on the model. We will try to do this analysis.

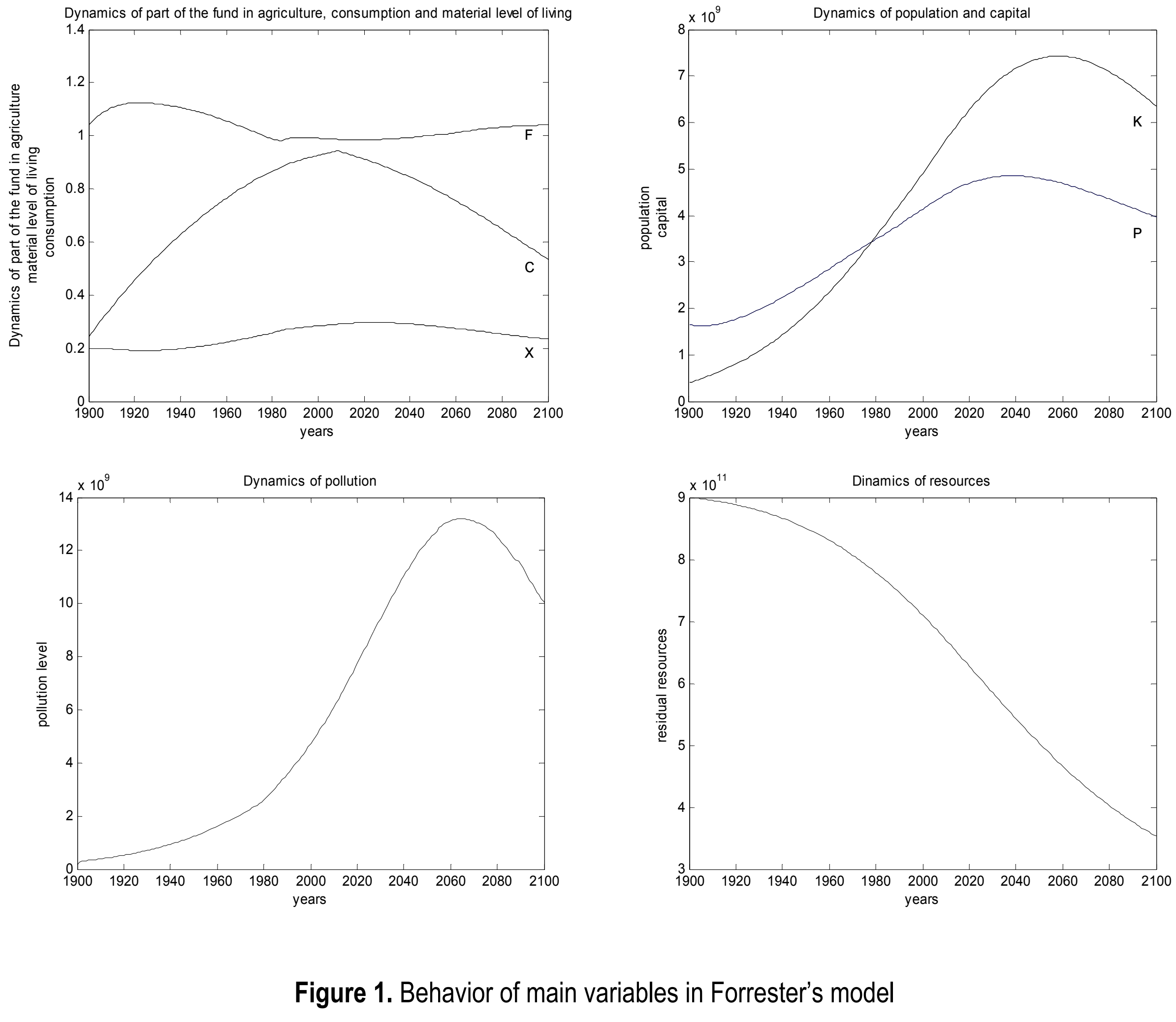


Figure 1. Behavior of main variables in Forrester’s model

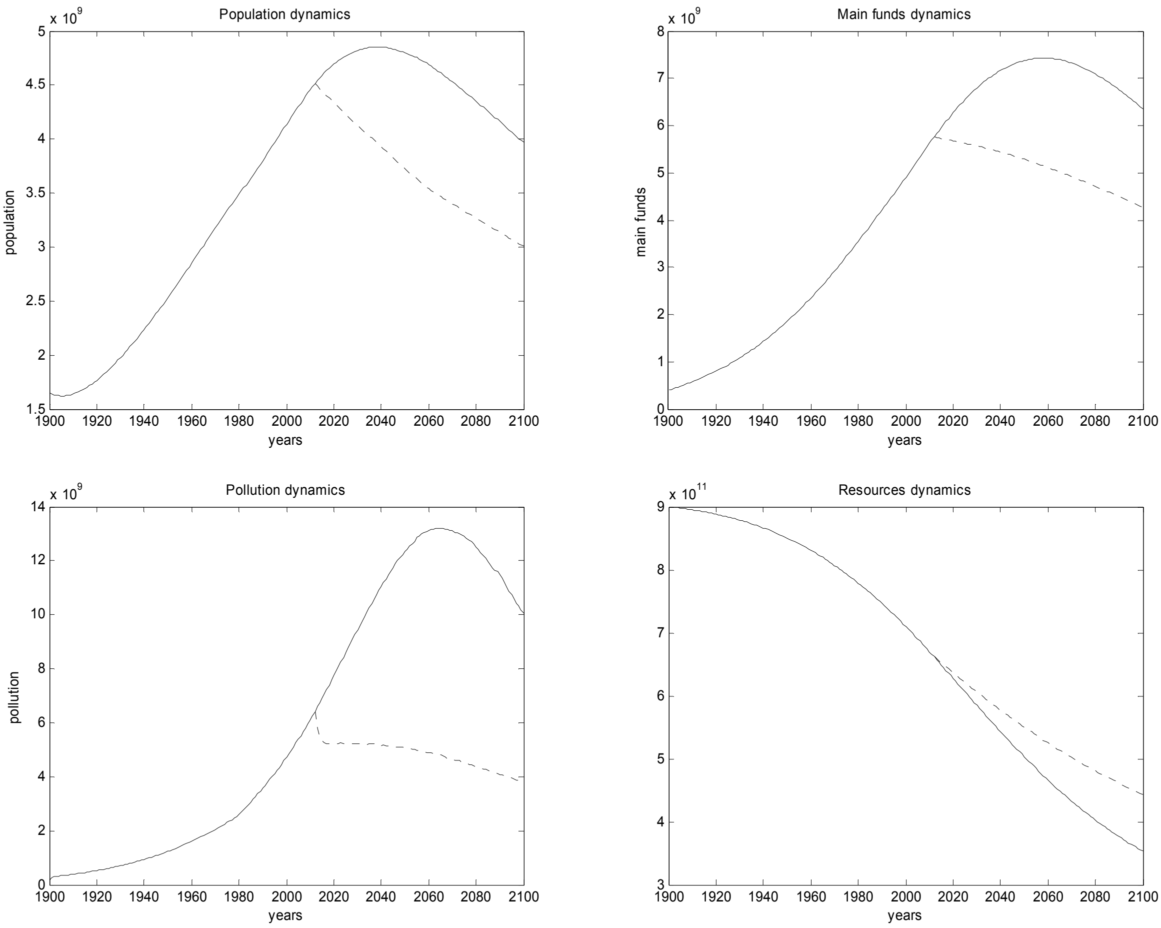


Figure 2. Behavior of main variables under assumptions 1-4. Dotted line: results of modeling under assumptions, firm line: initial behavior

2. Forrester’s model with noise

2.1. What means noise?

In this research we affect original Forrester’s model with additive and multiplicative noise. Under additive noise we understand external influences on the system (outside the scope of this level of consideration) that cannot be controlled. Under additive noise we understand disturbance within the system. We expect that influence of multiplicative noise will be less than influence of additive noise.

2.1. Modeling with additive noise

We appended white additive noise in 1970, in other worlds, on forecast. In this case the model before 1970 is setted by equations (3-7).

After 1970 the model is:

$$\frac{dP}{dt} = P(c_B B_C B_P B_F B_Z - c_D D_C D_P D_F D_Z) + \vartheta \tag{10}$$

$$\frac{dK}{dt} = c_K P K_C - \frac{K}{T_K} + \theta \tag{11}$$

$$\frac{dX}{dt} = \frac{X_F X_Q - X}{T_X} + \mu \tag{12}$$

$$\frac{dZ}{dt} = P Z_K - \frac{Z}{T_Z} + \sigma \tag{13}$$

$$\frac{dR}{dt} = -PR_C + \tau \tag{14}$$

where ϑ, θ, μ, σ, τ — stationary white noise.

When we find out the most powerful and the most sensitive variable, we append white noise only in one equation.

The noise was constructed as follows.

Signal strength was taken during the teaching phase, that is, for each variable was considered integral in the form:

$$\frac{1}{71}\int_{1900}^{1970} f^2(t)dt$$

Next, white noise was generated; in each time this value was multiplied by the noise signal power for each of the variables.

After that the system was solved 100 times, we got 100 "noise" functions. For each time point we calculated the arithmetic mean, the resulting function is the mathematical expectation of the process.

As a measure of stability, we used the relative standard deviation:

$$\sigma_i^{rat} = \frac{1}{131} * \sum_{t=1970}^{2100} \frac{1}{f_i(t)} * \sqrt{\sum_{k=1}^{100} (f_i^k(t) - f_i^{mean}(t))^2 * \frac{1}{100}} \tag{15}$$

where

$f_i(t)$ - noiseless value of the i-th variable (i = $\overline{1,5}$) - the number of variable in succession);

$f_i^k(t)$ – value at time t of i-th variable in the k-th realization;

$f_i^{mean}(t)$ – the mathematical expectation of k-th variable as defined above.

First of all we studied noise influence on the system, when noise affects all variables. We considered the case with 20% noise level. This level is close to the critical value, when some solutions become diverge. Figure 3 presents the results of modeling for all macroeconomics variables. There are 3 lines on the figure: thin uninterrupted line is the initial function, thick line is the forecast, and thin dotted line is the worst function.

One can see that the forecast is very close to the initial line, which reflects unnoised function. It means that our model is stable to the 20% noise. In this case all functions converge and the relative root-mean-square deviation for every variable is equal:

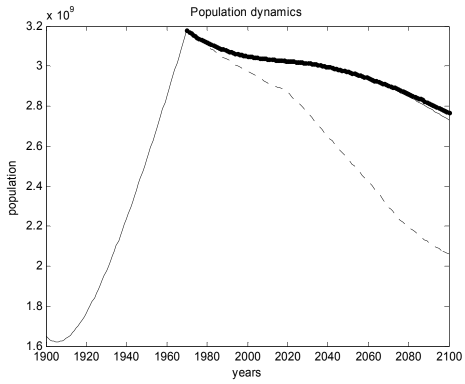
- 3,52% for population;
- 4,26% for main funds;
- 4,38% for agriculture;
- 6,00% for pollution;
- 6,45% for resources.

Generally speaking, the Forrester model is very stability to the noise, which acts on the forecast period. Even when the noise level reaches 50% we have 69% convergent functions [6]. This result also can be considered as the very good one.

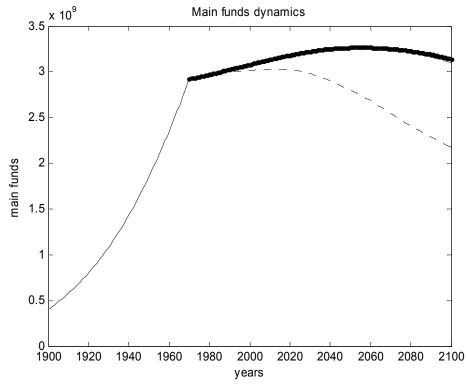
We also have done some experiments, when noise affects only one of variables. This way we wanted to find out the most sensitive and the most influential variable. When we talk about the most sensitive variable, we mean variable, which reacts most strongly to a noisy of the other variables. When we talk about the most influential variable, we mean variable, noisy of which influence the most other variables. We found out, that the most sensitive variable is pollution and the most influential variable is resources. Figure 4 illustrates the population dynamics given the influence of different variables.

In the case, when noise affects only resources, the relative root-mean-square deviation for every variable is equal:

- 0,47% for population;
- 0,78% for main funds;
- 0,41% for agriculture;
- 1,95% for pollution;
- 0,71% for resources;



(a)



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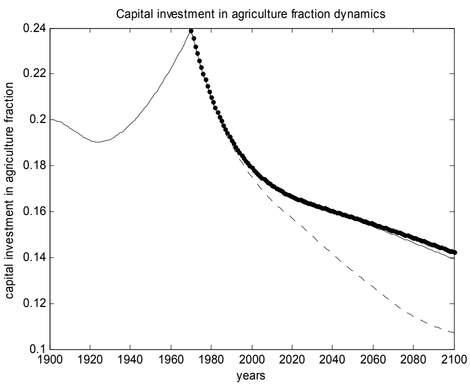

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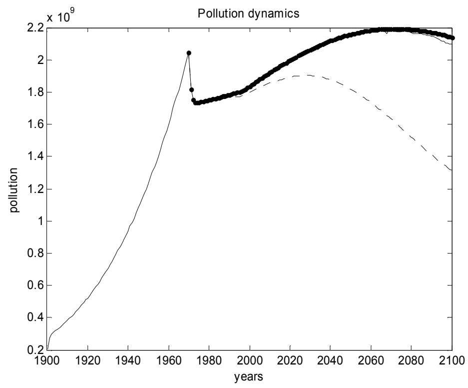

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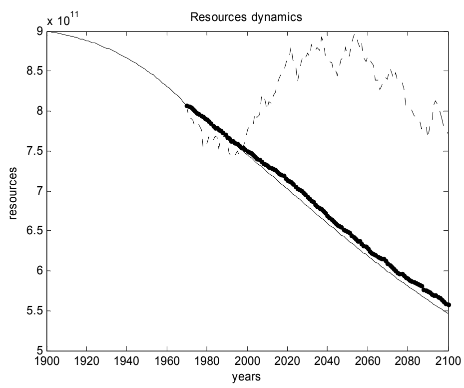


(e)

Figure 3. Results of modeling (a) population dynamics; (b) main funds dynamics; (c) dynamics of capital investment in agriculture fraction (d) pollution dynamics; (e) resources dynamics.

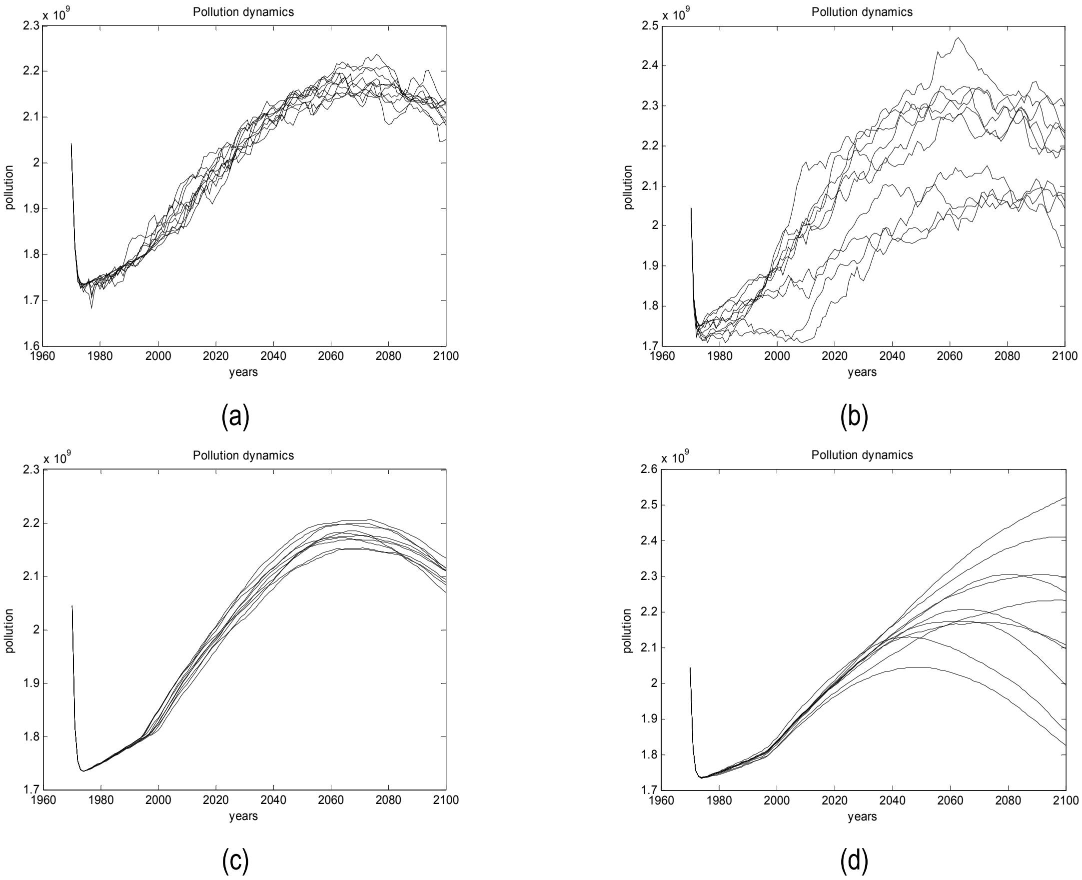


Figure 4. Results of modeling pollution dynamics (a) noise affects only population; (b) noise affects only main funds; (c) noise affects only pollution; (d) noise affects only resources

2.3. Modeling with multiplicative noise

We appended white multiplicative noise in 1970, in other worlds, on forecast. In this case the model before 1970 is setted by equations (3-7), after 1970 the model is:

$$\frac{dP}{dt} = P(c_B B_C B_P B_F B_Z - c_D D_C D_P D_F D_Z) * (1 + \vartheta) \tag{16}$$

$$\frac{dK}{dt} = (c_K P K_C - \frac{K}{T_K}) * (1 + \theta) \tag{17}$$

$$\frac{dX}{dt} = \frac{X_F X_Q - X}{T_X} * (1 + \mu) \tag{18}$$

$$\frac{dZ}{dt} = \left(P Z_K - \frac{Z}{T_Z}\right) * (1 + \sigma) \tag{19}$$

$$\frac{dR}{dt} = -P R_C * (1 + \tau) \tag{20}$$

where ϑ, θ, μ, σ, y,τ – stationary white noise. Each of these noises has power, it means, that we multiply it by its power.

After that the system was solved 100 times, we got 100 "noise" functions. For each time point we calculated the arithmetic mean, the resulting function is the mathematical expectation of the process.

As a measure of stability, we used (15).

We studied noise influence on the system, when noise affects all variables. We considered the case with 50% noise level. This level is close to the critical value, when some solutions become diverge. Figure 5 presents the results of modeling for all macroeconomics variables. There are 3 lines on the figure: thin uninterrupted line is the initial function, thick line is the forecast, and thin dotted line is the worst function.

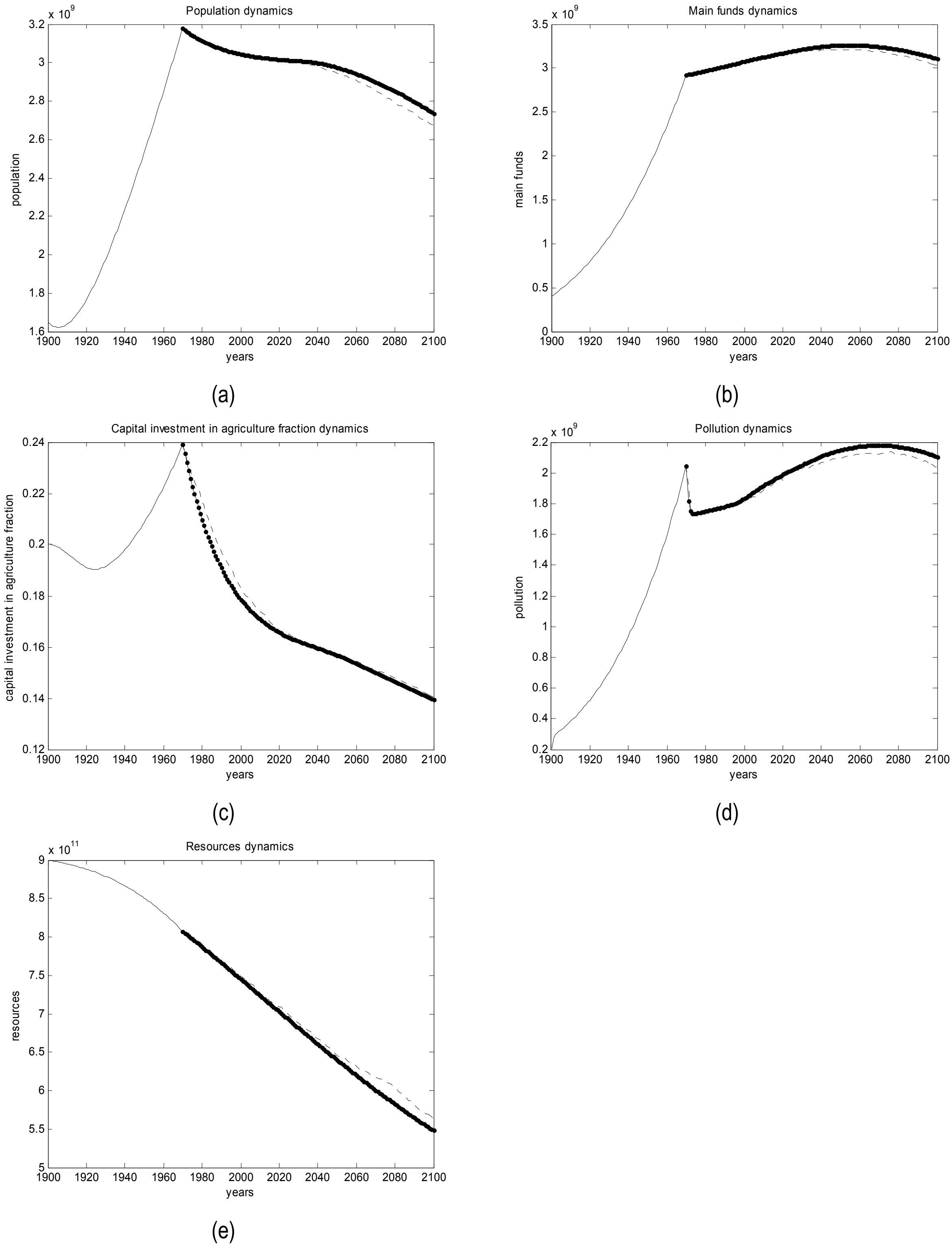


Figure 5. Results of modeling (a) population dynamics; (b) main funds dynamics; (c) dynamics of capital investment in agriculture fraction (d) pollution dynamics; (e) resources dynamics

We can see that the model is more stable to multiplicative noise, than to additive noise. In this case all functions converge and the relative root-mean-square deviation for every variable is equal:

- 0,23% for population;
- 0,28% for main funds;
- 0,48% for agriculture;
- -,50% for pollution;
- 0,49% for resources.

In the case with multiplicative noise we find out (as in the case with additive noise), that the most sensitive variable is pollution, and the most influential variable is resources.

Figure 6 illustrates the population dynamics given the influence of different variables.

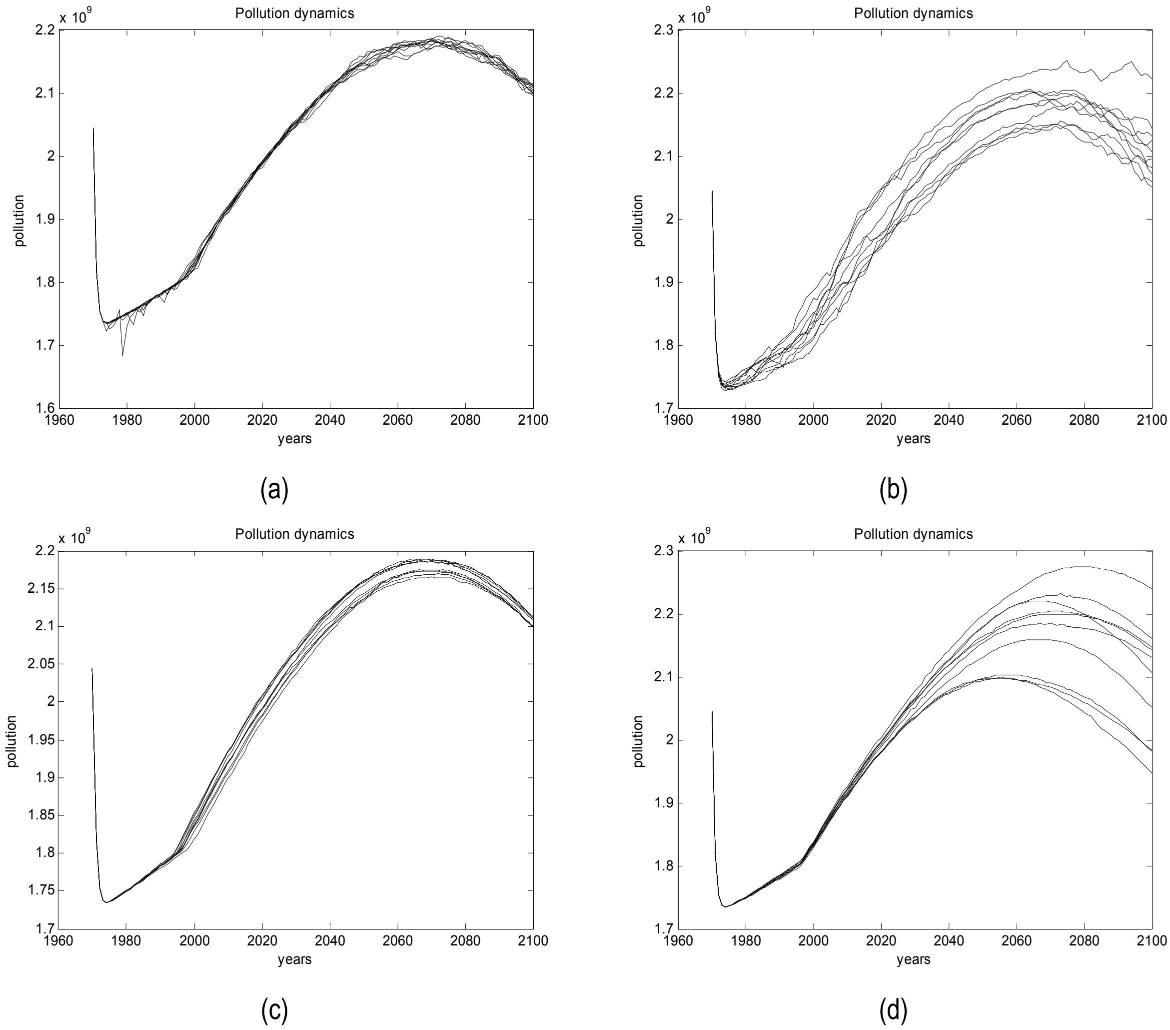


Figure 6. Results of modeling pollution dynamics (a) noise affects only population; (b) noise affects only main funds; (c) noise affects only pollution; (d) noise affects only resources

In the case, when noise affects only resources, the relative root-mean-square deviation for every variable is equal:

- 0,91% for population;
- 0,93% for main funds;
- 1,03% for agriculture;
- 1,13% for pollution;
- 1,83% for resources.

2.4. Comparative analysis

If we compare cases with additive and multiplicative noise, we can draw a conclusion, that, as we expected, multiplicative noise affect the system much less, than additive nose. And in both cases we get identical results about the most and the less sensitive and influential variable. We found out, that the most influential variable is resources. It means that we should pay large attention on resources, if we want to save stability of target system.

3. Research of crisis situation

3.1. What means “Research of crisis situation”?

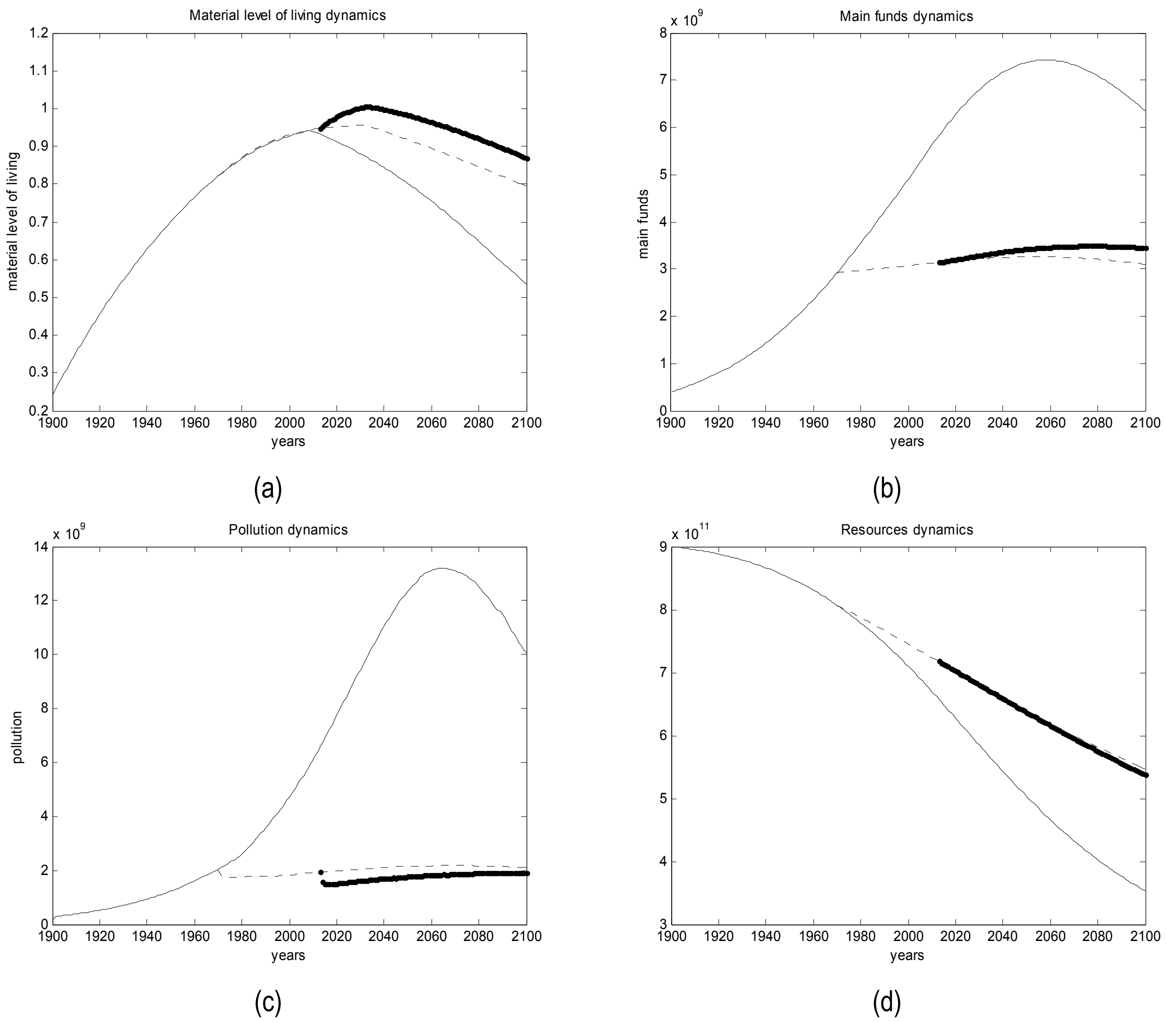
By the instrumentality of Forrester’s model, it is possible to predict world crises. And we can change parameters, as well as Forrester changed, to avoid crises. Parameter’s modifications mean different reforms, which can influence parameters of the model, such as birth rate or pollution rate restrictions.

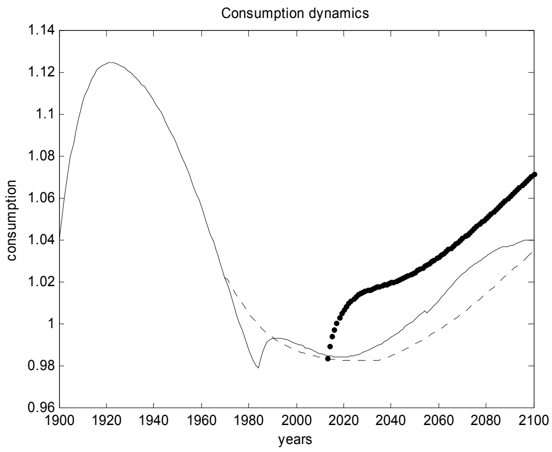
3.2. Experiment 1

For example, in addition to Forrester changes in 1970 we can append some changes in 2013 year. We suggest the following (in compare with 1970 year):

- To decrease birth rate on 10%;
- To decrease pollution on 20%.

In this case we get (figure 7):





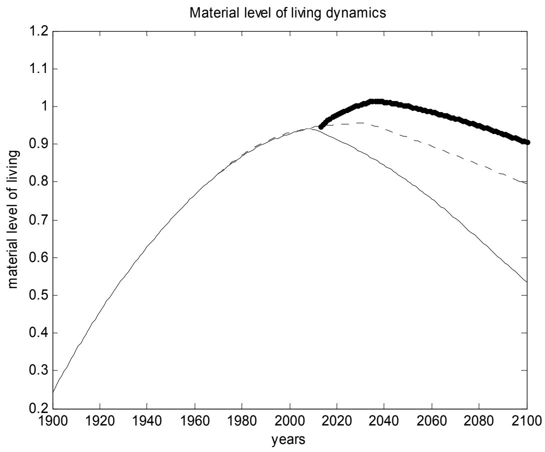
(e)

Figure 7. Results of modeling (a) material level of living dynamics; (b) main funds dynamics; (c) pollution dynamics; (d) resources dynamics; (e) consumption dynamics. Thin uninterrupted line is the initial function, thick line is the function after first break, and thin dotted line is the function after second break

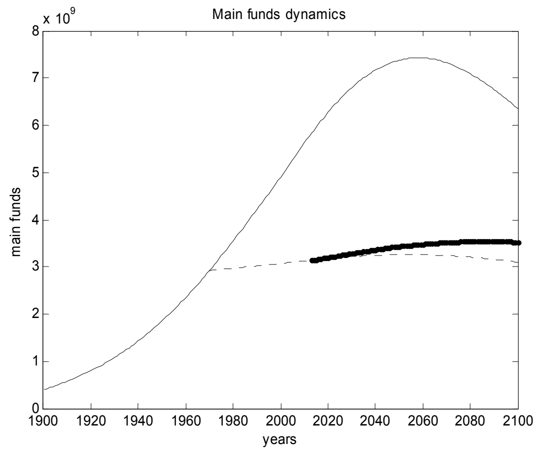
It is easy to see, that we improve material level of living and consumption, but don’t change situation with resources. So we should continue experiments with parameters.

3.3. Experiment 2

In addition to previous reforms we suggest to decrease resources consumption on 20%. The results one can see in figure 8:



(a)



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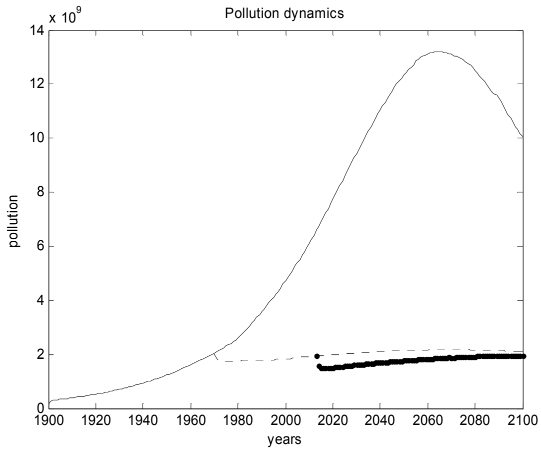
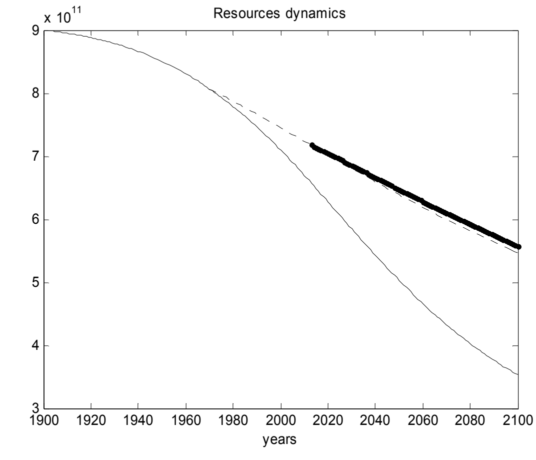
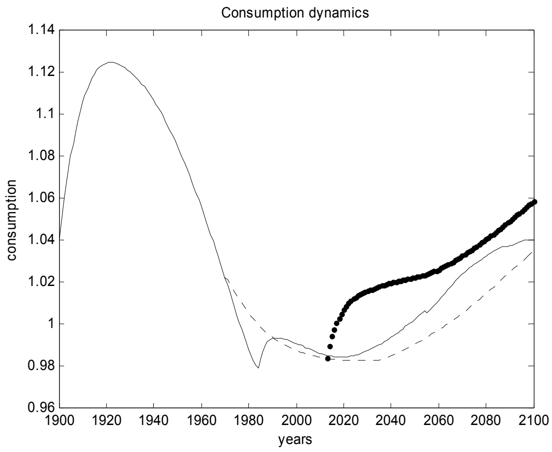




(c) (d)



(e)

Figure 8. Results of modeling (a) material level of living dynamics; (b) main funds dynamics; (c) pollution dynamics; (d) resources dynamics; (e) consumption dynamics. Thin uninterrupted line is the initial function, thick line is the function after first break, and thin dotted line is the function after second break

Unfortunately situation with resources doesn't change. But consumption is much better, even if compare it with initial dynamics.

4. WORLDDYN program

4.1. The development environment and the structure of the program

This program was developed in MATLAB [8]. The program is user-friendly, even if user doesn't have any skills in programming, he can work with WORLDDYN [7]. There are a several forms for each kind of experiments: adding additive or multiplicative noise, change parameters. One can see results of modeling in numerical forms (dispersions and percent of convergent functions) and in graphic form (figures).

4.2. Interface

A user can complete the experiments with noise. To make such an experiment in initial data the user should open "Options"-> "Noise settings (initial data)" (figure 9).

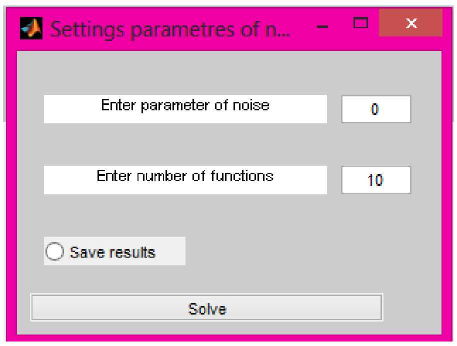


Figure 9. Menu for noise definition in the initial data

After that he chooses the noise level (parameter of noise) and the number of noise functions. To save the results the user points "Save results". The forecast will be save as "change_initial_*(name of variable)_expectation_*(noise level)noise_*(the number of functions)functions_*(the percent of converge functions)%_converge". And finally the user enters "Solve".

In order to complete the experiment with noise related with all variables on a forecast stage the user should open "Options"-> "Noise settings (forecast)" (figure 10):

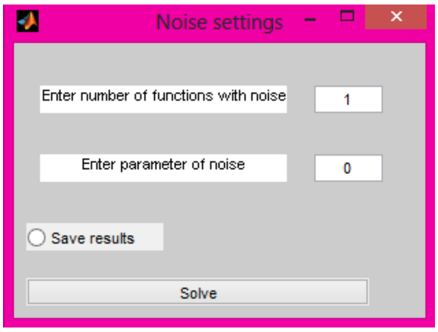


Figure 10. Menu for noise definition on the forecast stage

After that he chooses the noise level (parameter of noise) and the number of noise functions. To save the results the user points "Save results". The forecast will be saved as "change_all_*(name of variable)_expectation_*(noise level)noise_*(the number of functions)functions_*(the percent of converge functions)%_converge". And finally, the user enters "Solve".

In order to complete the experiment with noise related with one given variable on a forecast stage the user should open "Options"-> "Noise settings (only one function)" (figure 11):

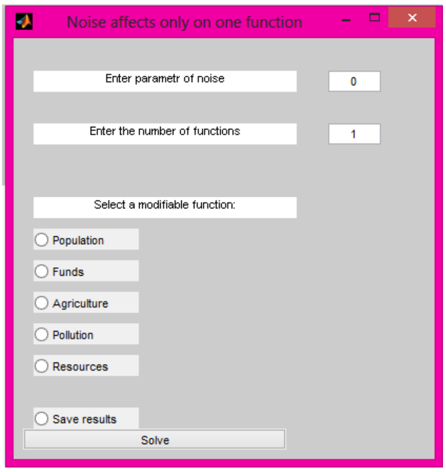


Figure 11. Menu for noise definition on the forecast stage (one variable)

After that he chooses the noise level (parameter of noise), the number of noise functions, and the name of modifiable variable. To save the results the user points "Save results". The expectation will be save as "change_only_*(name of modifiable variable) _*(name of variable)_expectation_*(noise level)noise_*(the number of functions)functions_*(the percent of converge functions)%_converge". And, finally, he can enter "Solve".

When the program World-Dyn works it can inform the user:

- About model solution (figure 12);
- About final of calculation (figure 13);
- About errors (figure 14 a, b).

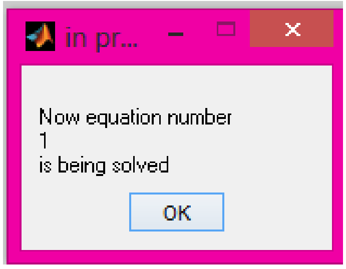


Figure 12. About model solution

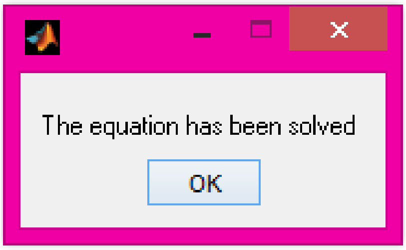


Figure 13. About final of calculation

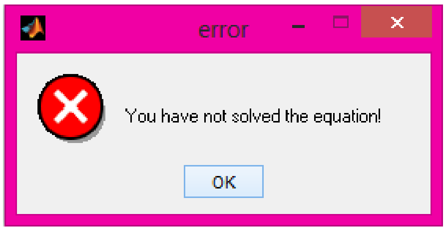


Figure 14. a) About error

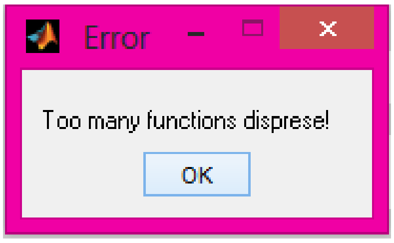


Figure 14. b) About error

4.3. Numerical methods

We used two algorithms in our research: Runge-Kutt method for modeling noise system and Adams-Bashwort-Multon method for modeling original system and system with modified parameters without noise. Consider each of these methods closer.

Runge-Kutt method

The Runge-Kutta method is as follows.

The following values are given for ordinary differential equations:

$$k_1 = f(t_n, u_n);$$
$$k_2 = f(t_n + \alpha_2\tau, u_n + \tau\beta_{21}k_1)$$
$$\dots$$
$$k_r = f(t_n + \alpha_r\tau,\ u_n + \tau(\beta_{r1}k_1 + \dots + \beta_{r,r-1}k_{r-1}))$$
$$u_{n+1} = u_n + \tau(\gamma_1 k_1 + \dots + \gamma_r k_r)$$

The factors determining the specific method can be presented as a Butcher table (table 1):

Table 1

0						
α_2	β_{21}					
α_3	β_{31}	β_{32}				
…	…	…	…			
α_r	β_{r1}	β_{r2}	…	$\beta_{r,r-1}$		
	γ_1	γ_2	…	γ_{r-1}	γ_r	

There is implemented method of third order accuracy in the program.

Adams method

Suppose we know the approximate solution of some of the computational grid nodes: $t_n, \dots, t_{n-m}$.. In the neighborhood of these nodes we replace the function of the interpolation polynomial, written in the form of Newton's:

$$f(t) = f(t_n) + f(t_n, t_{n-1})(t - t_n) + f(t_n, t_{n-1}, t_{n-2})(t - t_n)(t - t_{n-1}) + \cdots$$

In order to compute the solution at n +1, we write it in integral form:

$$u_{n+1} = u_n + \int_{t_n}^{t_{n+1}} f(t, u(t))dt = \int_{t_n}^{t_{n+1}} f(t)dt$$

4.4. Modeling protocol

User can see results in two forms: numerical and graphic. If user chooses graphic form, he can choose two modes: see all noise functions (figure 15, a) or 3 functions: the worth (the highest dispersion), mathematic expectation and unnoise function (figure 15, b). If user chooses numerical form, he will see next forms (figure 16 a, b):

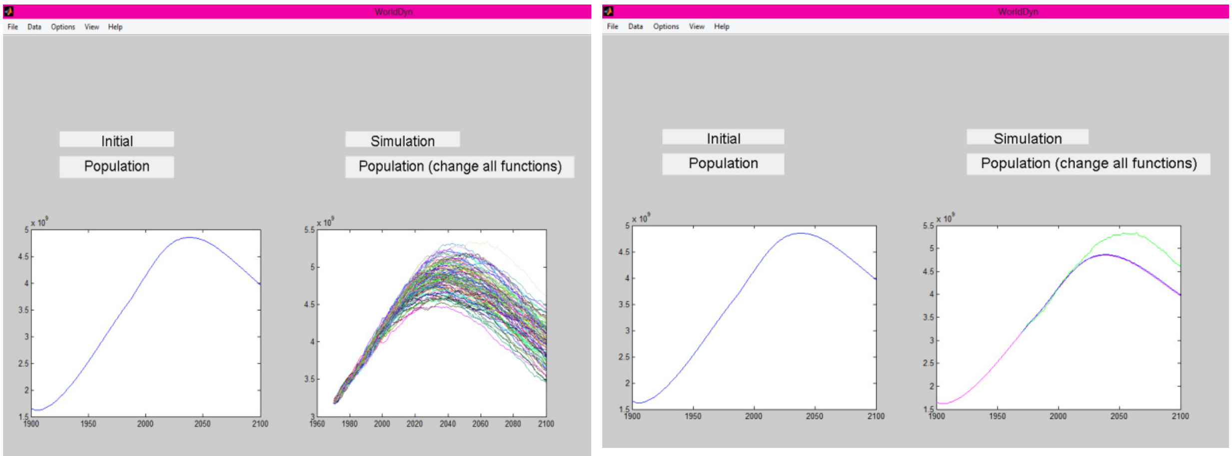


Figure 15. Graphic mode. a) All noise functions. b) 3 functions: the worth (the highest dispersion), mathematic expectation and unnoise function

Figure 16. Numerical mode a)Dispersion, b) Percent of converge functions

4.5. Rules of statement

User should choose the purpose of his research. Hence he will choose regimes of modeling and output.

If user has some problems with WORLDDYN, he can use help. There is workbook in Russian, soon will workbooks in English and Spanish.

5. Conclusion

In the paper we have studied the stability of the classical Forrester model to additive and multiplicative noise. The experiments show that

- Additive noise causes the essentially stronger effect then the same multiplicative noise on the stage of forecast;
- The most influential variable is resources; its changes provoke the strongest reaction of the model. The most sensitive variable is pollution.

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