ON PROBLEM OF ADEQUACY OF MULTISET MATHEMATICAL MODELS

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***Abstract**: The analysis of adequacy of competitiveness assessments based only on the values of multiplicity function of multiset is made. Such assessments are not adequate according to representational theory of measurement. An example of adequate assessments based on fuzzy multisets is given.*

***Keywords**: adequate assessment, competitiveness, representational theory of measurement, multiset.*

***ACM Classification Keywords**: F.4.1 Mathematical Logic, G.2.1 Combinatorics, I.2.4 Knowledge Representation Formalisms and Methods*

Introduction

Nowadays mathematical models based on multisets are often used for solving the practical problems [Syropoulos, 2012; Buy & Bogatyreva, 2010; Tarasov, 2008; Singh et al, 2007].

A set which can include the repeating elements is called as a (crisp) multiset or a bag. Currently, a multiset and a bag are being used interchangeably. Formalization of a multiset is reduced to definition of its multiplicity function.

Further the concept of a multiset was repeatedly generalized for the purpose of formalization of the description of certain problems. Fuzzy multisets (fuzzy bags) have been introduced in [Yager, 1986]. A concept of "Multi - Fuzzy Set" is entered in [Syropoulos, 2001]. Syropoulos also formulates L-fuzzy hybrid sets on fuzzyfying the objects of a hybrid multiset [Syropoulos, 2005]. A level fuzzy multiset is determined in [Tarasov, 2008]. There are generalizations such as rough fuzzy sets and fuzzy rough sets [Dubois & Prade, 1990], fuzzy soft sets [Maji et al, 2001].

One of important practical applications of the theory of multisets is connected with a problem of ordering of objects on the basis of a set of expert assessments of qualitative and quantitative characteristics of objects. One of the methods for solving such problem was stated in [Petrovsky, 2003]. For problem of ordering of such objects, the metric like Hamming in space of multisets is used, and objects are ordered by the value of distance in relation to the best (ideal) object. The offered method was much simpler in comparison with other well-known approaches for solving such problem of ordering of objects (for example, the method on the basis of Kemeni-Smell's median [Kemeni & Snell, 1962]). Since such approach started being used for solving many problems [Vovk & Gaidukova, 2008; Petrovsky et al, 2010; Demidova & Sokolova, 2014]. However, as shown below, in a number of enough simple practical situations this method leads to contradictory results. There is a natural question of adequacy of the offered method. The review of references showed that before now all studies didn't investigate the problem of adequacy of multiset models.

Currently, two approaches to the study of the adequacy of mathematical models are mainly used.

The first approach concerns the adequacy of the model determined by such concepts as verification and validation of the model [Thacker, 2004]. Adequacy of model is the decision that the model fidelity is sufficient for the intended use. The model fidelity is the difference between simulation and experimental outcomes. Model verification and validation is an enabling methodology for the development of computational models.

Verification is the process of determining that a model implementation accurately represents the developer’s conceptual description of the model and its solution. Validation is the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model.

In other word, adequacy of the model is the concordance of the model and the simulated objects or processes on the properties of the model, which are considered essential for the study. The expected outcome of the model verification and validation process is the quantified level of agreement between experimental data and model prediction, as well as the predictive accuracy of the model.

The second approach to a problem of adequacy of mathematical model develops within the representational theory of measurement. This theory was developed to formulate and solve problems of measurement for the nonphysical sciences, in which measurement has always been problematic. The representational theory of measurement explains why some attributes of objects, substances, and events reasonably be represented numerically [Luce et al, 2007]. An empirical system with relations, a numerical system with relations (in more general case, a mathematical system with relations), and also a concept of measurement scale are basic concepts of the representational theory of measurement. A mapping of empirical system with relations to numerical system with relations preserving the essential relations of empirical system is called as a scale of measurement.

In this paper we examine the adequacy of multiset model of assessments of competitiveness of enterprises [Vovk & Gaidukova, 2008]. On the base of representational theory of measurement we prove that such assessments are not adequate. Some results of research of the problem of assessment adequacy are given.

Multiset model of competitiveness assessments of enterprises

Let $\mathbf{A}=\{A_l\}_{l=1}^{p}$ be a set of enterprises, $\mathbf{Q}=\{Q_s\}_{s=1}^{m}$ be a set of criteria of assessments of competitiveness of enterprises. Consider that each criterion $Q_s=\left(q_s^{e_s}\right)_{e_{s=1}}^{h_s}$ is measured in the quantitative or qualitative scale [Luce et al, 2007] with strict ordered set of values $q_s^1>q_s^2>\ldots>q_s^{e_s}>\ldots>q_s^{h_s}$. Let n be the number of experts who gave the assessment to the enterprises for each of these criteria. Let a_l be the result of competitiveness assessment of the enterprise A_l . This result is based on the expert assessments of values of the set of criteria $\mathbf{Q}$, which is possible to represent as the multiset over domain $G=\{Q_1,\ldots,Q_s,\ldots,Q_m\}$:

$$a_l=\Big\{k_{a_l}\left(q_1^1\right)\circ q_1^1,\ldots,k_{a_l}\left(q_1^{h_1}\right)\circ q_1^{h_1},\ldots,k_{a_l}\left(q_s^1\right)\circ q_s^1,\ldots,k_{a_l}\left(q_s^{e_s}\right)\circ q_s^{e_s},\ldots,k_{a_l}\left(q_s^{h_s}\right)\circ q_s^{h_s},\ldots,$$
$$k_{a_l}\left(q_m^1\right)\circ q_m^1,\ldots,k_{a_l}\left(q_m^{h_m}\right)\circ q_m^{h_m}\Big\}, \tag{1}$$

where $k_{a_l}\left(q_s^{e_s}\right)\circ q_s^{e_s}$ is the ordered pair, $k_{a_l}\left(q_s^{e_s}\right)\in\{0,1,\ldots,n\}$ and $k_{a_l}\left(q_s^{e_s}\right)$ is the number of experts who gave the assessment $q_s^{e_s}$ for criterion Q_s to the enterprise A_l (or the value of multiplicity function [Petrovsky, 2003]).

Let $a_{\max}=\{n\circ q_1^1,\ldots,0\circ q_1^{h_1},\ldots,n\circ q_m^1,\ldots,0\circ q_m^{h_m}\}$ and $a_{\min}=\{0\circ q_1^1,\ldots,n\circ q_1^{h_1},\ldots,0\circ q_m^1,\ldots,n\circ q_m^{h_m}\}$ denote, respectively, the maximum and the minimum assessments. In the paper [Vovk & Gaidukova, 2008], an order relation on the set of assessments of competitiveness of enterprises $\mathbf{A}$ is determined on the basis of expert assessments of criteria $\mathbf{Q}$ as follows.

The metric like Hamming is determined on the set of assessments $\{a_l\}_{l=1}^{p}$ [Petrovsky, 2003]:

$$d\left(a_i,a_j\right)=\sum\nolimits_{s=1}^{m}\omega_s\sum\nolimits_{e_{s=1}}^{h_s}\left|k_{a_i}\left(q_s^{e_s}\right)-k_{a_j}\left(q_s^{e_s}\right)\right|\text{, for all } i,j\in\{1,\ldots,p\}, \tag{2}$$

where ω_s is the coefficient of the relative importance of criterion Q_s (i.e., the expression (1) is regarded as a crisp multiset). The enterprise A_i is better than the enterprise A_j ($A_i\succ A_j$), if $d\left(a_{\max},a_i\right)<d\left(a_{\max},a_j\right)$:

$$d\left(a_{\max}, a_i\right) < d\left(a_{\max}, a_j\right) \Rightarrow A_i \succ A_j . \tag{3}$$

If the equation $d\left(a_{\max}, a_i\right) = d\left(a_{\max}, a_j\right)$ holds, then the enterprises are equivalent $A_i \sim A_j$ ($a_i = a_j$) or non-comparable for competitiveness ($a_i \neq a_j$). The following condition is satisfied for each of criterion Q_s :

$$\sum_{e_s=1}^{h_s} k_{a_l}\left(q_s^{e_s}\right) = n , \quad l \in \{1,\ldots,p\} . \tag{4}$$

Therefore, $\sum_{e_s=2}^{h_s} k_{a_l}\left(q_s^{e_s}\right) = n - k_{a_l}\left(q_s^1\right)$. Then we obtain,

$$d\left(a_{\max}, a_l\right) = 2\sum_{s=1}^{m} \omega_s \left(n - k_{a_l}\left(q_s^1\right)\right). \tag{5}$$

From (3) – (5), the implication

$$\sum_{s=1}^{m} \omega_s k_{a_i}\left(q_s^1\right) > \sum_{s=1}^{m} \omega_s k_{a_j}\left(q_s^1\right) \Rightarrow A_i \succ A_j \tag{6}$$

follows. Thus, the ordering of enterprises is performed only on the base of the multiplicity of the best values of assessments for each of criteria.

For the first time, the implication (6) was published, possibly, in [Petrovsky, 2003, (9.23)], where the equality case $\sum_{s=1}^{m} \omega_s k_{a_i}\left(q_s^1\right) = \sum_{s=1}^{m} \omega_s k_{a_j}\left(q_s^1\right)$ is considered in more detail. In this case, it is proposed to continue the ordering on the basis of the implication $\sum_{s=1}^{m} \omega_s k_{a_i}\left(q_s^2\right) > \sum_{s=1}^{m} \omega_s k_{a_j}\left(q_s^2\right) \Rightarrow A_i \succ A_j$. The implication $\sum_{s=1}^{m} \omega_s k_{a_i}\left(q_s^3\right) > \sum_{s=1}^{m} \omega_s k_{a_j}\left(q_s^3\right) \Rightarrow A_i \succ A_j$ is used in the case $\sum_{s=1}^{m} \omega_s k_{a_i}\left(q_s^2\right) = \sum_{s=1}^{m} \omega_s k_{a_j}\left(q_s^2\right)$ etc.

Consider two simple examples when only one criterion Q_s ($m = 1$) is used for comparison of the enterprises, and the values of criterion are measured in absolute scale in which the only identical transformations are allowed [Luce et al, 2007]. Let $h_s = 5,\ q_s^1 = 5,\ q_s^2 = 4,\ q_s^3 = 3,\ q_s^4 = 2,\ q_s^5 = 1$, and $n = 20$.

Example 1. Let the expert assessments of the values of criterion Q_s to the enterprises A_1 and A_2 are equal $a_1 = \{2 \circ 5,\ 18 \circ 4,\ 0 \circ 3,\ 0 \circ 2,\ 0 \circ 1\}$, and $a_2 = \{2 \circ 5,\ 0 \circ 4,\ 0 \circ 3,\ 0 \circ 2,\ 18 \circ 1\}$, respectively.

It is obvious that the most of experts (18) are unanimous in an assessment: the enterprise A_1 is better than the enterprise A_2 (Figure 1).

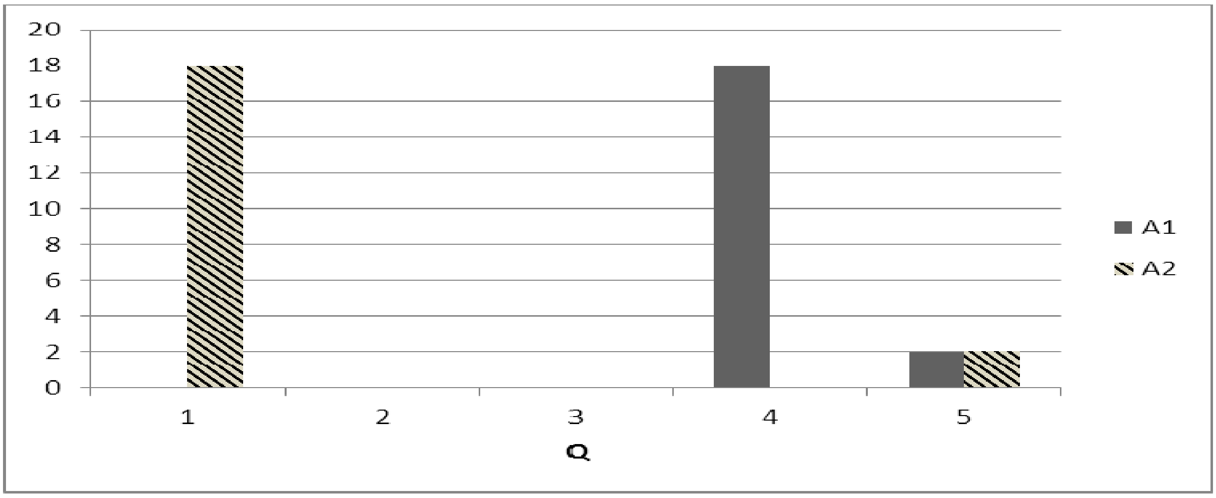

Figure 1. Results of expert assessments

However, from calculations on the above-stated formulas the equality $d(a_{\max}, a_1) = d(a_{\max}, a_2)$ follows, i.e., these enterprises are non-comparable for competitiveness ($a_1 \neq a_2$).

For this example, we get the right decision ($A_1 \succ A_2$) using the approach proposed in [Petrovsky, 2003].

Example 2. Let two of 20 experts have put the highest assessment for the enterprise A_1 $k_{a_1}(q_s^1) = 2$, and the other have put the lowest one, and for the enterprise A_2: $k_{a_2}(q_s^1) = 1$, $k_{a_2}(q_s^2) = 19$. Then, from (6) we conclude that $A_1 \succ A_2$. However, the majority of experts consider $A_2 \succ A_1$ (Figure 2), i.e., when comparing the competitiveness of enterprises on the basis of the implication (6), the opinion of one expert ($k_{a_1}(q_s^1) = 2$, $k_{a_2}(q_s^1) = 1$) has a decisive influence. This fact can't be regarded as the satisfactory result.

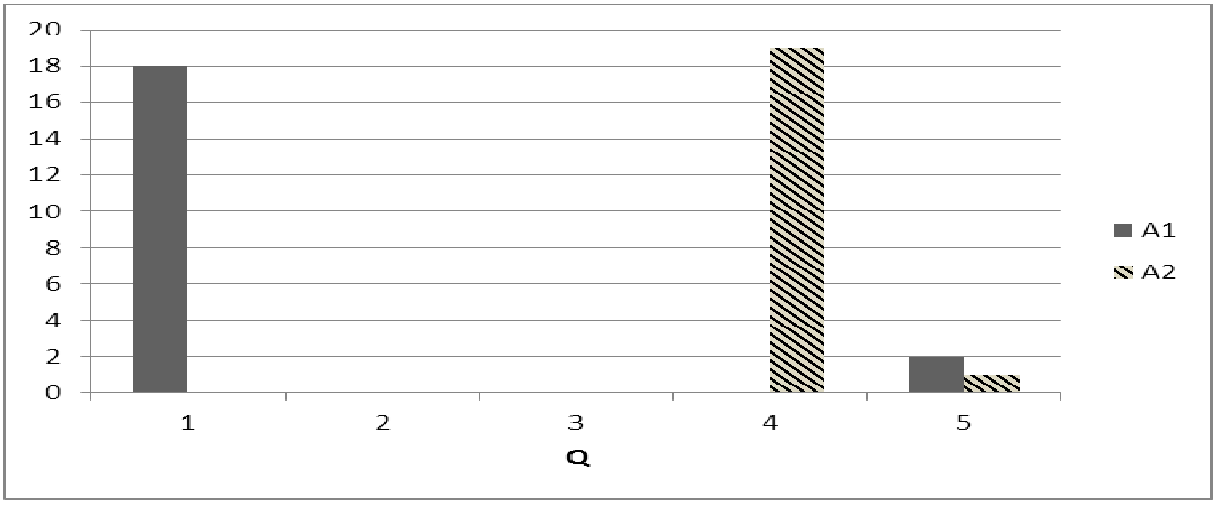

Figure 2. Results of expert assessments

Analysis of reasons of contradictions

1. Let $\mathbf{K} = \{K_j\}$ be the set of all finite (crisp) multisets over domain $G = \{Q_1,...,Q_m\}$, k_j be the multiplicity function of multiset K_j, $k_j : G \to N_n$, $N_n = \{0,1,...,n\}$. It is not difficult to prove that $(\mathbf{K}, \succeq)$ is the lattice, where $\succeq$ is a partial order of multiplicity functions, $\varnothing$ is the bottom element, $k_{\max} : G \to n$ and $k_{\max}$ is the top element of the lattice [Buy & Bogatyreva, 2010]. Then it is evidently that $k_i \succ k_j \Rightarrow |K_i| > |K_j|$ holds, where $\succ$ is the strict order relation, $|K_j|$ is the cardinality of multiset K_j:

$$|K_j| = \sum_{s=1}^{m} \sum_{e_s=1}^{h_s} k_j\left(q_s^{e_s}\right). \quad (7)$$

Theorem 1. A set of expert assessments $\{a_l\}_{l=1}^{p}$ is an anti-chain in the lattice $\mathbf{K}$.

Proof. Let $a, b \in \mathbf{K}$ be the assessments (1) of the enterprises A, B, respectively, and $a \neq b$. Suppose, that $a \succ b$ holds, then $a \succ b \Leftrightarrow k_a \succ k_b$ holds, and $k_a \succ k_b \Rightarrow |a| > |b|$. But according to (4) and (7) the equality $|a| = |b|$ holds for any assessments (1), i.e., $a \succ b$ is false. In other words, $\{a_l\}_{l=1}^{p}$ is an anti-chain in the lattice $\mathbf{K}$.

Thus, the order relation forming by metric (2) does not connect with the order relation in the lattice $\mathbf{K}$.

2. Let the assessments of competitiveness of enterprises, used in (3) and (6), are considered as a mapping $\varphi : \mathbf{A} \to R^1$, where R^1 is the set of real numbers. By $\varphi_1(A)$ we denote $d(a_{\max}, a)$, and by $\varphi_2(A)$ we denote $\sum_{s=1}^{m} \omega_s k_a(q_s^1)$. Then the implications (3) and (6) can be written, respectively, as follows:

$$\varphi_1(A) < \varphi_1(B) \Rightarrow A \succ B, \tag{8}$$

$$\varphi_2(A) > \varphi_2(B) \Rightarrow A \succ B. \tag{9}$$

There is a question, is it possible to use the assessments φ_1 and φ_2, and the implications (3) and (6) to compare the enterprises for competitiveness? In other words, what a necessary condition has to be carried out that a mapping $\varphi : \mathbf{A} \to R^1$ can be considered as the assessment of competitiveness of enterprises? Since any mapping φ, which to unequal assessments of competitiveness criteria of enterprises puts unequal numerical values, induces an order relation on the set $\mathbf{A}$. Such a necessary condition can be obtained by considering the competitiveness assessments on the base of representational theory of measurement: a mapping $\varphi : \mathbf{A} \to R^1$ must meet the definition of measurement scale. A measurement scale is a group of homomorphic mappings preserving relations in the empirical system [Luce et al, 2007].

Further, we show that the mapping φ_1 (φ_2) basing on the metric (2) is not a scale even when all criteria are measured in absolute scales.

The results of assessments by the set of experts X ($|X| = n$) of values of competitiveness criterion Q_s of enterprises can be represented as the set of functions $\{f_A^s : X \to Q_s \mid A \in \mathbf{A}\}$. Let the experts agree with the assessment to each enterprise, i.e., $\forall A \in \mathbf{A}\ \forall Q_s \in \mathbf{Q}\ f_A^s(x) = const(s)$. Obviously, the enterprise $A \in \mathbf{A}$ is better than the enterprise $B \in \mathbf{A}$ for competitiveness, if $\forall Q_s \in \mathbf{Q}\ \forall x \in X$ $f_A^s(x) \geq f_B^s(x)$, and if there exists at least one criterion $Q_g \in \mathbf{Q}$ for which $f_A^g(x) > f_B^g(x)$. Such assessments generate on $\mathbf{A}$ a strict order relation $\succ'$, which we will be called as a non-contradictory relation of strict domination. The corresponding relation $\succsim'$, which includes the case of equality, i.e., an equivalence relation $\sim'$ over the set of enterprises $\mathbf{A}$, will be called as a non-contradictory relation of domination.

Definition 1. A mapping $\varphi:\mathbf{A}\to R^1$ preserving a non-contradictory relation of domination in the empirical system will be called as an adequate assessment of competitiveness of enterprises *in a broad sense*, if

$$A \sim' B \Rightarrow \varphi(A)=\varphi(B) \ , \tag{10}$$

and when φ is an isotonic mapping,

$$A \succ' B \Rightarrow \varphi(A)>\varphi(B) \ , \tag{11}$$

and

$$A \succ' B \Rightarrow \varphi(A)<\varphi(B) \ , \tag{12}$$

when φ is an antitone mapping.

Definition 2. A mapping $\varphi:\mathbf{A}\to R^1$ will be called as an adequate assessment of competitiveness of enterprises *in a narrow sense*, if the implications (10, 11) or (10, 12) remain true under all admissible transformations of measurement scales of criteria of competitiveness $\mathbf{Q}$.

The kind of admissible transformations depends on the type of measurement scale [Luce et al, 2007].

Definition 3. A mapping $\varphi:\mathbf{A}\to R^1$ will be called as an admissible or adequate assessment of competitiveness of enterprises, if the assessment is adequate both in a broad and in a narrow sense.

Definition 4. A mapping $\varphi:\mathbf{A}\to R^1$ is called an invariant assessment of competitiveness of enterprises, if $\varphi(A\,|\,A\in\mathbf{A})=const$ under any admissible transformations of scale measurement of criteria of competitiveness.

Invariant assessments are adequate in the narrow sense, but these assessments can be inadequate in the broad sense.

Further, let all of the criteria are measured in absolute scales.

Theorem 2. A mapping φ_1 isn’t an adequate assessment.

Proof. Let $A \succ' B$ and none of the experts gave the highest assessment $q_s^1 \ \ \forall Q_s \in \mathbf{Q}$ to the enterprises A and B. In this case, using (5), we get $\varphi_1(A) = \varphi_1(B) = 2n\sum_{s=1}^{m} \omega_s$. Consequently, condition (12) is not satisfied for antitone mapping φ_1: from $A \succ' B$ the inequality $\varphi_1(A) < \varphi_1(B)$ doesn't follow. Thus, the mapping φ_1 doesn't preserve a non-contradictory relation of strict domination, i.e., this mapping is not adequate assessment in the broad sense.

In other word, the mapping φ_1 is not a scale of measurement.

Theorem 3. A mapping φ_2 isn’t an adequate assessment.

For the mapping φ_2, the proof of theorem is similar.

3. A *crisp* multiset is an *unordered* collection of elements, in which are allowed to repeat the elements. However, the domain $G = \{Q_1,...,Q_m\}$ contains the *ordered* subsets, thus, as we show, the assessment (1) should be considered as a fuzzy multiset.

Suppose $\vec{\mu}(A_l, x_r) = (\mu_s(l,r))_{s=1}^{m}$ is the vector assessment of expert $x_r \in X$ by criteria of competitiveness $\mathbf{Q} = \{Q_s\}_{s=1}^{m}$ for the enterprise $A_l \in \mathbf{A}$, where $\mu_s(l,r) \in Q_s = (q_s^{e_s})_{e_{s=1}}^{h_s}$, $q_s^1 > q_s^2 > ... > q_s^{e_s} > ... > q_s^{h_s}$, $l \in \{1,...,p\}$, $r \in \{1,...,n\}$. Obviously, $Q_1 \times ... \times Q_s \times ... \times Q_m$ is the lattice, $(q_1^{h_1},...,q_s^{h_s},...,q_m^{h_m})$ is the bottom element, and $(q_1^1,...,q_s^1,...,q_m^1)$ is the top element of the lattice. Then the set of such assessments of the enterprises $\mathbf{A}$ is the membership function of L-fuzzy set [Goguen, 1967]:

$$\Gamma_r = \{(A_l, \vec{\mu}(A_l, x_r)) \mid A_l \in \mathbf{A},\ \vec{\mu}(A_l, x_r) \in Q_1 \times ... \times Q_m,\ x_r \in X\}_{l=1}^{p}.$$

Therefore, the set of *n* expert assessments can be represented as:

$$\{\Gamma_1,...,\Gamma_r,...,\Gamma_n\} = \left\{\{(A_l, \vec{\mu}(A_l, x_r))\}_{l=1}^{p}\right\}_{r=1}^{n} = \{(A_l, \nu(A_l))\}_{l=1}^{p},$$

$$\nu(A_l) = \{\vec{\mu}(A_l, x_r)\}_{r=1}^{n} = \left\{(\mu_s(l,r))_{s=1}^{m}\right\}_{r=1}^{n} =$$

$$= \left\{k_{a_l}(q_1^1) \circ q_1^1, \ldots, k_{a_l}(q_1^{h_1}) \circ q_1^{h_1}, \ldots, k_{a_l}(q_s^{e_s}) \circ q_s^{e_s}, \ldots, k_{a_l}(q_m^1) \circ q_m^1, \ldots, k_{a_l}(q_m^{h_m}) \circ q_m^{h_m}\right\},$$

where $k_{a_l}(q_s^{e_s}) \circ q_s^{e_s}$ is the ordered pair, $k_{a_l}(q_s^{e_s})$ is the number of experts who gave the assessment $q_s^{e_s}$ for criterion Q_s to the enterprise A_l . Since $\vec{\mu}(A_l, x_r)$ is an element of the lattice $Q_1 \times \ldots \times Q_s \times \ldots \times Q_m$ then $\nu(A_l)$ is the fuzzy multiset over domain $G = \{Q_1, \ldots, Q_s, \ldots, Q_m\}$, and $\nu(A_l) = a_l$, i.e., the assessment (1) is a fuzzy multiset. The cardinality of such fuzzy multiset is $|a_l| = \sum_{s=1}^{m} \sum_{e_s=1}^{h_s} q_s^{e_s} k_{a_l}(q_s^{e_s})$ (consider that the values of all criteria of competitiveness $\mathbf{Q}$ are measured in absolute scales).

At the equivalence of all criteria the adequate assessment of competitiveness of the enterprise A_l on the basis of expert assessments can be defined as $\varphi_3(A_l) = |a_l|$, and then we determine $\varphi_3(A_l) = \varphi_3(A_r) \Rightarrow A_l = A_r$, $\varphi_3(A_l) > \varphi_3(A_r) \Rightarrow A_l \succ A_r$.

Evidently, the assessment $\varphi_3(A_l)$ is measured in the absolute scale. It is easy to show that $A_l \succsim' A_r \Rightarrow \varphi_3(A_l) \geq \varphi_3(A_r)$, where $\succsim'$ is the non-contradictory relation of domination. Hence, the assessment φ_3 is adequate assessment both in the broad sense and in the narrow sense.

Using data of Example 1, we obtain $\varphi_3(A_1) = 82$, and $\varphi_3(A_2) = 28$, i.e., $A_1 \succ A_2$. This is consistent with the opinions of the most experts (Figure 1). For Example 2, we calculate $\varphi_3(A_1) = 28$, and $\varphi_3(A_2) = 81$, i.e., $A_2 \succ A_1$ and this also is consistent with the opinion of the most experts (Figure 2).

Conclusion

On the base of representational theory of measurement the approach to investigating the adequacy of assessments basing on multiset models is suggested. It is shown that:

- The set of assessments of competitiveness of enterprises, which use only the values of multiplicity functions, is the anti-chain in the lattice of crisp multisets.
- Such assessments are not adequate and, therefore, they can lead to incorrect results when the competitiveness of enterprises is analyzed.

As a rule, the concept of an adequate assessment is considered from the point of view of invariance of conclusions at admissible transformations of results of measurements. Such estimates we call adequate in a narrow sense. From positions of the representational theory of measurement the measurement problem, first of all, is reduced to a problem of representation or construction of a scale of measurement that saves substantial relations between empirical objects. The assessment preserving substantial relations we call adequate in a broad sense. Adequate or admissible assessments must be adequate both in a broad and in a narrow sense. The assessments of competitiveness of enterprises, which use only the values of multiplicity functions of multisets, are invariant. However such assessments can lead to mistakes in ordering of enterprises for competitiveness because these assessments don't consider the order relation for scale values of criteria of competitiveness. In other words, on the basis of a metric in space of crisp multisets [Petrovsky, 2003] it is impossible to construct a competitiveness measurement scale, consequently, it is impossible to get the adequate assessment of competitiveness of enterprises.

The example of adequate assessment based on fuzzy multisets is given for the case when all criteria are measured in absolute scales. For criteria, measured in scales that are not absolute (as ordinal scales, ratio scales, and interval scales), methods of construction of adequate assessments of competitiveness will be discussed in following publications.

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