PHYSICAL PHENOMENON OF STATISTICAL STABILITY

Igor Gorban

Abstract: *The article presents new monograph dedicated to the researching of physical phenomenon of statistical stability and exposure of basics of physical-mathematical theory of hyper-random phenomena, the latter describing physical events, variables and processes with consideration of violation of statistical stability. In contrast to two previous author's monograph devoted to the same subject in which the main attention was paid to the mathematical aspects of the theory of hyper-random phenomena, in the new book the accent is made on physical headwords. The book is oriented on scientists, engineers, and post-graduate students researching in statistical laws of natural physical phenomena as well as developing and using statistical methods for high-precision measuring, prediction and signal processing on long observation intervals. It may also be useful for high-level courses for university students majoring in physical, engineering, and mathematical fields.*

Keywords: *phenomenon of statistical stability, theory of hyper-random phenomena, physical process, violation of convergence.*

ACM Classification Keywords: *G.3 Probability and Statistics*

Introduction

In 2014 it has been published a new monograph [Gorban, 2014 (1)] dedicated to the research of physical phenomenon of statistical stability and exposure of basics of physical-mathematical theory of hyper-random phenomena, the latter describing physical events, variables and processes with consideration of violation of the statistical stability. In contrast to previous monographs [Gorban, 2007 (1), 2011 (1)] mainly devoted to mathematical aspects of the theory of hyper-random phenomena in the new book the accent is made on physical questions.

The aim of the current article is presentation of the new monograph. It is written on the base of the original author's researches published in Russian and English in different scientific issues from 2005 to 2014 [Gorban, 2005-2014].

Questions regarded in the monograph

The phenomenon of statistical stability

One of the surprising physical phenomena is the phenomenon of statistical stability, consists of the *stability of statistics* (that are the functions of the sample), in particular a frequency of mass events, averages, etc. This phenomenon is widespread and therefore *it can be regarded as fundamental natural phenomena*.

The statistical stability phenomenon the first noticed in 1669 John Graunt (who was a cloth merchant) [Graunt, 1939]. Researches of this phenomenon led to the development of the probability theory, widely used now in different areas of science and technology.

Axiomatization of the probability theory

Prior to the beginning of XX century probability theory was regarded as a *physical theory*, which describes the phenomenon of statistical stability.

At the beginning of the last century, the problem of probability theory axiomatization was raised. David Hilbert formulated this problem as a part of axiomatization problem of physics [Hilbert’s problems, 1969].

Many famous scientists have made great efforts to solve the problem. Different approaches have been proposed. At present, recognized approach is the set-theoretic one proposed by A.N. Kolmogorov, raised even to the rank of the international standard ISO [International standard, 2006].

The concept of random phenomenon

In accordance with the Kolmogorov’s approach a *random event* is described by using the probabilistic space defined by the triad of ($\Omega, \Im, P$), where Ω is the space of elementary events $\omega \in \Omega$, $\Im$ is the Borel field (σ-algebra of subsets of events) and P is the probability measure (probability) of subsets of events.

A *random variable* is regarded as a measurable function defined on the space Ω of the elementary random events ω, and a *random function* — as a function of the independent argument, the value of which is a random variable when its argument is fixed.

Under a *random phenomenon* is understood the mathematical object (a random event, a variable or a function), which is exhaustively characterized by certain concrete probability distribution law.

In the book, a *phenomenon* or a *mathematical model*, not being described by a concrete distribution law does not consider as a random one. This is an extremely important position that must be taken into account.

The probability concept

In probability theory a concept of probability event is a key one. Draw attention, in Kolmogorov’s definition the probability has not physical interpretation.

With more visual statistical determining of probability concept (by R. von Mises [Mises, 1964]), the probability $P(A)$ of a random event A is interpreted as a limit of the event frequency $p_N(A)$ when the experiments are led under the identical statistical conditions and the experiment quantity N tends to infinity: $P(A) = \lim_{N \to \infty} p_N(A)$.

At small values N the frequency $p_N(A)$ can fluctuate greatly, but with increasing of N it gradually stabilizes and under $N \to \infty$ tends to a definite limit $P(A)$.

Physical hypotheses of the probability theory

All mathematical theories, including probability theory based on the Kolmogorov’s axioms are related to the abstract mathematical concepts which *are not associated with the actual physical world*. In practice, correct application of these theories is possible if some *physical hypotheses* asserting the adequacy of description of real world objects by the relevant mathematical models are accepted.

For probability theory such *physical hypotheses* are the following ones:

- The *hypothesis of perfect statistical stability* (*ideal statistical predictability*) of parameters and characteristics of real physical phenomena (real events, variables, processes, and fields), signifying the *presence of convergence of any statistic to a constant value*;
- The *hypothesis of adequate describing of real physical phenomena by random* (*stochastic*) *models.*

It is assumed that the hypothesis of perfect statistical stability is valid for a lot of physical mass phenomena. In other words, the *stochastic concept of world’s building is accepted.*

The hypothesis of perfect statistical stability

One of the main requests to the physical hypotheses is that *they are harmonized with the experimental data*.

For many years, the hypothesis of ideal statistical stability was not doubtful one, although some scholars (even Kolmogorov and such famous scientists as A. A. Markov [Markov, 1924, p. 67] A. V. Skorokhod [Ivanenko, Labkovsky, 1990, p. 4], E. Borel [Borel, 1956, pp. 28-29] V. N. Tutubalin [Tutubalin, 1972 (2), pp. 6-7] and others) noticed that, in the real world this hypothesis is valid only with certain reservations.

Violation of statistical stability in the real world

Experimental researches on large observation intervals of various processes of different physical nature show that the *hypothesis of perfect statistical stability is not confirmed.*

The real world is continuously changing. Changes occur at all levels, including statistical one. Statistical assessments formed at relatively small observation intervals are relatively stable. The stability is manifested in decreasing of fluctuation of statistical assessments when the number of statistical data is raised. This creates the illusion of a perfect statistical stability. However, starting from a certain critical volume, the level of fluctuations practically does not change (sometimes even grows) when the amount of the data is raised. This indicates that the statistical stability is not perfect.

Violation of statistical stability in the real world means that the *probability concept has not physical interpretation. The probability is a mathematical abstraction.*

Violation of statistical stability in deterministic and stochastic models

Violation of statistical stability is observed in different models, even deterministic and random ones.

A typical example is a random variable described by the Cauchy distribution. This distribution has not the moments and therefore any assessments of its moments are *statistically unstable* (*inconsistent*) ones.

Causes of statistical instability in the real world

Violation of statistical stability is called by a lot of reasons. They are inflow in open system matter, energy and (or) information, feeding non-equilibrium processes, various nonlinear transformations, low-frequency linear filtering of special type, etc.

It was founded that *statistical stability of the process was determined by its power spectral density.* Therefore, a broadband stationary statistically stable noise in a result of low-frequency filtration can be transformed into statistically unstable process.

Investigation of violations of statistical stability and looking for effective ways for adequate description of real world phenomena accounting these violations led to building of new *physical-mathematical theory of hyper-random-random phenomena* [Gorban, 2007 (1), 2011 (1), 2014 (1)].

The concept of hyper-random-random phenomenon

In the probability theory the basic mathematical objects (models) are random phenomena (random event, variable, and function); in the theory of hyper-random phenomena such objects are hyper-random phenomena (hyper-random event, variable, and function) representing the set of unlinked random objects regarded in complex as comprehensive whole.

A *hyper-random event* can be described by the tetrad ($\Omega, \Im, G, P_g$), where Ω is a space of elementary events $\omega \in \Omega$, $\Im$ is a Borel field, G is a set of conditions $g \in G$, P_g is a probability measure of subsets of events, depending on conditions g. Thus, the probability measure is defined for all subsets of events and all possible conditions $g \in G$. Mark, the measure for conditions $g \in G$ does not exist.

Using a statistical approach, hyper-random event A can be interpreted as an event, the frequency $p_N(A)$ of which *does not stabilize with rising of the number N and under $N \to \infty$ this frequency has not a limit.* So in this case, the property of statistical stability is not intrinsic for event frequency. However, the property of statistical stability may be intrinsic for other statistics, for instance ones describing *bounds of event’s frequency oscillation*.

A random phenomenon is exhaustively described by the probability distribution, and a hyper-random phenomenon — by the set of conditional probability distributions.

A random variable X, for example, is completely characterized by the distribution function $F(x)$, and a hyper-random variable $X = \{X / g \in G\}$ — by the *set of conditional distribution functions* $F(x/g)$, $g \in G$.

A hyper-random variable can be presented not only by such set, but also by other characteristics and parameters, in particular by the upper $F_S(x) = \sup_{g \in G} F(x/g)$ and the lower $F_I(x) = \inf_{g \in G} F(x/g)$ bounds of the distribution function, by the central and crude moments of these bounds, by the bounds of the moments, etc.

The link of hyper-random models with others ones

A random variable can be interpreted as the hyper-random variable with coincided bounds of the distribution function: $F_S(x) = F_I(x) = F(x)$.

A *deterministic variable* (*constant*) can be approximately regarded as a degenerated random (or hyper-random) variable with distribution function $F(x)$ having a single jump at the point x_0.

An *interval variable* characterized by the borders x_1, x_2 of the interval can be represented by the hyper-random variable with bounds of the distribution function $F_S(x)$, $F_I(x)$ that have unit jumps at the points x_1 and x_2.

Thus, a *hyper-random variable is a generalization of the concepts of deterministic, random, and interval variables.* Therefore hyper-random models can be used for modeling of different physical phenomena that have various type and degree of uncertainty.

Determinism and uncertainty

For centuries it was believed that the world is based on deterministic principles. Discovery of the phenomenon of statistical stability shook these representations. It turned out that it was essential not only determinism, but also uncertainty too.

An important form of uncertainty is *many-valuedness*. Many-valued mathematical objects are random phenomena, interval variables and functions, as well as hyper-random phenomena. In all of them there is an uncertainty, though of different types. *Uncertainty of random phenomena has a probability measure* and *interval variables and functions have not such measure. Hyper-random phenomena contain uncertainty of both types.*

Object and subject of investigation of the theory of hyper-random phenomena

The *study object* of the theory of hyper-random phenomena are real physical phenomena — events, quantities, processes and fields. The *study subject* is a violation of statistical stability of characteristics and parameters of real physical phenomena.

General characteristic of the theory of hyper-random phenomena

The theory of hyper-random phenomena has mathematical and physical components. The mathematical component is based on the classical Kolmogorov’s axioms of probability theory, the physical one — on two hyper-random physical adequacy hypotheses:

- The *hypothesis of imperfect statistical stability of real events, quantities, processes, and fields*, and also;
- The *hypothesis of adequate description of real physical phenomena by hyper-random models.*

The assumption that these hypotheses are valid for a wide range of mass phenomena leads *to accepting of a new concept of the world’s building: it builds on hyper-random principles*. Fundamental role in this concept plays imperfect statistical stability.

From the mathematic point of view the theory of hyper-random phenomena is a *branch of the probability theory*; from the physics point of view it is a *new theory* based on the new concepts of the world’s building.

The law of large numbers and the central limit theorem in case of violation of statistical stability

A violation of statistical stability is reflected in the statistical properties of the physical phenomena, in particular ones described by the law of large numbers and the central limit theorem.

Investigations show that *both in the absence and in the presence of violation of statistical stability, the sample mean of a random sample tends to average of the mathematical expectations*. However, in the absence of violation of statistical stability the sampling mean converges to a certain number, and *in violation of stability it tends to infinity (plus or minus) or fluctuates within a certain range*. In general, the sample mean in limit may be a number, a random variable, interval or hyper-random variable with continuous area of uncertainty, bounded by the curves consisting of fragments of Gaussian curves.

Sample mean of a hyper-random sample converges to fixed value (*number*)*, to a set of the fixed values* (*numbers*)*, fluctuates in one or more disjoint intervals or tends to infinity.* At the same time the sample mean in limit can be a number, interval, multiinterval, random variable, or hyper-random variable with optional continuous area of uncertainty, bounded by the curves consisting of fragments of Gaussian curves.

Potential accuracy in case of violation of statistical stability

One of the most important questions is the potential accuracy of the measurement.

According to classical conception developed yet by Galileo Galileo the estimated physical quantity can be represented by the *single-valued deterministic volume* and the result of measurement — by the *random variable*. Measurement error has two components: *systematic* and *random* ones.

According to the probability theory when the sample size follows to infinity, the random component tends to zero and the whole error — to the systematic component. However, in practice, as is well known, it does not occur. The cause of that is in violation of statistical stability.

Within the hyper-random paradigm, the *error has hyper-random nature* and *describes by hyper-random variables*.

In general, it is impossible to divide the error in some components. In one of the simplest cases (when the bounds of the distribution function of hyper-random error differ only on mathematical expectations) the *error can be* *divided in a systematic, a random and an uncertain* (*an unpredictable*) *components*, the latter is described by the interval value.

While the sample size follows to infinity the *hyper-random error keeps hyper-random character*.

This explains many well-known but for a long time incomprehensible facts, in particular, why the accuracy of any physical measurements is limited, why in case of a large number of experimental data the accuracy does not depend on the volume of data, etc.

How the uncertainty is formed

There are many pathways of uncertainty forming. The simplest of them is a non-linear transformation, generating many-valuedness. The averaging of the determined data in the absence of convergence leads to uncertainty too.

Efficiency of different models

Different models describe the indeterminate properties of the visual environment with various accuracy and in different ways.

Since the concept of probability has not physical interpretation, we must recognize that stochastic models describe these properties approximately. An adequate description can provide *interval* and *hyper-random models*.

This circumstance, however, does not mean that the stochastic model and other simple ones are useless. Not a complete correspondence the model to simulated object is important only for large sample sizes. Often the sample sizes are small. Then the description error of the real objects by stochastic and by other approximate models *is negligible*. Typically, these models are simpler than the interval and hyper-random models, and therefore in many cases *they are preferred*.

The necessity in more complex interval and hyper-random models arises when *gives evidence the restricted character of the phenomenon of statistical stability*. This usually there is in case of large observation intervals and large sample sizes.

Using scope of hyper-random models

The primary using scope of the hyper-random models is associated with the statistical processing of various physical processes (electrical, magnetic, electromagnetic, acoustic, hydroacoustic, seismic, meteorological, etc.) *of long duration*, as well as *high-precision measurements of physical quantities* and *forecasting physical processes on the basis of statistical processing of large data sets*.

Hyper-random models can also be used for simulation of physical events, variables, processes and fields, for which, due to the extreme smallness of the statistical data it is impossible to obtain well assessments of the parameters and characteristics, and it is possible only to point their borders.

The problem for formalization of physical concepts

Using non-stochastic models (in particular, interval and hyper-random models) exacerbates the underlying *problem of correct formalization of physical concepts defined now by using stochastic models*, for instance the concept of entropy.

The difficulty is that the probability has not physical interpretation, and therefore all physical concepts using the concept of probability, are actually uncertain. But this difficulty, as it turns out, can be overcome.

Mathematical analysis of divergent and many-valued functions

The theory of hyper-random phenomena touches a *little-studied field of mathematics concerning violations of convergence and many-valuedness*.

Modern mathematics is built on mathematical analysis deal with single valued sequences and functions which have single valued limits.

The development of the theory of hyper-random phenomena led to the formation of the *foundations of mathematical analysis of divergent and many-valued functions.* Limit concept is extended to the case of diverging (in usual sense) sequences and functions, and the concepts of convergence, continuity, differentiability, primitive, indefinite and definite integrals — to many-valued functions.

These questions are researched in the monograph.

The structure of the book

The monograph consists of five parts. The first part (Chapters 1-8) is devoted to discussion of phenomenon of statistical stability and developing of the methodology for researching of violations of statistical stability, in particular in case of limited amount of data. The second part (Chapters 9-13) contains a description of a set of experimental researches that examine violations of statistical stability of the various processes of different physical nature. The third part (Chapters 14-21) presents a shot description of mathematical foundations of the theory of hyper-random phenomena. The fourth part (Chapters 22-25) is devoted to the mathematical generalization of the results of the theory of hyper-random phenomena and formation of foundations of the mathematical analysis of divergent and many-valued functions. The fifth part (Chapters 26-33) contains theoretical and experimental researches of the statistical regularities in case of violation of statistical stability.

Summaries of the chapters

Chapter 1

The main manifestations of the phenomenon of statistical stability are examined, namely the statistical stability of the event's frequency and the sample averages. Mark that phenomenon of statistical stability has emergent property and it not inherent only to physical phenomena of stochastic type. The hypothesis of perfect (absolute) statistical stability, assuming the convergence of event frequencies and averages is discussed. The examples of statistically unstable processes are presented. The terms "identical statistical conditions" and "unpredictable statistical conditions" are discussed.

Chapter 2

The sixth Hilbert's problem concerning the axiomatization of physics is described. Generally recognized mathematical axiomatization principles of probability theory and mechanics are presented. New approach for solving the sixth problem based on the spreading the set of mathematical axioms by adding the physical adequacy hypotheses establishing a connection between the existing axiomatic mathematical theories and the real world is proposed. Fundamental concepts of probability theory and the theory of hyper-random phenomena are considered. The adequacy hypotheses for the probability theory and the theory of hyper-random phenomena are stated. Attention is drawn that the probability has not physical interpretation in the real world.

Chapter 3

Various conceptual views on the structure of the world from the standpoint of determinism and of uncertainty are examined. A classification of uncertainties is presented. The uniform method for presentation of the models using the distribution function is described. The classification of mathematical models is proposed.

Chapter 4

Random variables and stochastic processes, statistically unstable with respect to different statistics are examined. Various types of non-stationary processes are analyzed in respect to statistical stability.

Chapter 5

The notion of the statistical stability is formalized. The parameters of statistical instability are introduced. Measuring units of the statistical instability parameters are proposed. The concepts of the statistical stability/instability of the processes in narrow and wide senses are introduced. Statistical stability of the several models of the processes is researched.

Chapter 6

Dependence of statistical stability of the process from particularities of its temporal characteristics is researched (in particular the dependence from fluctuation parameters of expectation and correlation samples).

Chapter 7

Wiener — Khinchin transformation is examined. Attention is drawn that there are stochastic processes, which have not the correlation function typical for stationary process and the power spectral density together. Interaction between the statistical stability and power spectral density of the continues process is found. The statistical stability of the process, power spectral density of which is described by power function is investigated.

Chapter 8

Interaction between statistical stability and power spectral density of discrete process is found. The simulation results that confirm the correctness of formulas describing the dependence of the statistical instability parameters from the spectral power density of the process are presented.

Chapter 9

The results of experimental studies of the statistical stability of various physical processes are presented. It is researched the intrinsic noise of the amplifier, hydroacoustic noise of the ship, urban voltage oscillations, height and period of sea heaving, Earth's magnetic field variations, currency fluctuations. Attention is drawn to that in the small observation intervals the violations of statistical stability are not detected, but in the large observation intervals they become explicit.

Chapter 10

The results of experimental studies of statistical stability of air temperature and precipitation in Moscow and Kiev areas, as well as the wind speed in Chernobyl are presented. It is shown that all of these processes are statistically unstable. The degree of their instability is different. It is found that the temperature fluctuations much more unstable than the precipitation oscillations.

Chapter 11

The results of experimental studies on large observation intervals of the statistical stability of temperature and sound speed variations in the Pacific Ocean are presented. A statistical instability of these processes is found.

Chapter 12

The results of experimental studies on large observation intervals of the statistical stability of radiation of the astrophysical objects in the X-ray band are presented. All investigated fluctuations have been found statistically unstable. The most stable oscillation is the intensity of pulsar PSRJ 1012+5307. It is found that in the whole observation interval, its oscillations are statistically stable with respect to the average, but unstable with respect to the standard deviation.

Chapter 13

Different types of noise are researched, in particular, color noise, flicker noise, self-similar (fractal) one. The results of studies of statistical stability of various noises and processes are generalized. The causes of the violation of statistical stability are researched. It is found that statistically unstable processes can be formed in different ways: as a result of revenues from the outside in an open system of matter, energy and (or) information, as a result of nonlinear and even linear transformation.

Chapter 14

The notion of hyper-random event is introduced. To describe the hyper-random event the conditional probabilities and probability borders are used. The properties of these parameters are presented.

Chapter 15

The concept of hyper-random scalar variable is introduced. To describe it, the conditional distribution function (giving an exhaustive description of the hyper-random variable), the bounds of the distribution function, and their moments, as well as the bound of the moments are used. The properties of these characteristics and parameters are presented.

Chapter 16

The notion of hyper-random vector variable is introduced. Methods describing the scalar hyper-random variables are extended to the case of vector hyper-random variables. Properties of the characteristics and parameters of vector hyper-random variables are given.

Chapter 17

The notion of hyper-random scalar function is introduced. Various ways of its presentation are examined. To describe them, the conditional distribution functions (giving the most complete characterization of hyper-random function), the bounds of the distribution function, the density distribution of the borders, the moment borders, and borders of the moments are used.

Chapter 18

The bases of mathematical analysis of random functions are presented. The notions of the convergence of the sequence of the random variables and functions, the derivative and the integral of a random function are determined. It is introduced the concepts of the convergence of a sequence of hyper-random variables and functions, as well as of the concepts of the continuity, the differentiability and the integrability of hyper-random functions.

Chapter 19

Known for stochastic functions concepts such as stationarity and ergodicity are generalized to hyper-random functions. The spectral methods for describing of stationary hyper-random functions are regarded. The properties of stationary and ergodic hyper-random functions are presented.

Chapter 20

Different description methods of hyper-random variables and processes in a respect to appropriateness of their using in different types of transformations are analyzed. Relationships linking the characteristics and the parameters of transformed and primary hyper-random variables and processes are presented. Recommendations on using of different description ways of hyper-random variables in case of linear and nonlinear transformations, as well as of hyper-random processes in case of inertialess and inertial transformations are developed.

Chapter 21

The notion of hyper-random sample and its properties are formalized. The forming methodology of the assessments of the characteristics of the hyper-random variable is described. It is focus on that there is violation of the convergence of real assessments and that hyper-random models give adequate description of such assessments.

Chapter 22

The notion of a limit of a convergent numerical sequence is generalized to the case of divergent sequence and function. Unlike a usual limit necessarily receiving a single value, a generalized limit receives a set of values. For the divergent numerical sequence the concept of the spectrum of the limit points is introduced. The theorem on average sequence is proved.

Chapter 23

For description of divergent sequences and functions the approach based on the tool of distribution function is presented. The theorem concerned the spectrum of value frequency of sequence discharge is proved. The examples of description of the divergent functions are presented.

Chapter 24

Different variants for description of many-valued variables and functions are regarded. Using the mathematical tool developed in the theory of hyper-random phenomena, the notions of many-valued variable and many-valued function are formalized. The link between multiple meaning and the violation of the convergence is found. The concepts of the spectrum and the distribution functions of multi-valued variables and functions are introduced.

Chapter 25

For many-valued functions the concepts of the continuous function, the derivative, indefinite and definite integrals, as well as the spectrum of the principal values of the definite integral are introduced.

Chapter 26

It is established that the law of large numbers, known for the sequence of random variables, is valid both in the presence and absence of the convergence of the sample mean. In the absence of the convergence, the sample average tends to the average of expectations, fluctuating synchronously with it in a certain range. The law of large numbers is generalized to the case of the sequence of hyper-random variables. The particularities of the generalized law of large numbers are investigated.

Chapter 27

Peculiarities of the central limit theorem for a sequence of random variables in case of presence and absence of convergence of the sample mean to the fixed number are researched. The central limit theorem is generalized for a sequence of hyper-random quantities. The experimental results demonstrating the lack of convergence of sample means of real physical processes to fixed numbers are presented.

Chapter 28

Two concepts for evaluation the accuracy of the measurement are analyzed: the error concept and the uncertainty one. A number of measurement models are considered.

Chapter 29

The deterministic — hyper-random measurement model is studied. For the point hyper-random estimates the concepts of unbiased, consistent, effective, and sufficient estimates are introduced; for the interval hyper-random estimates the concepts of confidence interval and bounds of confidence probability are introduced. Theorems defining bounds of upper limits of accuracy of the point estimate and bounds of confidence interval of the interval estimate are proved. It is shown that hyper-random estimates of deterministic variables are not consistent and therefore the accuracy of any measurement is limited.

Chapter 30

The hyper-random — hyper-random measurement model is studied. The formulas describing the error of hyper-random estimate of hyper-random variable in general and particular cases are obtained. The relations that gives possibility to calculate the error of hyper-random estimate in case of indirect measurements of hyper-random variable are obtained.

Chapter 31

For point hyper-random estimates of hyper-random variables the concepts of unbiased, consistent, efficient and sufficient estimates are introduced. The theorems defining the upper limit of the accuracy of the point estimate and bounds of confidence interval of the interval estimate are proved. The fact well known from the practice that the accuracy of any actual physical measurement has a limit that can not be overcame even in case of very large data amount is explained.

Chapter 32

Different definitions of the entropy concept are analyzed. The concept of Shannon entropy for random variables is disseminated on uncertain variables that have no probability measure. The entropy concept for hyper-random and interval variables is introduced.

Chapter 33

Different ways of uncertainty formation are researched. It is found that uncertainty may arise as a result of a certain type of nonlinear transformation and in the process of the averaging the deterministic variables in the absence of convergence. It is explained why the interval, multiinterval, and hyper-random models can adequately reflect the world realities, and the random models are mathematical abstractions.

In **Appendix 1** quotes by famous scientists about the phenomenon of statistical stability are presented, in **Appendix 2** — a practical guidance for studies of statistical stability, and in **Appendix 3** — a brief history of the theory of hyper-random phenomena formation.

Conclusion

Hypothesis of absolute (ideal) statistical stability of the physical phenomena generated the classical theory of probability and mathematical statistics does not find experimental confirmation. Studies show that the *probability is the mathematical abstraction, which has not a physical interpretation. In the real world there is not absolute* *statistical stability.* However, *there is a limited statistical stability, manifested in the absence of convergence of statistics* (*their inconsistency*).

Looking for the adequate means for description of the real physical events, variables, processes and fields, taking into account the statistical violations of stability led to a new physical-mathematical theory of hyper-random phenomena, offering a new view on the world and new ways for its learning.

Theory of hyper-random phenomena does not cancel the achievements of the classical probability theory and mathematical statistics. It complements these achievements, extending the statements of these disciplines to the case, they are not considered: the lacking of the statistics convergence.

Limitations of statistical stability are manifested at large sample sizes and at the passage to the limit. Since the sample sizes are often small, stochastic models can provide a solution of many practical tasks with acceptable accuracy. Typically, these models are easier than hyper-random models and therefore for not very large sizes of the samples are preferred.

Hyper hyper-random models have clear advantages over stochastic and other relatively simple models in case when it is manifested the violation of phenomenon statistical stability – usually in large observation intervals and large sample sizes.

Therefore, the primary application area of hyper-random models is linked with statistical processing of various long-duration physical processes (electrical, magnetic, electromagnetic, acoustic, hydroacoustical, seismic, meteorological, etc.), as well as with high accuracy measurements of physical quantities and forecasting of physical processes on the basis of statistical processing of large data sets.

It is reasonable to use the hyper-random model for modeling of physical events, variables, processes, and fields, for which due to the extremely small volume of the statistical material it is impossible to obtain high quality estimates of the parameters and characteristics, and it is possible only to specify the boundaries in which these estimates are located.

The theory of hyper-random phenomena touches little-studied *mathematical* sphere concerning convergence violation and multivaluedness. Approaches developed in the monograph can be used for forming the *mathematical analysis of divergent and many-valued functions*. The scope of this new theory seems to be quite broad, going too far the statistics.

Limited character of statistical stability indicates the need to revise a number of statements of *physical disciplines*, in which the concepts of probability and convergence play a key role: it is primarily a *statistical mechanics, statistical physics and quantum mechanics*. Accounting violations of statistical stability may lead to new scientific results interesting for both theory and practice.

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Author's Information

Igor Gorban *– Principal scientist of the Institute of Mathematical Machines and Systems Problem, National Academy of Sciences of Ukraine, Glushkov ave., 42, Kiev, 03187, Ukraine; e-mail: igor.gorban@yahoo.com*

Major Fields of Scientific Research: Phenomenon of Statistical Stability, Mathematical statistics, Probability theory, Physical-mathematical theory of hyper-random phenomena, Space-time signal processing in complicated dynamic conditions, Fast multichannel space-time signal processing.