CONSTRUCTING AN OPTIMAL INVESTMENT PORTFOLIO BY USING FUZZY SETS THEORY

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Abstract: The problem of constructing an optimal portfolio of securities under uncertainty was considered.

The global market crisis of recent years has shown that the existing theory of optimization of investment portfolios and forecasting stock indices exhausted and the revision of the basic theory of portfolio management is strongly needed. Therefore the fuzzy sets theory was used for getting an optimal portfolio.

In this paper direct, dual and multicriteria problems with the use of triangular membership functions work were considered. The problem of portfolio optimization during the time period also was described in this article. In direct task we define structure of a portfolio, which will provide the maximum profitableness at the set risk level. In dual task we define structure of a portfolio, which will provide the minimum risk level at the set level of critical profitableness. In multicriteria problem we simultaneously maximize profitability and minimize risk level. The input data for the optimization system were predicted by using the Fuzzy Group Method of Data Handling (FGMDH). The optimal portfolios for assets were determined. The comparative analysis of optimal portfolios obtained by different methods and approaches was fulfilled.

***Keywords**: membership function, fuzzy sets theory, optimal portfolio, investments, stock securities, fuzzy number, FGMDH*

***ACM Classification Keywords**: G.1.0 Mathematics of Computing– General – Error analysis; G.1.6 Mathematics of Computing – Numerical Analysis – Optimization - Gradient methods, Least squares methods; I.2.3 Computing Methodologies - Artificial Intelligence - Uncertainty, "fuzzy", and probabilistic reasoning.*

Introduction

The problem of investment in securities arouse with appearance of the first stock markets. The main objective of portfolio investment is to improve the investment environment, giving securities such investment characteristics that are only possible in their combination. Careful processing and accounting of investment risks have become an integral and important part of the success of each company. However, more and more companies have to make decisions under uncertainty, which may lead to unintended consequences and, therefore, undesirable results. Particularly serious consequences may have the wrong decisions at long-term investments. Therefore, early detection, adequate and the most accurate assessment of risk is one of the biggest problems of modern investment analysis.

Historically, the first and the most common way to take account of uncertainty is the use of probability theory. The beginning of modern investment theory was in the article H. Markowitz, "Portfolio Selection", which was released in 1952. In this article mathematical model of optimal portfolio of securities was first proposed. Methods of constructing such portfolios under certain conditions are based on theoretical and probabilistic formalization of the concept profitability and risk. For many years the classical theory of Markowitz was the main theoretical tool for optimal investment portfolio construction, after which most of the novel theories were only modifications of the basic theory [Burenin, 1998].

However, the global market crisis of recent years has shown that the existing theory of investment portfolio optimization and forecasting stock indices exhausted itself and a revision of the basic theory of portfolio management is strongly needed.

New approach in the problem of investment portfolio construction under uncertainty is connected with fuzzy sets theory. Fuzzy sets theory was created about half a century ago in the fundamental work of Lotfi Zadeh [Zadeh, 1999]. This theory came into use in the economy in the late 70's. By using fuzzy numbers in the forecast parameters decision - making person was not required to form probability estimates.

The application of fuzzy sets technique enabled to create a novel theory of fuzzy portfolio optimization under uncertainty and risk deprived of drawbacks of classical portfolio theory by Markovitz. In this work we use fuzzy sets theory for getting an optimal investment portfolio. Firstly, portfolio optimization problem in this formulation was considered by O.A. Nedosekin [Nedosekin, 2002]. But in his work only direct problem was considered. The investigations were continued by Esfandiyarfard Maliheh. In [Zaychenko & Maliheh, 2008] the direct optimization problem using different membership functions was considered. However, in these studies, the investor can't determine an optimal portfolio during the time period. Therefore, in this study multiobjective optimization problem in which an investor can prefer risk or profitability using weights coefficients and solve portfolio optimization problem at the chosen time period is considered and analyzed.

Problem statement of portfolio optimization

The purpose of the analysis and optimization of an investment portfolio is research in area of portfolio optimization, and also the comparative analysis of structure of the effective portfolios received using the model Markovitz and fuzzy-set model of a share portfolio optimization.

Let us consider a share portfolio from N components and its expected behavior at time interval $[0, T]$. Each of a portfolio component $i = \overline{1, N}$ at the moment T is characterized by its financial profitableness r_i (evaluated at a point T as a relative increase in the price of the asset for the period) [Zaychenko, 2008].

The holder of a share portfolio – the private investor, the investment company, mutual fund – operates the investments, being guided by certain reasons. On the one hand, the investor tries to maximize the profitableness. On the other hand, it fixes maximum permissible risk of an inefficiency of the investments. We will assume the capital of the investor be equal 1. The problem of optimization of a share portfolio consists in a finding of a vector of share price distribution of papers in a portfolio $x = \{x_i\}, i = \overline{1, N}$ of the investor maximizing the income at the set risk level (obviously, that $\sum_{i=1}^{N} x_i = 1$).

In process of practical application of Markovitz model its lacks were found out:

- The hypothesis about normality profitableness distributions in practice does not prove to be true;
- Stationary of price processes also not always is confirmed in practice;
- At last, the risk of assets is considered as a dispersion (or standard deviation) of the prices of securities from expected value i.e. as decrease in profitableness of securities in relation to expected value, and profitableness increase in relation to an average are estimated absolutely the same.

Though for the proprietor of securities these events are absolutely different.

These weaknesses of Markovitz theory define necessity of use of essentially new approach of definition of an optimum investment portfolio.

Let review the main principles and idea of a method.

The risk of a portfolio is not its volatility, but possibility that expected profitableness of a portfolio will appear below some pre established planned value:

- Correlation of assets in a portfolio is not considered and not accounted;
- Profitableness of each asset is not random, but a fuzzy number. Similarly, restriction on extremely low level of profitableness can be both usual scalar and fuzzy number of any kind. Therefore optimize a portfolio in such statement may mean, in that specific case, the requirement to maximize expected profitableness of a portfolio in a point of time T at the fixed risk level of a portfolio;

– Profitableness of a security on termination of ownership term is expected to be equal r and is in a settlement range. For i -th security let’s denote:
  - $\bar{r}_i$ – Expected profitableness of *i-th* security;
  - r_{1i} – The lower border of profitableness of *i-th* security;
  - r_{2i} – The upper border of profitableness of *i-th* security;
  - $r_i = \left(r_{1i}, \bar{r}_i, r_{2i}\right)$ – Profitableness of *i-th* security is triangular fuzzy number.

Then profitableness of a portfolio:

$$r = (r_{\min} = \sum_{i=1}^{N} x_i r_{1i}; \bar{r} = \sum_{i=1}^{N} x_i \bar{r}_i; r_{\max} = \sum_{i=1}^{N} x_i r_{2i}) \tag{1}$$

where x_i is a weight of i -th asset in portfolio, and

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1, N} \tag{2}$$

Critical level of profitableness of a portfolio at the moment of T may be fuzzy triangular type number $r^* = \left(r_1^*; \bar{r}^*; r_2^*\right)$.

Direct portfolio optimization problem with triangular membership functions

To define structure of a portfolio which will provide the maximum profitableness at the set risk level, it is required to solve the following problem (3):

$$\{x_{opt}\} = \{x\} \mid r \to \max,\ \ \beta = const \tag{3}$$

where r is profitableness, β is a desired risk, vector's components x satisfy (2).

The most expected value risk degree of a portfolio is defined:

$$\beta = \begin{cases} 0, & if\ \ r^* < r_{\min} \\ R\left(1 + \dfrac{1-\alpha_1}{\alpha_1}\ln(1-a_1)\right), & if\ \ r_{\min} \leq r^* \leq \tilde{r} \\ 1-(1-R)\left(1 + \dfrac{1-\alpha_1}{\alpha_1}\ln(1-a_1)\right), & if\ \ \tilde{r} \leq r^* < r_{\max} \\ 1, & if\ \ r^* \geq r_{\max} \end{cases} \tag{4}$$

where

$$R = \begin{cases} \dfrac{r^* - r_{\min}}{r_{\max} - r_{\min}}, & if \quad r^* < r_{\max} \\ 1, \quad if \quad r^* \geq \mathrm{r}_{\max} \end{cases}$$

$$\alpha_1 = \begin{cases} 0, & if \quad r^* < r_{\min} \\ \dfrac{r^* - r_{\min}}{\tilde{r} - r_{\min}}, & if \quad r_{\min} \leq r^* < \tilde{r} \\ 1, & if \quad r^* = \tilde{r} \\ \dfrac{r_{\max} - r^*}{r_{\max} - \tilde{r}}, & if \quad \tilde{r} < r^* < r_{\max} \\ 0, & if \quad r^* \geq r_{\max} \end{cases} \tag{5}$$

Having recollected also, that profitableness of a portfolio is:

$$r = (r_{\min} = \sum_{i=1}^{N} x_i r_{1i};\ \tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i;\ r_{\min} = \sum x_i r_{2i}),$$

where $(r_{1i}, \overline{r_i}, r_{2i})$ is the profitableness of the i -th security, we obtain the following problem of optimization (6) - (8):

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max \tag{6}$$

$$\beta = const \tag{7}$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1, N} \tag{8}$$

At a risk level β variation 3 cases are possible. We'll consider in detail each of them.

1. $\beta = 0$

From (4) it is evident, that this case is possible when $r^* < \sum_{i=1}^{N} x_i r_{i1} \cdot$.

We obtain the following problem of linear programming:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max \tag{9}$$

$$\sum_{i=1}^{N} x_i r_{i1} > r^* \tag{10}$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1, N} \tag{11}$$

Found result of the problem solution (9) - (11) vector $x = \{x_i\},\ i = \overline{1, N}$ is a required structure of an optimum portfolio for the given risk level.

2. $\beta = 1$

From (4) it follows, that this case is possible when $r^* \geq \sum_{i=1}^{N} x_i r_{i2}$.

We obtain the following problem

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max, \quad \sum_{i=1}^{N} x_i r_{i2} \leq r^*,$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1, N}$$

Found result of the problem decision (9) - (11) vector $x = \{x_i\},\ i = \overline{1, N}$ is a required structure of an optimum portfolio for the given risk level.

3. $0 < \beta < 1$

From (4) it is evident, that this case is possible when $\sum_{i=1}^{N} x_i r_{i1} \leq r^* \leq \sum_{i=1}^{N} x_i \tilde{r}_i$, or when $\sum_{i=1}^{N} x_i \tilde{r}_i \leq r^* \leq \sum_{i=1}^{N} x_i r_{i2}$.

a) Let $\sum_{i=1}^{N} x_i r_{i1} \leq r^* \leq \sum_{i=1}^{N} x_i \tilde{r}_i$ .Then using (4) - (5) problem (6) - (8) is reduced to the following problem of nonlinear programming [Zaychenko, 2006]:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max,$$

$$\left(\left(r^* - \sum_{i=1}^{N} x_i r_{i1} \right) + \left(\sum_{i=1}^{N} x_i \tilde{r}_i - r^* \right) \cdot \ln \left(\frac{\sum_{i=1}^{N} x_i \tilde{r}_i - r^*}{\sum_{i=1}^{N} x_i \tilde{r}_i - \sum_{i=1}^{N} x_i r_{i1}} \right) \right) \cdot \tag{12}$$

$$\cdot \frac{1}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} = \beta, \tag{13}$$

$$\sum_{i=1}^{N} x_i r_{i1} \leq r^* \tag{14}$$

$$\sum_{i=1}^{N} x_i \tilde{r}_i > r^* \tag{15}$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \ge 0,\ i=\overline{1,N} \quad (16)$$

б) Let $\sum_{i=1}^{N} x_i \tilde{r}_i \le r^* \le \sum_{i=1}^{N} x_i r_{i2}$. Then the problem (6) - (8) is reduced to the following problem of nonlinear programming:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max \quad (17)$$

$$\left(\left(r^* - \sum_{i=1}^{N} x_i r_{i1} \right) - \left(r^* - \sum_{i=1}^{N} x_i \tilde{r}_i \right) \cdot \ln \left(\frac{r^* - \sum_{i=1}^{N} x_i \tilde{r}_i}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} \right) \right) \cdot \frac{1}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} = \beta \quad (18)$$

$$\sum_{i=1}^{N} x_i r_{i2} > r^* \quad (19)$$

$$\sum_{i=1}^{N} x_i \tilde{r}_i \le r^* \quad (20)$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \ge 0,\ i=\overline{1,N} \quad (21)$$

The R-algorithm of minimization of not differentiated functions is applied to the decision of problems (12) - (16) and (17) - (21). Let both problems: (12) - (16) and (17) - (21) solvable. Then to the structure of a required optimum portfolio will correspond a vector – $x = \{x_i\},\ i=\overline{1,N}$ the decision of that problem (12) - (16), (17) - (21) the criterion function value of which will be greater.

The dual optimization problem

It is necessary to determine the structure of the portfolio, which will provide a minimum level of risk for a given level of the portfolio profitability.

We obtain the following optimization problem:

$$\min \beta(x),\ \tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \ge r_{зад} = \mathrm{r}^*,$$
$$\sum_{i=1}^{N} x_i = 1,\ x_i \ge 0,\ i=\overline{1,N}, \quad (22)$$

where r and β is determined by the used membership function.

Consider optimization problem with the triangular MF.

It is necessary to solve optimization problem (22) where $\beta(x)$ is determined from the formula (4) and (5).

Multicriteria optimization problem

Now consider multicriteria fuzzy portfolio optimization problem in which portfolio profitableness should be maximized and risk should be minimized.

In order to find the structure of corresponding fuzzy portfolio the following problem is to be solved:

$$\{x_{opt}\} = \{x\} \quad | \quad r \to \max, \quad \beta \to \min, \tag{23}$$

where r and β are determined by formulas (4) and (5) and vector *X* components satisfy (8).

For simplifying problem solution transfer it to single criterion. Normalize the value of profitableness as follows:

$$\tilde{r}_H = \frac{r_{\max} - \tilde{r}}{r_{\max} - r_{\min}}, \quad \tilde{r}_H \in [0;1], \tag{24}$$

Using formulas (23) and (24), we obtain the optimization problem in such form:

$$\{w_1 \tilde{r}_H + w_2 \beta(x)\} \to \min$$
$$w_1 \geq 0,\ w_2 \geq 0,\ w_1 \neq w_2,\ w_1 + w_2 = 1$$
$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1, N}$$

Optimization problem during the time period

In this case we must define the structure of the portfolio, which provides the maximum average return for a given level of risk. So we calculate the profitableness from (3) as:

$$\tilde{r} = \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} x_{ti} \tilde{r}_{ti}, \tag{25}$$

where $\tilde{r}_{ti}$ - the expected profitableness of *i*-th security in time unit t. T - the length of time period. We should find an optimal portfolio from the (3), using (4), (5), (25).

Analysis of the results

The profitableness values of leading companies in the period from 01.12.2014 to 10.04.2015 were used as the input data. The companies: Google Inc (GOOGL), Walt Disney Co (DIS), The Coca Cola Co (KO), Kimberly Clark Corp (KMB), Seagate Technology PLC (STX), Tesla Motors Inc (TSLA). The corresponding data is presented in the Table 1.

Table 1. The profitableness

Companies / Dates	**GOOGL**	**DIS**	**KO**	**KMB**	**STX**	**TSLA**
05.12.2014	-0,0214	0,0114	-0,0229	-0,0161	-0,0192	-0,0418
12.12.2014	-0,0174	-0,0246	-0,0517	-0,0126	-0,0312	-0,0143
12.12.2014	0,0081	0,0219	0,0340	0,0311	0,0847	0,0389
26.12.2014	0,0173	0,0088	0,0144	0,0157	-0,0067	-0,0284
02.01.2015	-0,0144	-0,0183	-0,0168	-0,0161	-0,0235	-0,0163
09.01.2015	-0,0361	0,0202	0,0211	0,0091	0,0185	-0,0452
16.01.2015	0,0270	0,0076	-0,0026	0,0163	-0,0207	0,0488
23.01.2015	0,0628	-0,0002	0,0035	-0,0500	0,0136	-0,0143
30.01.2015	0,0015	-0,0422	-0,0426	-0,0172	-0,0444	0,0304
06.02.2015	0,0032	0,1098	-0,0034	-0,0096	0,0370	-0,0630
13.02.2015	0,0413	0,0240	0,0184	0,0257	0,0238	0,0624
20.02.2015	-0,0059	0,0041	0,0038	-0,0028	0,0168	-0,0173
27.02.2015	0,0516	-0,0087	0,0346	-0,0120	-0,0140	-0,0186
06.03.2015	-0,0037	-0,0195	-0,0389	-0,0325	-0,0757	-0,0174
13.03.2015	-0,0368	0,0112	-0,0358	-0,0115	-0,0565	-0,0115
20.03.2015	0,0059	0,0099	0,0089	0,0193	0,0305	0,0124
27.03.2015	**-0,0138**	**-0,0253**	**-0,0133**	**-0,0240**	**-0,0553**	**-0,0733**
02.04.2015	**-0,0353**	**-0,0011**	**0,0042**	**-0,0067**	**-0,0119**	**0,0023**
10.04.2015	**0,0084**	**0,0125**	**-0,0070**	**-0,0056**	**0,0538**	**0,0384**

For getting an input data for the optimization system we used the Fuzzy GMDH method with triangular membership functions, linear partial descriptions, training sample of 70% size. The profitableness values were forecasted for each of 3 weeks (Table 2).

Table 2. Forecasted profitableness

Companies	Profitableness				MAPE test sample	MSE test sample
	Real value	Low bound	Forecasted value	Upper bound		
27.03.2015						
GOOGL	-0,0138	-0,0408	-0,0116	0,0176	1,5116	0,0265
DIS	-0,0253	-0,0130	0,0087	0,0304	2,0415	0,0412
KO	-0,0133	-0,0629	-0,0160	0,0309	1,0619	0,0131
KMB	-0,0240	-0,1520	-0,0303	0,0914	2,3122	0,0136
STX	-0,0553	-0,0618	-0,0344	-0,0070	2,5215	0,0462
TSLA	-0,0733	-0,1919	-0,0391	0,1137	1,459	0,0078
02.04.2015						
GOOGL	-0,0353	-0,0672	-0,0328	0,0016	2,0537	0,0226
DIS	-0,0011	0,0082	0,0299	0,0516	1,7141	0,0371
KO	0,0042	-0,0418	0,0021	0,0460	1,9743	0,0159
KMB	-0,0067	-0,1342	-0,0125	0,1092	1,7452	0,0245
STX	-0,0119	-0,0164	0,0090	0,0344	1,8243	0,0172
TSLA	0,0023	-0,0999	0,0349	0,1697	2,101	0,0241

10.04.2015						
GOOGL	0,0084	-0,0215	0,0109	0,0433	1,439	0,0162
DIS	0,0125	0,0308	0,0425	0,0542	3,103	0,0215
KO	-0,007	-0,0506	-0,0087	0,0332	1,037	0,0107
KMB	-0,0056	-0,1136	-0,0119	0,0898	2,014	0,0135
STX	0,0538	0,0483	0,0747	0,1011	2,855	0,0178
TSLA	0,0384	0,0362	0,069	0,1018	2,014	0,0194

In this way the portfolio optimization system stops to be dependent on factor of expert subjectivity. Besides, we can get data for this method automatically, without expert's estimates.

Let the critical profitableness level set by 7,5 %. Varying the risk level we obtain the following results for triangular MF (10.04.2015). The results are presented in the Tables 3, 4 and Figure 1.

Table 3. Direct problem - distribution of components of the optimal portfolio with critical level r *=7,5 %

GOOGL	DIS	KO	KMB	STX	TSLA
0,00065	0,00025	0,00137	0,00423	0,99017	0,00333
0,00159	0,00157	0,00066	0,00323	0,9923	0,00065
0,00435	0,00307	0,00058	0,00271	0,98533	0,00396
0,00018	0,00008	0,00082	0,00291	0,99363	0,00238
0,0049	0,00394	0,00071	0,00244	0,97964	0,00837
0,00287	0,00525	0,00296	0,00506	0,97678	0,00708
0,0033	0,00187	0,00279	0,00301	0,9707	0,01833

0,00188	0,00232	0,00153	0,0125	0,97369	0,00808
0,00439	0,00297	0,00238	0,0152	0,94177	0,03329
0,00314	0,00317	0,0036	0,02553	0,93466	0,0299
0,00339	0,00579	0,00405	0,03727	0,93124	0,01826

Table 4. Direct problem - parameters of the optimal portfolio with critical level r *=7,5 %

Low bound	**Expected profitableness**	**Upper bound**	**Risk level**
4,732	**7,412**	**10,093**	**0,25**
4,756	7,421	10,085	0,3
4,74	7,402	10,064	0,35
4,77	7,435	10,1	0,4
4,732	7,394	10,056	0,45
4,681	7,362	10,043	0,5
4,705	7,383	10,061	0,55
4,586	7,325	10,064	0,6
4,484	7,262	10,04	0,65
4,317	7,172	10,026	0,7
4,131	7,063	9,995	0,75

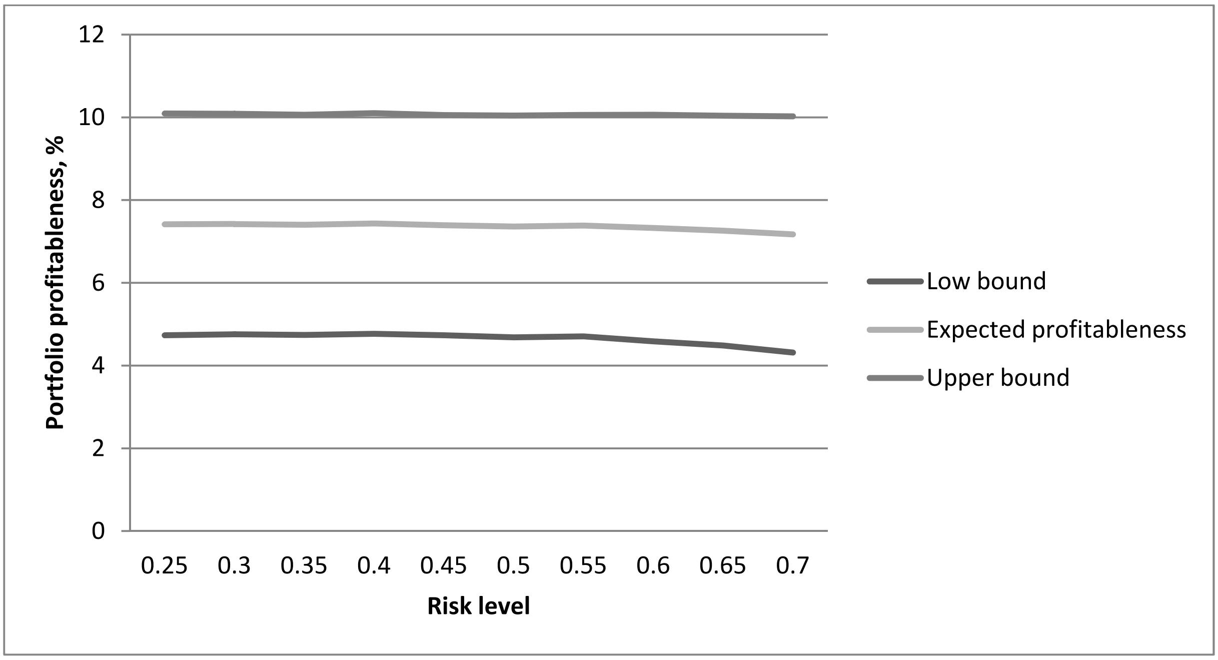

Figure 1. Dependence of expected portfolio profitableness on risk level

As we can see on Figure 1 the dependence profitableness - risk has a descending type, the greater risk the lesser is profitableness opposite to classical probabilistic methods. It may be explained so that at fuzzy approach by risk is meant the situation when the expected profitableness happens to be less than the given criteria level. When the expected profitableness decreases, the risk grows.

The profitableness of the real portfolio is 5,34 %.This value falls in results calculated corridor of profitableness [4,732; 7,412; 10,093], indicating the high quality of the forecast. The main portfolio portion in this case goes to company Seagate Technology PLC that can be explained by the high level of its profitableness in comparison with other companies.

Now consider in Tables 5, 6 and Figure 2, the results obtained by solving multicriteria problem.

Table 5. Multicriteria problem - distribution of components of the optimal portfolio with critical level r *=7,5 %

GOOGL	DIS	KO	KMB	STX	TSLA
0,01069	0,02289	0,00316	0,00117	0,92946	0,03263
0,0121	0,02234	0,0058	0,00414	0,92512	0,0305
0,01265	0,02131	0,00733	0,00596	0,92451	0,02824

0,01418	0,02124	0,00982	0,00871	0,91908	0,02697
0,01478	0,02049	0,01124	0,01038	0,918	0,02511
0,0154	0,01989	0,01265	0,01199	0,91656	0,02351
0,01603	0,01929	0,01402	0,01355	0,91518	0,02193
0,01668	0,0188	0,01537	0,01505	0,91356	0,02054
0,01733	0,01838	0,01669	0,01655	0,91182	0,01923
0,01069	0,02289	0,00316	0,00117	0,92946	0,03263
0,0121	0,02234	0,0058	0,00414	0,92512	0,0305

Table 6. Multicriteria problem - parameters of the optimal portfolio with critical level r *=7,5 %

Low bound	Expected profitableness	Upper bound	Risk level	w1
4,626	**7,273**	**9,92**	**0,65253**	**0,1**
4,545	7,219	9,893	0,67539	0,2
4,501	7,192	9,882	0,69878	0,3
4,423	7,138	9,854	0,71007	0,4
4,381	7,112	9,842	0,72732	0,5
4,34	7,085	9,83	0,74142	0,6
4,3	7,059	9,818	0,75483	0,7
4,26	7,033	9,806	0,76619	0,8
4,221	7,007	9,793	0,77641	0,9

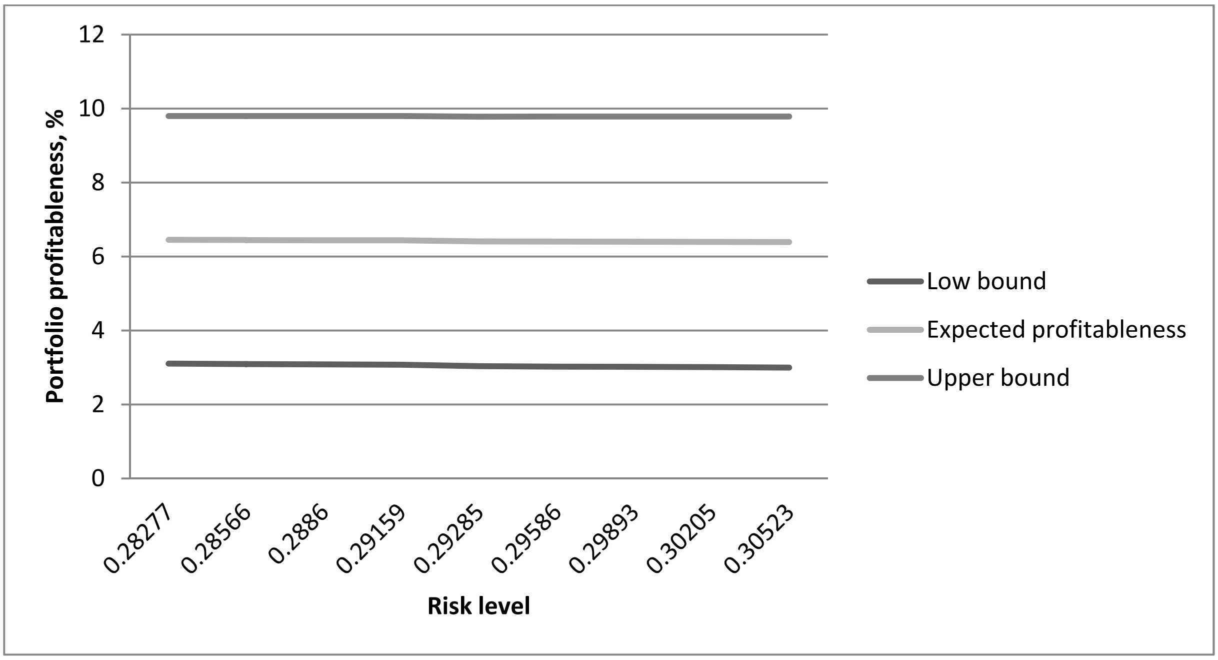

Figure 2. Dependence of expected portfolio profitableness on risk level

The profitableness of the real portfolio is 5,16 %. This value falls in results calculated corridor of profitableness [4,626; 7,273; 9,92]. The dependence profitableness - risk also has descending type.

Let’s consider an optimization portfolio at the time interval of 3 weeks. We have obtained profitableness values for all weeks by using the Fuzzy GMDH [Zaychenko, 2000] (Tables 7, 8 and Figure 3).

Table 7. Optimization problem during the period - distribution of components of the optimal portfolio for triangular MF with critical level r *=7,5 %

GOOGL	DIS	KO	KMB	STX	TSLA
0,01964	0,92474	0,02535	0,00102	0,02878	0,00047
0,01965	**0,92309**	**0,02536**	**0,00287**	**0,02876**	**0,00027**
0,02066	0,92203	0,02637	0,00066	0,02975	0,00053
0,01968	0,92274	0,02538	0,00069	0,02873	0,00278
0,01969	0,92329	0,02539	0,00206	0,02871	0,00086

0,0197	0,92068	0,0254	0,00208	0,02869	0,00345
0,01972	0,92355	0,02541	0,00231	0,02867	0,00034
0,01973	0,9228	0,02542	0,00223	0,02865	0,00117
0,02075	0,9212	0,02643	0,00124	0,02962	0,00076
0,01976	0,92152	0,02544	0,00205	0,0286	0,00263

Table 8. Optimization problem during the period - parameters of the optimal portfolio for triangular MF with critical level r *=7,5 %

Low bound	Expected profitableness	Upper bound	Risk level
4,732	7,412	10,093	0,25
4,756	**7,421**	**10,085**	**0,3**
4,74	7,402	10,064	0,35
4,77	7,435	10,1	0,4
4,732	7,394	10,056	0,45
4,681	7,362	10,043	0,5
4,705	7,383	10,061	0,55
4,586	7,325	10,064	0,6
4,484	7,262	10,04	0,65
4,317	7,172	10,026	0,7

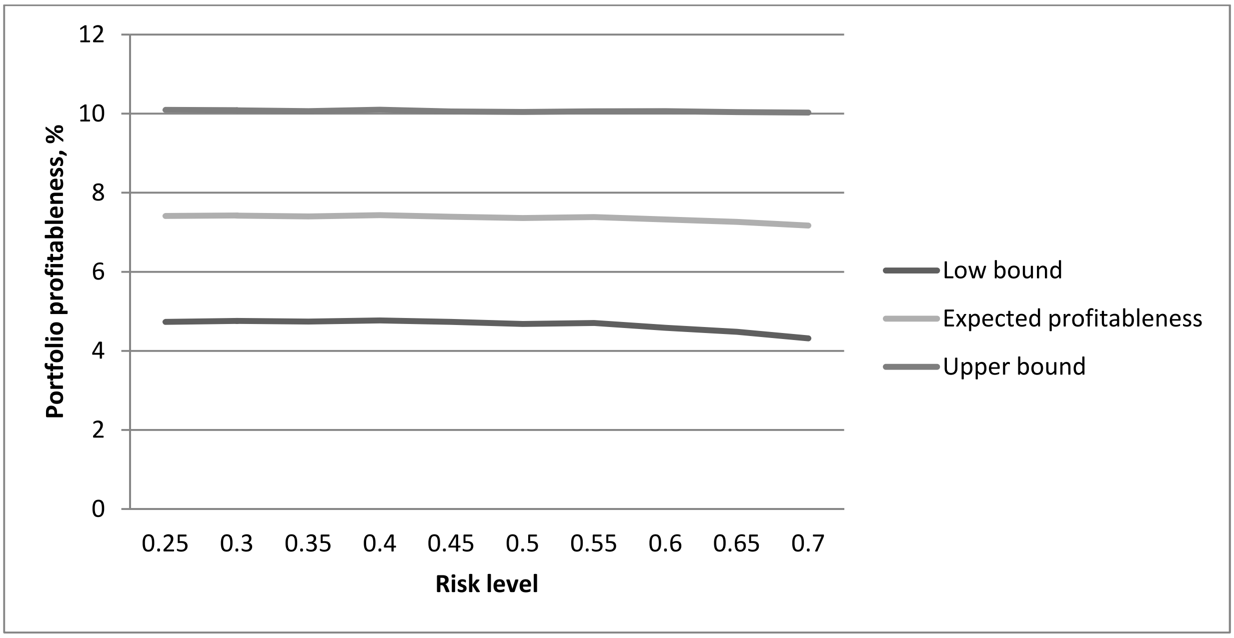

Figure 3. Dependence of expected portfolio profitableness on risk level

Let’s consider the results obtained by solving the dual problem using triangular MF. In this case, the investor sets the rate of return, and the problem is to minimize the risk. The main portfolio portion in this case goes to company Walt Disney Co that can be explained by the high level of its average profitableness in comparison with other companies.

The optimal portfolio is listed in Tables 9, 10 and the dependence of the risk level on a given critical return is presented in the Figure 4.

Table 9. Dual problem. Distribution of components of the optimal portfolio

GOOGL	DIS	KO	KMB	STX	TSLA
0,01756	0,01918	0,01664	0,01503	0,91202	0,01956
0,01752	0,02002	0,01609	0,01389	0,91166	0,02082
0,01666	0,02021	0,01459	0,01194	0,91494	0,02166
0,01606	0,02093	0,01318	0,01032	0,91606	0,02345
0,01404	0,02119	0,00973	0,00691	0,92231	0,02582
0,00768	0,02313	0,00185	0,00015	0,93191	0,03528
0,00951	0,02323	0,0009	0,0006	0,93027	0,0355

Table 10. Dual problem. Parameters of the optimal portfolio

Low bound	Expected profitableness	Upper bound	Risk level	Critical rate of return
0,07016	0,04242	0,0979	0,01821	5
0,07027	0,04263	0,09791	0,05381	5,5
0,07061	0,04315	0,09808	0,11501	6
0,07088	0,04356	0,0982	0,21157	6,5
0,07157	0,04456	0,09857	0,37038	7
0,0731	0,04673	0,09947	0,6253	7,5
0,07302	0,04661	0,09942	0,87555	8

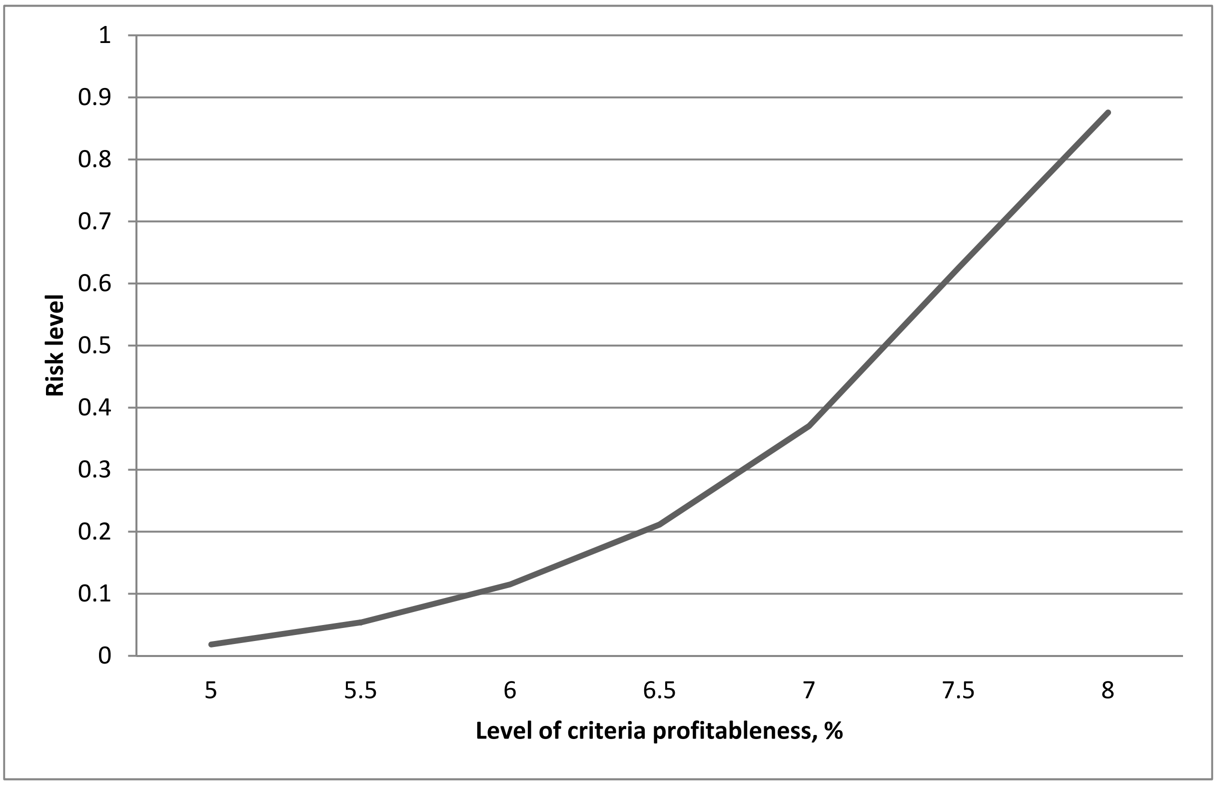

Figure 4. Dependence of the risk level on a given critical return

From these results we can see that the dependence risk - given critical level of profitability takes a growing character, because at the growth of the critical profitability the probability that the expected return will be lower than a given critical value also increases.

Conclusion

In this work the research in the field of portfolio management was carried out. Fuzzy sets theory was used as a tool for getting an optimal portfolio. As a result of research the mathematical model based on the fuzzy-set approach for a finding of structure of the optimal investment portfolio has been obtained. On the basis of the theory of fuzzy sets the algorithm of optimization of a share portfolio has been developed. As a result of research the following conclusions were made:

- The dependence profitableness - risk has a descending type, the greater risk the lesser is profitableness opposite from classical probabilistic methods. It may be explained so that at fuzzy approach by risk is meant the situation when the expected profitableness happens to be less than the given criteria level. When the expected profitableness decreases, the risk grows;
- Portfolios during the time period and at the end of period have different structure and characteristics, that can be explained by the variations of average profitableness;
- For improving the accuracy of the suggested fuzzy portfolio model, the fuzzy GMDH method was applied for profitableness forecasting. The experimental investigations have proved its high efficiency.
- The dependence risk - given critical level of profitability has a growing character in the dual task, because with the growth of the critical profitability increases the probability that the expected return will be lower than a given critical value.

Thus, we have developed a system that not only automates the search of the optimal portfolio, but also provides a flexible and effective management of portfolio investments.

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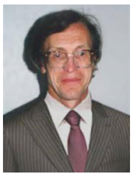

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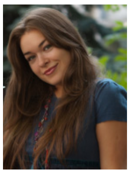

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