ALGORITHMIZATION PROCESS FOR FRACTAL ANALYSIS IN THE CHAOTIC DYNAMICS OF COMPLEX SYSTEMS STRUCTURED TYPE

Igor Skiter, Elena Trunova

***Abstract**: This work deals in the usage of fractal analysis to determine the trend of dynamic characteristics of chaotic series based on R/S-analysis. We propose an algorithm using the method of dynamic roles with large dimensions, which can be specified Hurst at every step of aggregation without storing values of data stream based on neural network technology.*

***Keywords**: Complex System of Structured Type (CSST), chaotic dynamic series, fractal analysis, Hausdorff rate, Hurst rate, algorithm R/S -analysis, persistence, antipersistence, prognosis, neural network.*

***ACM Classification Keywords**: G.1.2 Approximation, F.1.1 Models of Computation (F.4.1)*

Introduction

The existence of complex systems, is usually due not only to their internal structure, but to a large extent by their interconnection with the environment. That is why the development of such systems in time, their dynamics contains a significant amount of uncertainty, which is defined by the influence of uncontrollable factors. Dynamic states, parameters, characteristics of such systems can be described on the basis of dynamic series analysis having chaotic nature.

Modern science has a wide selection of instruments for the study of the dynamic parameters of complex systems structured type (CSST). The problem lies in the fact that classical statistical methods used in the study of CSST, in most cases, are inadequate. Classical mathematical statistics is based on the central limit theorem (the law of large figures), which states that as an increasing number of trials, the limited distribution of the random values to be normally distributed. This means that the event must be independent, i.e. should not interfere with each other, and thus they must all be equally probable. Chaotic behavior of the series is due to growth and decline of levels. And because of crises occur much more frequently than this theory provides, then the distribution describing the state of the system, although visually similar to the normal or log-normal distributions, actually have the Pareto distribution with "fat tails", which explains the frequency of crises.

From the perspective of rigorous scientific approach the above facts discrepancy normally distributed random variables are fundamental in the sense that there is a problematic issue of the illegality of the use of a greater part of the statistical analysis methods, including methods for the diagnosis of conditions CSST developed in classical statistics. Since the majority of real-time series are not fulfilled by the “normal” conditions, the new and different standard statistical analysis techniques to study the state CSST are necessary. All this led to the replacement in the future linear paradigm by the nonlinear components of which is chaos theory, fractal statistics, discrete nonlinear dynamics and other areas. With respect to the development of CSST, time can speak on fractal characteristics of such systems, as in the case where dynamic series combines deterministic and stochastic properties - its fractal nature.

Assessment of the nature of the dynamics of systems based on fractal analysis

Let us focus on the possibility of applying research and prognosis of CSST, the mathematical theory of fractals. Modern science makes extensive use of the theory of fractals for the study of complex systems in various sectors in order to increase the reliability of the prognosis of their functioning. In the construction of fractals are implemented the principles of non-linearity are realized in the selection process of the system. The non-linearity in the world outlook means multiple variants of development, availability of choice among alternatives and a certain tempo of evolution, as well as the irreversibility of the evolutionary process. The non-linearity in the mathematical sense, means the corresponding type of mathematical equations (nonlinear differential equations) containing sought out quantities in the extent greater than one or coefficients depending on the properties of the environment. That is, fractals are used in the case where the object has several variants for the development and the state of the system is determined by the position in which it is now – this is an attempt to model the chaotic development. The fractal structure of the object provides immutability complexity of its structure with increasing scale consideration.

The dynamic series data that describe the behavior of complex systems structured type, at different scales have approximately the same form. To characterize the fractal structure using the fractal index D, introduced by Hausdorff [Hausdorff, 1919; Mandelbrot, 1982; Feder, 1988], which numerically determines the size of a compact set in an arbitrary metric space:

$$D = \lim_{\delta \to 0} \frac{\ln N(\delta)}{\ln(\frac{1}{\delta})}, \tag{1}$$

where - $N(\delta)$ the number of layers of the radius, which cover the compact sets in an arbitrary metric space.

To characterize the dynamics of the properties such as persistence antipersistence ("mean reversion") recurrence. It is therefore an important indicator of the fractal in the sense that it is closely related to the Hurst exponent [Frost & Prechter, 2001], which determines the properties of the dynamics of the system or the dynamic series. For a Gaussian process, the Hurst exponent is defined as:

$$H = 2 - D. \tag{2}$$

But in practice, Hurst rarely calculated by the formula (2), since it is strictly proved only for certain processes. In addition to its definition uses of random data sets containing up to several thousands of values [Kronover, 2000]. Reducing the amount of array data is possible by using the method of rescaled range (R/S-analysis). It has been proved that the Hurst exponent related to the coefficient of the normalized amplitude (R/S) [Skiter & Trunova, 2013]. For the majority of dynamic series, the equation:

$$R / S = (a \cdot N)^H \tag{3}$$

where *R/S* - normalized scope of the accumulated mean;

S - Standard deviation;

a - A constant for each individual process;

N - The number of observations;

H - Hurst exponent (0 <H <1), which characterizes the fractal dimension of the process.

For the dynamic series of finite duration is the process of determining the Hurst exponent can algorithmize as follows:

1. Calculate the arithmetic mean of a set of observations:

$$\overline{X} = \frac{1}{N} \sum_{i=1}^{N} x_i \,, \tag{4}$$

where x_i - series levels.

2. Calculation of the standard deviation of the series:

$$S = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{X})^2} \tag{5}$$

3. Calculation of the accumulated deviation from the mean number of members

$$Z_N = \sum_{i=1}^{N}(x_i - \overline{X}) \tag{6}$$

4. Calculation of the scope of accumulated deviations:

$$R = \max_{1\le i\le N}\{Z_N\} - \min_{1\le i\le N}\{Z_N\} \tag{7}$$

5. Calculation of the Hurst exponent:

$$\log(R/S) = H\log a + H\log N\ , \tag{8}$$

from where

$$H = \frac{\log(R/S)}{\log a + \log N} \tag{9}$$

Along with the use of the Hurst exponent for trend analysis is the correlation ratio, this characteristic is used to assess the impact of previous values of the autocorrelation of the dynamic series at its next value and define the future trends, using the correlation ratio [Chabak, 2011; Kronover, 2000]:

$$r = 2^{2H-1} - 1, \tag{10}$$

where *r* - the measure of correlation, *H* – Hurst exponent.

Depending on the range, which includes the value of the Hurst exponent, there are three main features of the functioning of CSST:

1. If *0,5 <H <1 (1 <D <1,5)*, systems refer to trend stability, that is, the tendency shown time series will continue in the future for a certain length of time, to the same exponent *H* is direct dependence on the strength of trends (the larger the index of *r* - the higher the index of *H*).
2. If *H* = *0,5* (respectively, *D* = *1,5*) state of the system is totally independent random character without any correlation (*r* = *0*): the current state of the system is in no way related to its future state.
3. If the range of *H* <*0,5 (D*> *1,5),* it refers to a sequence belonging to the "antipersistence series", i.e. the system will show the future trend opposite to that which has been characteristic of the previous period of development.

It should be noted that the method of determining the fractal characteristics of dynamic series based on *R/S*-analysis using the algorithm based on (4)-(10) can only be used for enormous datasets. In the case when the sample value is of significant quantity (several thousand items), it is expedient to use a modification of the proposed algorithm, wherein:

1. Dynamic series *N (t)* is divided by *A* contiguous periods of length *n*.
2. Determined the average value of the normalized amplitude:

$$(R/S)_n = \frac{1}{A}\sum_{i=1}^{A}(R_i/S_i)_n \, , \tag{11}$$

where R_i - the maximum range of the *i*-period, S_i - selective deviation calculated for each period.

3. Definition $\log(R/S)_n$ and $\log(n)$.
4. Construction of linear regression $\log(R/S)_n = f(\log(n))$, in which the estimate of the parameter "slope" will be equal to the assessment of the Hurst exponent *H*.

Prediction based on the definition of fractal characteristics of the dynamic series

To define the parameters for the operation of Hurst CSST, you can make the necessary parameters prognosis system [Feder, 1988]. But the classic method of Hurst has several disadvantages, including the inability to calculate the exponent in real time because of the large amounts of computation. One of the options for escaping this shortcoming would be to use the incremental recursive algorithm

$$H(k+1) = \frac{\ln\left(\dfrac{R(k+1)}{S(k+1)}\right)}{(\ln(k+1) + \ln\alpha)} \, , \tag{12}$$

where $k = 1,2,...$ - which corresponds to time intervals of data aggregation process of observation of the real functioning of CSST.

From the equation (12) that the Hurst exponent can be recalculated at every step of aggregation without storing the values of the data stream.

The implementation of the algorithm determining the *H* can be simplified by using neural network technology. The result of Hurst parameter calculation heavily depends on the sample volume and parameters that may lead to the fact that for the same data stream realization can produce different and sometimes opposite effects. This problem can be solved by the use of neural network technologies associated with training algorithms and analysis of real systems. Then, by rewriting (12) in the form

$$\ln\frac{R(k)}{S(k)} = H\ln\alpha + \ln k\,, \tag{13}$$

will introduce a training signal

$$z(k) = \ln\frac{R(k)}{S(k)}\,, \tag{14}$$

for a linear straightforward neural network ADALINE type, for which we use a training algorithm to obtain estimates of the unknown parameters in the form:

$$\begin{pmatrix} h(k+1) \\ H(k+1) \end{pmatrix} = \begin{pmatrix} h(k) \\ H(k) \end{pmatrix} + \frac{z(k+1) - h(k) - H(k)\ln(k+1)}{1 + (\ln(k+1))^2} \cdot \begin{pmatrix} 1 \\ \ln(k+1) \end{pmatrix}, \tag{15}$$

$$\ln\alpha(k+1) = \frac{h(k+1)}{H(k+1)} \tag{16}$$

In this case, to calculate the Hurst exponent and network parameters can be applied to the appropriate architecture of artificial neural network, which will operate in parallel to the controlled process to identify changes that occur in real-time.

In the second embodiment, the neural network approach to prognosis parameters CSST as the essential element is used neural network with radial basis functions excitation (see Figure 1) [Skiter & Trunova, 2013].

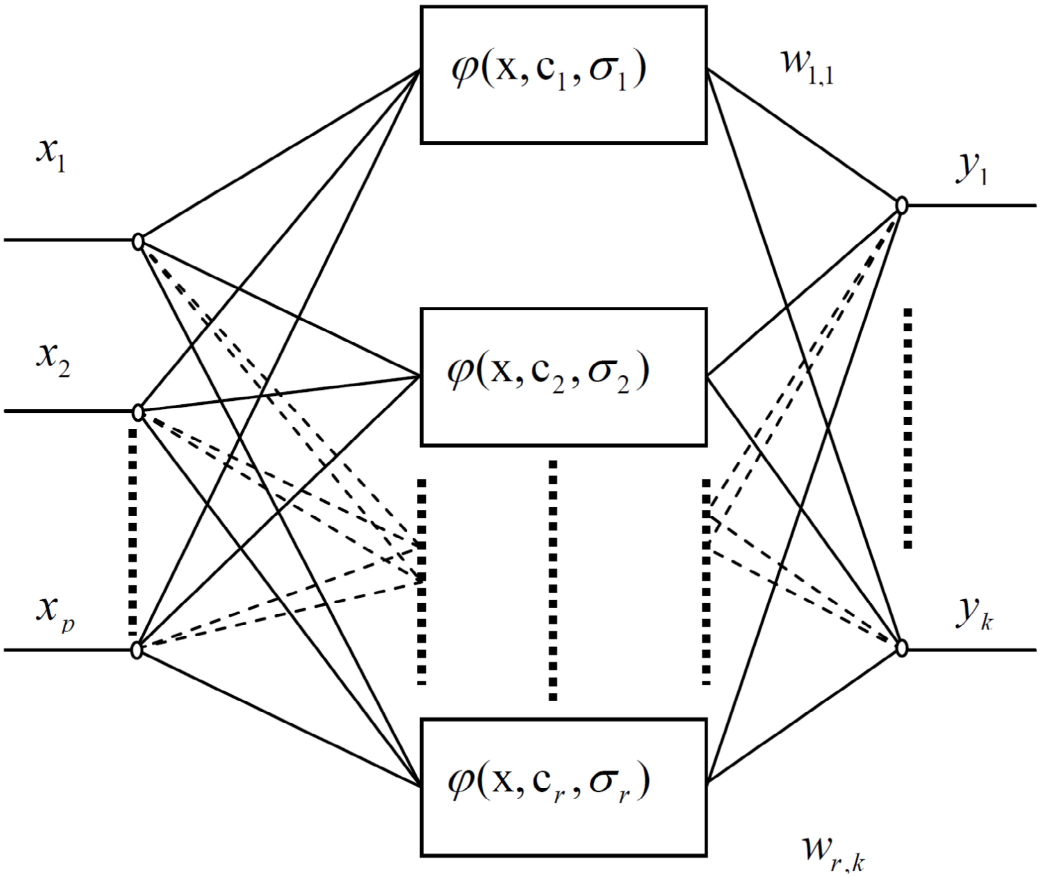

Figure 1. Architecture of *p-r-k* network with radial basis functions

The output signal of such a network has the form

$$y = \varphi \cdot W \ , \tag{17}$$

where $y = [y_1, ..., y_k]$ - the output of neural network (projected parameters);

k - The dimension of the output vector;

φ - vector of radial basis functions (RBF) neurons in the obscure layer. Vector of RBF neurons in the hidden layer.

Elements of the obscure layer are:

$$\varphi_1 = e^{-\frac{x - c_i}{\sigma_i}} \ , \tag{18}$$

where $x = [x_1, ... x_M]$ - the input of neural network (parameters for the current data stream);

M - The dimension of the input vector;

$c_i = [c_{i1},...c_{iM}]$ - Coordinates of the centers of activation functions $i = 1,...N$;

r - The number of hidden units in the network;

σ_i - The width of the activation functions;

W - Initial weight matrix network (dimension $r \cdot k$).

For such a network one can use the following the training algorithm:

1. Choose the size of the obscure layer *r* which is equal to the number of training patterns. Synaptic weights of the obscure layer neurons assumed to be 1.
2. Place the center of activation functions of the obscure layer neurons in the space of points *x* of input network, which includes a set of training patterns Ξ , $c_j = x_j$, $j = 1,2,...,r$.
3. Choose the window width activation functions of neurons in the obscure layer σ_j , $j = 1,2,...,r$, sufficiently large such that they do not overlap in space output signals.
4. To determine the scale of the neurons of the output layer network w_{ij} , $i = 1,2,...,r$, $j = 1,2,...,k$, for which the network will provide the entire set of training patterns Ξ , and as a result we obtain a set of linear equations that can be written in matrix form $\Phi \cdot w = D$, where D - matrix size $r \cdot k$ of the expected output patterns; *Φ* - interpolation matrix, in size $r \cdot r$ the elements of which:

$$\varphi_{ij} = e^{-\frac{(x-c_i)^2}{\sigma_i^2}}, \tag{19}$$

where $i = 1,2,...,r$, $j = 1,2,...,r$.

The form of system of equations $w = \Phi^{-1} \cdot D$ allows the passage of the interpolation surface through all points of the training set patterns.

Conclusion

The proposed method of Hurst parameter estimation can be used for mathematical modeling and prognosis of CSST. The algorithm for dynamic series with a large dimension in which Hurst exponent can be specified at each step of aggregation without storing the values of the data stream based on neural network technology, which makes it possible to classify the functioning of the CSST and to increase the reliability of predicting the behavior of systems in real-time.

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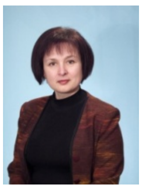

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