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# RANDOMIZED SET SYSTEMS CONSTRAINED BY THE DISCRETE TOMOGRAPHY

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***Abstract***: *This article, in general, is devoted to a set of discrete optimization issues derived from the domain of pattern recognition, machine learning and data mining - specifically. The global objectives are the compactness hypotheses of pattern recognition, and the structural reconstruction of the discrete tomography.*

*The driving force of the current research was the proof technic of the discrete isoperimetry problem. In proofs by induction the split technique was applied and then it is important to have some information about the sizes of the split compounds. Isoperimetry itself is a formalism of the compactness hypotheses. From one side knowledge on split sizes helps to find the compact structures and learning sets based on this, from the other side – split sizes help to prove the necessary relations. The pure combinatorial approaches [22-77] are not able at the moment to give an efficient description of the split sizes and – the weighted row-different matrices. The probabilistic method, as it is well-known, gives additional knowledge about the random subsets, and this may be useful as a complementary knowledge about a different objects or a situations concerned the properties of discrete structures – isoperimetry and tomography.*

*The discrete mathematical science deals with different types of discrete structures, studying their transformations and properties. In some problems we face the issues about the existence of structures under some special constraints, about the enumeration of structures under these constraints, and – on algorithmic optimization. Given a simple structure – in some cases, it can be even hard to compute some basic properties of it. Such are for example the graph chromatic number, the minimal set cover, the solution of the well-known SAT and plenty of other NP-complete problems. When structures are given, the mentioned parameters may be easily computable. To find a structure by the given parameters often becomes hard. We call such problems – inverse problems. Our special interest is in considering of simple (0,1) matrices and their row and column weights. Given a matrix we can compute the mentioned weights (direct problem). The inverse problem, -- when it is to find the construction with the given weights is not simple. At least there is not known polynomial algorithms for this problem. Moreover, the problem is known as the hypotheses posted by famous graph theorist C. Berge so that the problem is well known and unsolved.*

*Besides the logical and combinatorial analysis of the inverse type problems of discrete optimization, in several cases the probabilistic models were applied successfully. The idea of this paper is to use the probabilistic theory of combinatorial analysis to the discrete tomography problem given in terms of the (0,1) matrices. The paper tries to outline the models, relations and the methodology. Our research priority interest is to understand the opportunities, similarities and perspectives in this broad research area. The study is ongoing and the follow up publication will come soon.*

***Keywords**: discrete tomography, discrete isoperimetry, probabilistic theory of combinatorial analysis, (0,1) matrices, discrete optimization.*

***ACM Classification Keywords**: G.2.1 Combinatorics*

**Content of the paper**

## Introduction

Our aim is to present the work done on composition and analysis of appropriate probabilistic models that support investigations of a group of combinatorial problems related to the basic one – the Discrete Tomography. The first articles in the domain of probabilistic theory of combinatorial analysis [1-4] by Erdős, Rényi and Gilbert, and [5-7] by Glagolev, Kospanov and Nechiporuk considered graph models with random vertices and edges, and random Boolean schemes and formulas -- evaluating the frequencies of appearance of different configurations given the specific properties. Since then, thousands of publications appeared in this domain. Today also the systematic presentation of the subject is accessible [8-10].

Our consideration is focused on study of different specifically constrained set systems, which are known alternatively as the problems of hypergraph theory, or the Boolean functions or the $n$-cube geometrical studies. As usual, we denote the set of all vertices of the $\mathrm{n}$-dimensional unite cube by $\mathrm{E^n}$. Subsets of $\mathrm{E^n}$ compose set systems with the base $n$-set, that represents the set of Boolean variables. Alternatively, we will also consider multi-sets, defined over the set of vertices of $\mathrm{E^n}$.

Within the set theory, formally, a multi-set (or bag) is a 2-tuple $(\mathrm{A}, \mathrm{r})$ where A is some set of elements and $\mathrm{r}\colon \mathrm{A} \to \mathrm{Z_+}$ is a function from A to the set $\mathrm{Z_+} = \{1,2, \dots\}$ of positive natural numbers. For each $\mathrm{a} \in \mathrm{A}$ the multiplicity of a in $(\mathrm{A}, \mathrm{r})$ is the number $\mathrm{r}(\mathrm{a})$, that is the number of its occurrences. For a finite A multi-set $(\mathrm{A}, \mathrm{r})$ can be given by a list. If an underlying set U, wherefrom the elements of A are obtained is specified, then the definition can be simplified to just a multiplicity function $\mathrm{r}\colon \mathrm{U} \to \mathrm{Z_{\geq 0}}$ obtained by extending $A$ to U with the use of the values $0$ for all elements not in A.

The multi-sets $M = (A, r)$ with the universal set $E^n$ and a multiplicity function $\mathrm{r}\colon E^n \to Z_{\geq 0}$ will be considered. Different interpretations concerning the structure $(A, \mathrm{r})$ will be considered and used throughout this paper. In particular, $A$ may denote the pairs of rows in a (0,1) matrix. The different rows compose a set of vertices of $E^n$. The number of repeated rows (or the number of pairs of equal rows) will denote multiplicity.

The probabilistic method used throughout this paper basically deals with the discrete probability spaces. We suppose that it is given a finite space $A$ of elementary events (outcomes of a certain processes or experiments/trials) and that there are probabilities related to these events. If the total number of outcomes of experiment is $n$, then the probabilities $p_1, p_2, \dots, p_n$ are allocated to them so that $\sum p_i = 1$. The set theoretical relations such as union and intersection, and the inclusion exclusion

principle regulate more complex events $X \subseteq A$ under this scheme. The probability space provides probability measures of these events, $p\,(X) = \sum_{a \epsilon X} p\,(a)$. Let us remind some of the basic probabilistic relations. Let $A$ be the finite set of elementary events, $X_1$, and $X_2 \subseteq A$. Then $p\,(X_1 \cup X_2) = p\,(X_1) + p\,(X_2) - p\,(X_1 \cap X_2)$. Form the expression $p\,(X_1 \cup X_2) \leq p\,(X_1) + p\,(X_2)$ that is known as the Boole's inequality. If here we have the equality, $p\,(X_1 \cap X_2) = 0$, then we call $X_1$ and $X_2$ ***mutually exclusive***. $X_1$ and $X_2$ are ***independent***, iff $p\,(X_1 \cap X_2) = p\,(X_1)p\,(X_2)$. Further:

for arbitrary events $X_i$

$$p\,(X_1 \cup X_2 \cup \ldots \cup X_m) \leq p\,(X_1) + p\,(X_2) + \cdots + p\,(X_m), \tag{2.1}$$

for independent events $X_i$

$$p\,(X_1 \cap X_2 \cap \ldots \cap X_m) = p\,(X_1)p\,(X_2) \ldots p\,(X_m) \tag{2.2}$$

and

$$p\,(X_1 \cup X_2 \cup \ldots \cup X_m) = 1 - (1 - p\,(X_1))(1 - (1 - p\,(X_2)) \ldots (1 - (1 - p\,(X_m)). \tag{2.3}$$

Repeated independent trials when there are only two outcomes in each trial and the probabilities of outcomes remind unchanged is known as the Bernoulli model.

In a succession of $n$ Bernoulli trials number of successes can be $0, 1, \ldots, n$. If $p$ and $q = 1 - p$ are the probabilities of success and failure correspondingly, then the probability that the $n$ Bernoulli trials result in t success, exactly, equals $b(t, n, p) = C_n^t p^t q^{n-t}$. By this we obtained the probability of the random number $t$ of successes in $n$ Bernoulli trials that is known as the binomial probabilistic model. The maximum of $b(t, n, p)$ by $t$ we call the central term. It is easy to check that when $t$ goes from 0 to $m$ the probability $b(t, n, p)$ first increase monotonically to the central term and then it decrease. The central term reaches at $t = (n + 1)p$ and $t - 1$ when these are integer, and at $[(n + 1)p]$ otherwise.

In numerical applications of Bernoulli trials, as a rule, $n$ is large and $p$ is small with the product $\lambda = np$ of a moderate magnitude. Consider the formula

$$C_n^t p^t q^{n-t} = \frac{n(n-1) \ldots (n-t+1)}{t!} p^t (1-p)^{n-t}.$$

Let $t^2 = o(n)$, then $n(n-1) \ldots (n-t+1) \sim n^t$ and $(1-p)^{n-t} \sim (1-p)^n \sim e^{-np}$, and

$$b(t,n,p) = C_n^t p^t q^{n-t} \sim \frac{\lambda^t}{t!} e^{-\lambda}.$$

The right part of the last line determines the well-known Poisson probability distribution. The distribution, as demonstrated, is a convenient approximation to the binomial distribution.

Let $\xi$ be the random integer variable. The expected value $E\ (\xi)$ or mean and the variance $Var\ (\xi)$ can be computed as

$E\ [\xi] = \sum_t \mathrm{t} \cdot \Pr\ (\xi = t)$ and $Var\ [\xi] = \sum_t (\mathrm{t} - \mathrm{E}\ [\xi])^2 \cdot \Pr\ (\xi = t)$.

We differentiate several types of convergences of random variables. Let $\xi_l$ be a sequence of random variables, and let their distribution functions be $F_l(x)$, respectively.

The first notion of convergence of a sequence of random variables is known as convergence in probability. The sequence $\xi_l$ converges to a random variable $\xi$ in probability, denoted $\xi_l \xrightarrow{p} \xi$ if for any $> 0\ \lim_{l\to\infty} \Pr\ (|\xi_l - \xi| < \varepsilon) = 1$.

Note that this does not say that the difference between $\xi_l$ and $\xi$ becomes very small. What converges here is the probability that the difference between $\xi_l$ and $\xi$ becomes very small. It is, therefore, possible, although unlikely, for $\xi_l$ and $\xi$ to differ by a significant mount and for such differences to occur infinitely often.

A stronger kind of convergence, which does not allow such behavior, is called almost sure convergence or strong convergence. A sequence of random variables $\xi_l$ converges to a random variable $\xi$ almost surely, denoted $\xi_l \xrightarrow{s} \xi$ if for any $\varepsilon > 0\ \lim_{L\to\infty} \Pr\ (\sup_{l\geq L} |\xi_l - \xi| < \varepsilon) = 1$.

Finally we like to remind the classical continuity theorem of distributions.

Suppose that for any fixed $n$ the sequence

$$a_{0,n}, a_{1,n}, a_{2,n}, \ldots$$

is a probability distribution of $n$-th trial, that is, $a_{k,n} \geq 0$ and $\sum_{k=0}^{\infty} a_{k,n} = 1$.

In order that a limit $a_k = \lim_{n\to\infty} a_{k,n}$ exists for every $k \geq 0$ it is necessary and sufficient that the limit

$$A(s) = \lim_{n\to\infty} \sum_{k=0}^{\infty} a_{k,n} \cdot s^k$$

exists for each $s$ in the open interval $0 < s < 1$. In this case

$$A(s) = \sum_{k=0}^{\infty} a_k \cdot s^k.$$

*Inequalities: First and Second Moment Methods.*

We review some inequalities that play a considerable role in probabilistic analysis of algorithms. In particular, we discuss first and second moment methods that are 'bread-and-butter' of a typical probabilistic analysis.

*Markov Inequality:* For a nonnegative random variable $\xi$ and $\varepsilon > 0$ the following holds: $\Pr(\xi \geq \varepsilon) \leq \frac{E(\xi)}{\varepsilon}$.

Indeed: let $I(A)$ be the indicator function of $A$ (i.e., $I(A) = 1$ if $A$ occurs, and zero otherwise). Then, $E[\xi] \geq E(\xi \cdot I(\xi \geq \varepsilon)) \geq \varepsilon \mathrm{E}(I(\xi \geq \varepsilon) = \varepsilon \Pr(\xi \geq \varepsilon)$.

*Chebyshev Inequality:* If one replaces in the Markov inequality $\xi$ by $|\xi - \mathrm{E}[\xi]|$ then $\Pr(|\xi - \mathrm{E}[\xi]| > \varepsilon) \leq \frac{Var(\xi)}{\varepsilon^2}$.

### *A.* The problem

Our investigation, in general, belongs to the combinatorial theory of finite and constraint set systems described above. Doing this r we will construct and investigate probabilistic models complementary to the combinatorial analysis, intending to achieve in an alternative way to gain the necessary knowledge on structures and properties of these set systems – of subsets of $\mathrm{E}^n$ representing the set systems of a universal $n$-element set. In this way we face 2 basic problems: first is how to obtain meaningful postulations over the probabilistic models, and second is how to transfer this knowledge to the

combinatorial domain of the basic addressed problems. To give an expression about the structures and properties it is enough to note the following. Erdős and Rényi [1-2] initiated the domain of random graphs but they considered a larger domain of set systems with different properties: constrained intersection, special coverage and others.

Glagolev [5] initiated the probabilistic study of Boolean functions through the description of sub-cubes of the truth domain.

Aslanyan and Akopova (Arsenyan) [11-21] applied the probabilistic technique to the domain of discrete isoperimetry.

The Hungarian and Russian scientific schools were the well-recognized centers of intensive use of probabilistic models of combinatorics.

### *B.* The model

Probabilistic model of a combinatorial problem consists the set of all structures of the problem and extends probabilities on this set. Allocate probabilities to the variables that are generating the $n$-dimensional unite cube. Boolean functions will appear with corresponding probabilities. If we allocate probabilities to the vertices of $E^n$ to be the truth-value of random Boolean function $f$, then Boolean functions will appear with different probabilities. The probabilistic models that are used in combinatorics is very large. For the beginning, in our study of random discrete tomography we will use one of these models.

The probabilistic model $\mathfrak{M}_{\mathrm{pq}}$.

Consider random Boolean $m \times n$ matrix $R_{mn}$ generated in the result of evaluation of column variables $x_j, j = 1, \cdots, n$, that independently and identically (i.i.) attain values $1$ and $0$ with respective none-zero probabilities $p_j$ and $q_j = 1 - p_j$. The number of all matrices that will be generated in this scheme is $2^{mn}$ (this is the set of all possible $m \times n$ matrices) and the probability of a particular matrix being generated is tightly related to the number of "1" values in its columns. Consider a matrix, and let its column weights are as $s_1, s_2, \dots, s_n$, then, the probability of this matrix in $\mathfrak{M}_{\mathrm{pq}}$ is equal to $\prod_{j=1}^{n} p_j^{s_j} q_j^{m-s_j}$. There are $\prod_{j=1}^{n} C_m^{s_j}$ different matrices with column weights $s_1, s_2, \dots, s_n$ so that the probability that the random matrix obey column weights $s_1, s_2, \dots, s_n$ equals

$$\prod_{j=1}^{n} C_m^{s_j} p_j^{s_j} q_j^{m-s_j}. \tag{2.4}$$

It is convenient to consider this probabilistic model as a process, where $m$ vertices i.i. distributed, as it is defined above, are dropped onto the $E^n$. Vertices can appear repeatedly and only in cases when there are no vertex repetitions we receive an $m$-subset of the $E^n$ (vertices given by the rows of the matrix). In an approach, -- the indicator to the existence of such row different matrices can be the related nonzero probability - in the given model $\mathfrak{M}_{\mathrm{pq}}$.

In a special case we will suppose that $p_j = s_j/m$ and intend to prove asymptotically, when $n, m \to \infty$ the following:

$Z1$. Probabilities that column sums are equivalent/equal to $s_1, s_2, \dots, s_n$ are positive and/or tend to 1.

$Z2$. Probability that all rows are different tends to 1, or is straightly grader than $0$.

Consider an arbitrary column $j$. Let $M_j$ and $D_j$ be the average value and the dispersion of the random weight of column $j$. We will treat $Z1$ and $Z2$ on this probabilistic basis, and will also combine the problems $Z1$ and $Z2$.

Additionally we will consider the issue:

$Z3$. Probability that a random set of $\mathfrak{M}_{\mathrm{pq}}$ is a Sperner Family (SF) is positive under the special constraints.

Alternatively, several more probabilistic models besides the $\mathfrak{M}_{\mathrm{pq}}$ can be involved into this study but we postpone this for the continuation.

### *C.* The technique

The technique used in this domain is diverse. The first step in many studies is the calculation of averages for the target properties. The main value shows the existence of a value that is greater (smaller) than this value. Proving the existence of certain types of structures is achieved by showing that the averages/probabilities are positive. Next group of considerations are in computation and use of the second moments. By the Chebyshev inequality, when the main value and the variance are appropriate, conclusions are made in terms of probabilistic convergences, in particular in terms of "almost all"

structures. Another scheme is in use of the means of convergences of probabilistic distributions. In particular, this is through the estimation of factorial moments of random numbers, and then the continuity of distributions in a couple of cases brings us to the resulting Poisson or some other distribution. For example, this is the case of the random number of isolated truth-values of the random Boolean functions.

*D.* **Example 1. Positive probability implies the existence.**

In a simplest application of the probabilistic method to prove that a structure with the desired properties exists we define an appropriate probability space and then show that the desired properties hold in this space with positive probability. Our first example belongs to this case. We also decided the first example to be graph theoretical.

- Let $G = (V, E)$ be a graph with $n$ vertices and $m$ edges. Then $G$ contains a bipartite sub-graph with at least $m/2$ edges.

The proof of such proposition is very simple. It is to generate random subsets $X$ of vertices with $pr(v \epsilon X) = 1/2$. Then the average of number of edges linked to $X$ is equal to $m/2$, which implies the existence of the desired bipartite graph. Continue applying this construction recursively, we see that the graph edges will expire in $logm$ steps. We see an interesting extension of the "bisection" principle in a form - applicable to the arbitrary graphs.

*E.* **Example 2. Probability distributions with the Chebyshev inequality give the asymptotics.**

Next to the "positive probability" level model it comes the model based on second probabilistic moments. Let us bring an example from the field of Boolean functions. Consider ordinary Boolean functions. We intend to derive the complexity asymptotic formula $s(f)$ of reduced disjunctive normal form of random Boolean functions. Consider the appropriate model. Let the "thru" values are generated randomly and uniformly on Boolean vertices by probability $p$. For the average number of the $k$ dimensional prime sub-cubes we obtain the formula

$$i_k(n,p) = m(i_k(f)) = C_n^k 2^{n-k} p^{2^k} (1 - p^{2^k})^{n-k}.$$

$C_n^k$ denote the so called "directions" of the $k$-sub-cubes. On these directions the second probabilistic moments are calculated:

$$d(i_k(f)) = 2^{n-k} p^{2^k} (1 - p^{2^k})^{n-k} + C_{n-k}^2 2^{n-k} p^{2^{k+1}} (1 - p^{2^k})^{2(n-k-1)}$$
$$- (1 + (n-k) + C_{n-k}^2) C_{n-k}^2 2^{n-k} p^{2^{k+1}} \left(1 - p^{2^k}\right)^{2(n-k)}.$$

Then, continuing with the use of the Chebyshev inequality we obtain that simultaneously in all $C_n^k$ directions with the probability tending to 1, and $n \to \infty$, $i_k(f) \sim m(i_k(f))$.

Let $p = 1/2$. There is a unique integer point $k_0$ of maximum of the function $i_k(n,p)$. The result achieved says that for a random Boolean function, with probability tending to 1 with $n \to \infty$

$$s(f) \sim m\big(i_{k_0}(f)\big) + m\big(i_{k_0+1}(f)\big).$$

*F.* **Example 3. Continuity theorem helps to describe the types of the Boolean functions.**
Consider the next model according to the continuity theorem of the generating functions of factorial moments. We construct the model around the well-known discrete isoperimetry problem.

As usual, we denote the set of points of the $n$-dimensional unite cube by $E^n$. For a subset $A \subseteq E^n$ we call a point $\alpha$ *interior* if $S_1^n(\alpha) \subseteq A$ where $S_1^n(\alpha)$ is the sphere of radius 1 with centre $\alpha$ in Hamming metrics $\rho$. Let $B(A)$ denote the set of all interior points of the subset $A$. $\Gamma(A) = A - B(A)$ is called the *boundary* of the subset $A$.

Discrete isoperimetry is the problem of finding the value

$$\Gamma(\mathrm{a}) = \min_{B \subseteq E^n, |B| = a} |\Gamma(\mathrm{B})|$$

for given $a$, $0 \le a \le 2^n$.

For the ratio $\tau_n$ of the subsets $A \subseteq E^n$ with $|A| = (1 - \alpha(n)n)2^{n-1}$ and $k$ interior points the following holds:

$$\tau_n \to \frac{1}{k!}\left(\frac{e^{\lambda}}{2}\right)^k e^{-\frac{e^{\lambda}}{2}}$$

if $\lim_{n\to\infty} \alpha(n)n = \lambda$ $(k = 0,1,2, \dots)$.

## Random Discrete Tomography

*G.* **Series of compound trials.**

$\mathfrak{M}_{\mathrm{pq}}$, is one of considered probabilistic distribution schemes over the set $E^n$. Let $x_1, x_2, \dots, x_n$ be the generating binary variables of $E^n$. Consider a compound trial, where the variables $x_1, x_2, \dots, x_n$ independently of each other accept values 1 by corresponding probabilities $p_1, p_2, \dots, p_n$. By the complementary probabilities $q_i = 1 - p_i, i = \overline{1,n}$ these variables accept the value 0. Series of $m$ such compound trials will be considered. Here, in particular, the random $x_i$ remain under the same distribution throughout the sequence of compound trials, and, because of in each trial there are only two

possible outcomes and their probabilities are unchanged, we deal with the Bernoulli trials at $i = 1,2, \dots, n$. We use a compound trial consisting of $n$ variables and get one vertex of $E^n$ as the outcome of each compound trial.

Consider $x \in E^n$. Which is the probability of appearance of this vertex $x$ in the model $\mathfrak{M}_{pq}$? Since the individual variable trials in a compound trial are independent, the probabilities of individual variables multiply. In order to calculate the probability of the vertex $x$ it is to take the product obtained on replacing the symbols 1 and 0 by p and q (depending on $i$), respectively. Doing $m$ sequential compound trials generates a distribution over the multi-sets on $E^n$.

*H.* **The column weight probability and the average column weight.**

Consider the random variable $\xi_j$ representing the number of 1's in the column $j$ (we call it also column sum, weight, projection) in the model $\mathfrak{M}_{\mathrm{pq}}$. The probability of the value $\xi_j$ in the random matrix $R_{mn}$ equals

$$C_m^{\xi_j} p_j^{\xi_j} q_j^{m-\xi_j} \tag{3.1}$$

On this base for the expectation of the weight of column $j$ we obtain the formula

$$M_j = M(\xi_j) = \sum_{t=0}^{m} t C_m^t p_j^t \, q_j^{m-t} = \sum_{t=1}^{m} t \frac{m!}{t!\,(m-t)!} p_j^t \, q_j^{m-t}$$
$$= m \sum_{t=1}^{m} \frac{(m-1)!}{(t-1)!\,(m-1-(t-1))!} p_j^t \, q_j^{m-t}.$$

Substituting $u = t - 1$ we obtain

$$M_j = m \sum_{u=0}^{m-1} C_{m-1}^u \, p_j p_j^u q_j^{m-1-u} = m p_j.$$

For example, when $p_j = q_j = 1/2$ we receive parameters of the usual homogeneous model of random Boolean functions on $E^n$ that are easy interpretable. In the weighted model, the overall average

sums by the set of coordinates/columns will be $mp_1, mp_2, \dots, mp_n$. And in utilization of $p_j = \frac{s_j}{m}, j = 1, \cdots, n$ we obtain that the average columns sums vector equals to $s_1, s_2, \dots, s_n$. In this concern the model $\mathfrak{M}_{\mathrm{pq}}$ is the most convenient to the problem $Z1$. But it does not fit well to the requirements of $Z2$.

At this point we have 2 known parameters – the probability of the given column weight and the average value of the weight. The column weight $t$ probability $C_m^t p_j^t q_j^{m-t}$ for $p_j = \frac{s_j}{m}$ is straightly greater than $0$, which indicates the existence of construction with this weight $t$. Although trivial, this same postulation does not follow from the notion of the main value. The main value instead shows that a greater and/or lesser values (an integrative event) exist. These notes are applicable also to compound values because of the independency of the coordinates.

*I.* **The column weight variance.**

The additional use of the variance in model $\mathfrak{M}_{\mathrm{pq}}$ brings more points. Combined with the Chebyshev inequality this gives intervals around the values $s_j$ with a property that the random sums belong to these intervals (to polyhedrons) with a strongly positive probability.

Let us compute the variance of $\xi_j$. There are 2 ways. One is in direct use of formula $D_j = M(\xi_j^2) - M_j^2$. But, there is an easier way of computing $M(\xi_j(\xi_j - 1))$, having in mind that $M(\xi_j(\xi_j - 1)) = M(\xi_j^2) - M(\xi_j)$ and that we already computed the $M(\xi_j)$. The benefit of this choice is seen below.

$$M(\xi_j(\xi_j - 1)) = \sum_{t=0}^{m} t(t-1) C_m^t p_j^t \, q_j^{m-t} = \sum_{t=2}^{m} t(t-1) \frac{m!}{t!\,(m-t)!} p_j^t \, q_j^{m-t}$$

$$= m(m-1) \sum_{t=2}^{m} \frac{(m-2)!}{(t-2)!\,(m-2-(t-2))!} p_j^t \, q_j^{m-t}.$$

Performing substitution $u = t - 2$ we obtain

$$M(\xi_j(\xi_j - 1)) = m(m-1) \sum_{u=0}^{m-2} C_{m-2}^u \, {p_j}^2 p_j^u q_j^{m-2-u} = m(m-1) p_j^2.$$

Substituting the values finally we obtain that

$$D_j = m(m-1)p_j^2 + mp_j - \left(mp_j\right)^2 = mp_j\left(1-p_j\right) = mp_jq_j.$$

By the Chebyshev inequality we obtain the following probability estimate:

$$P\left(\left|\xi_j - M_j\right| \geq \varepsilon M_j\right) \leq \frac{D_j}{\varepsilon^2 M_j^2} = \frac{mp_jq_j}{\varepsilon^2 m^2 p_j^2} = \frac{q_j}{\varepsilon^2 mp_j}. \quad (3.1)$$

Let $\varepsilon \to 0$ with $m \to \infty$. $q_j$ and $p_j$ may depend on $m$, but let they have the same order. It is easy to choose $\varepsilon$ in a way that $\frac{1}{\varepsilon^2 m} \to 0$. So the probability of the event $P\left(\left|\xi_j - M_j\right| < \varepsilon M_j\right)$ tends to 1 and the relation achieved says, "with probability tending to 1 the random weight $\xi_j$ is equivalent to $M_j$". So, the probability, that the random variable $\xi_j$ of the component $j$ is equivalent to its average value $mp_j$ when $m \to \infty$, is tending to 1.

### *J.* **Analysis of compound trials.**

Consider series of $m$ compound trials by the model $\mathfrak{M}_{\mathrm{pq}}$. Because of the component probabilities $p_1, p_2, \ldots, p_n$ in $\mathfrak{M}_{\mathrm{pq}}$ act separately/independently, we have that any set of events, each defined in terms of one individual coordinate probability are totally independent. Denote the event $\left|\xi_j - M_j\right| \geq \varepsilon M_j$ by $\mho_j$ and consider the compound event $\underline{\mho} = \mho_1 \cup \mho_2 \cup \ldots \cup \mho_n$. Then, by (2.1) $P\left(\underline{\mho}\right) \leq \sum_{j=1}^{n} P\left(\mho_j\right)$. Denote the event $\left|\xi_j - M_j\right| < \varepsilon M_j$, complementary to $\mho_j$ as $\neg\mho_j$. Then for $\underline{\neg\mho} = \neg\mho_1 \cup \neg\mho_2 \cup \ldots \cup \neg\mho_n$ it is true that $P\left(\underline{\neg\mho}\right) \leq \sum_{j=1}^{n} P\left(\neg\mho_j\right)$. These relations are true for arbitrary sets of events.

Events $\mho_j$ are independent by the notion above, and all they have the same structure of the probability. In this way, if $\overline{\mho} = \mho_1 \cap \mho_2 \cap \ldots \cap \mho_n$, then $P\left(\overline{\mho}\right) = \prod_{j=1}^{n} P\left(\mho_j\right)$. Also the events $\neg\mho_j$ are totally independent so that for $\overline{\neg\mho} = \neg\mho_1 \cap \neg\mho_2 \cap \ldots \cap \neg\mho_n$ it is valid the expression $P\left(\overline{\neg\mho}\right) = \prod_{j=1}^{n} P\left(\neg\mho_j\right)$.

By $\mathfrak{M}_{\mathrm{pq}}$ we have a very peculiar model that gives us more useful relations:

$$P\left(\underline{\mho}\right) = 1 - \prod_{j=1}^{n}\left(1 - P\left(\mho_j\right)\right) = 1 - \prod_{j=1}^{n} P\left(\neg\mho_j\right)) = 1 - P(\overline{\neg\mho})$$

and

$$P(\underline{\neg\mho}) = 1 - \prod_{j=1}^{n}(1 - P\left(\neg\mho_j\right)) = 1 - \prod_{j=1}^{n} P\left(\mho_j\right) = 1 - P\left(\overline{\mho}\right).$$

We proceed to derive the "compound" generalization of point III.C. We use the set of Chebyshev inequalities for coordinates:

$$P\left(\mho_j\right) = P\left(\left|\xi_j - M_j\right| \geq \varepsilon M_j\right) \leq \frac{D_j}{\varepsilon^2 M_j^2} = \frac{q_j}{\varepsilon^2 m p_j}.$$

Let us mention also the equivalent forms

$$P\left(\neg\mho_j\right) = P\left(\left|\xi_j - M_j\right| < \varepsilon M_j\right) \geq 1 - \frac{D_j}{\varepsilon^2 M_j^2} = 1 - \frac{q_j}{\varepsilon^2 m p_j}.$$

Note that $\underline{\mho}$ means that there exists a more that $\varepsilon$ divergence from the main value at least in one of the coordinates. And $\overline{\neg\mho}$ denotes the event wherein simultaneously in all coordinates deviations are less than $\varepsilon$. Concluding, -- our aim is to find the conditions when $P\left(\underline{\mho}\right)$ is small and/or when $P\left(\overline{\neg\mho}\right)$ is large (which is the same in our case of $\mathfrak{M}_{\mathrm{pq}}$).

Start with $P\left(\underline{\mho}\right) \leq \sum_{j=1}^{n} P\left(\mho_j\right)$. Substituting inequalities $P\left(\mho_j\right) \leq \frac{q_j}{\varepsilon^2 m p_j}$ into this formula we receive

$$P\left(\underline{\mho}\right) \leq \frac{1}{\varepsilon^2 m}\left(\frac{q_1}{p_1} + \frac{q_2}{p_2} + \cdots + \frac{q_n}{p_n}\right) \leq \frac{n}{\varepsilon^2 m}\frac{\max\limits_{1\leq i\leq n} q_i}{1 - \max\limits_{1\leq i\leq n} q_i}.$$

To get an applicable result let us suppose that $\frac{n}{\varepsilon^2 \cdot m} \to 0$ asymptotically and that all factors $\frac{q_i}{p_i}$ are limited. In these conditions we receive that in the model $\mathfrak{M}_{\mathrm{pq}}$, having a series of $m$ trials, the probability of a sensitive deviation from the mean value, -- at least in one of the coordinates tends to $0$.

We may also use the formula $P\left(\underline{\mho}\right) = 1 - \prod_{j=1}^{n} P\left(\neg\mho_j\right))$ complemented with the $P\left(\neg\mho_j\right) \geq 1 - \frac{q_j}{\varepsilon^2 m p_j}$. To get an applicable postulation from this, it is to require that

$$\lambda = \prod_{j=1}^{n} (1 - \frac{q_j}{\varepsilon^2 m p_j}) \to 1.$$

$$\lambda \geq \prod_{j=1}^{n} \left(1 - \frac{\max\limits_{1 \leq i \leq n} q_i}{\varepsilon^2 m \left(1 - \max\limits_{1 \leq i \leq n} q_i\right)}\right) = \left(1 - \frac{\max\limits_{1 \leq i \leq n} q_i}{\varepsilon^2 m \left(1 - \max\limits_{1 \leq i \leq n} q_i\right)}\right)^n.$$

Apply the following formula: if $0 \leq x \leq 1/2$ and $0 \leq y$, then $\exp(-x(1-x)y) \leq (1-x)^y$. We get

$$\lambda \geq \exp\left(-\frac{n \cdot \max\limits_{1 \leq i \leq n} q_i}{\varepsilon^2 m \left(1 - \max\limits_{1 \leq i \leq n} q_i\right)} \left(1 - \frac{\max\limits_{1 \leq i \leq n} q_i}{\varepsilon^2 m \left(1 - \max\limits_{1 \leq i \leq n} q_i\right)}\right)\right).$$

We arrived to the same condition. To get an interpretable result it is to suppose that $\frac{n}{\varepsilon^{2 \cdot} m} \to 0$ asymptotically, and that all factors $\frac{q_i}{p_i}$ are limited. In these conditions we receive that in the model $\mathfrak{M}_{\mathrm{pq}}$, having a series of $m$ trials, the probability of a sensitive deviation from the mean value in at least in one of the coordinates tends to $0$.

Recall out main target. Our interest is in a situation when column weights of an $\mathfrak{M}_{\mathrm{pq}}$ random matrix are close to the given $s_1, s_2, \ldots, s_n$, and the rows of the matrix are all different. Ideally, $n \geq$logm is "satisfactory" for the row difference. And $\frac{q_i}{p_i}$ limited is also an acceptable condition (but not necessary), because of this is the case when each column participates in row differentiation.

This is our result for the point $Z1$. The domain described by the above intervals is a rectangular area in the space of all sum vectors space $\Xi_m^n$ and the achieved property insists that there exist a proper random sum vector that belongs to the indicated rectangular area. Setting $s_1, s_2, \ldots, s_n$ arbitrarily, we receive corresponding rectangular area of different size and probability (it can be also empty). Unless attractive, the property in this form is not yet useful, because of we do not know if the rows of random matrix that are different in this case.

The strategy at this point is:

$\alpha$: $n$ independent coordinates run in $m$-dimensional unit cubes each. If to consider the most transparent case $p = \frac{1}{2}$, then $m$-column-evaluations are equally probable with probability $\frac{1}{2^m}$. If $s = m/2$, then there are $\sim 2^m/\sqrt{m}$ evaluations with this $s$ and the concluding probability is $1/\sqrt{m}$ in one direction, and, -- $m^{-n/2}$ in integration by the $n$ coordinates. This is a small value, but still positive, that shows the existence of $s$-weighted vectors. All weights are possible but probabilities are different. And $s$ is the central term of this distribution.

$\beta$: The situation with low probabilities can be softened a bit. The way is in considering the equivalency classes to the $s$ vectors [12-29]. For simplicity consider the n-cube when m is odd. This gives a $2^n$ multiplier (the size of the equivalency class) to the probability that now becomes $m^{1-n/2}$. Here we suppose (by bisection) that that the most acceptable value is close to the $n = \log_2 m$. This is a higher probability but the difference is not sensitive. The next step forward is:

$\gamma$: For each coordinate consider an interval of length $\sqrt{m}$. In the composed rectangular area there will be $m^{n/2}$ points and this gives a constant probability to the considered event. But this may only speak about the existence of a point nearby the vector $s$, in a rectangular area.

The further idea is to find a type of independency between the events related to the weights $s$, and the events of the row-difference. It is also to be able to apply the part of proof on row-difference to the parts of distribution by the weights, or to its central term, which we adopt to be the compound weight $s$.

### *K.* The row differences model.

This point deals with the model $\mathfrak{M}_{\mathrm{pq}}$ considering random matrices $R_{mn}$ obtained by $\mathfrak{M}_{\mathrm{pq}}$, evaluating probability of matrices under the constraint of having no repeated rows. For an arbitrary matrix $R_{mn}$ we generate a correlated with it matrix $D_{C_m^2 n}$, that consists of all comparisons of pairs of rows of $R_{mn}$ in the following way [6]. $D_{C_m^2 n}$ consists of $m-1$ separate parts, $n$-column sub-matrices, which are concatenated vertically. First sub-matrix has $m-1$ rows that represent coordinate wise $mod2$ summations of the first row of $R_{mn}$ with the reminding rows $2,3,\dots,m$. Denote this sub-matrix by $D_{(m-1)\,n}$. The next sub-matrix $D_{(m-2)\,n}$ is composed by $m-2$ rows generated from $D_{(m-1)\,n}$ in the same way (first row with other rows). The last group $(m-1)$ will be a 1-row matrix, $D_{(m-(m-1))n} = D_{1n}$.

$R_{mn}$ and $D_{C_m^2 n}$ are straightly related to each other by the following important properties. Rows $r \epsilon D_{C_m^2 n}$ correspond to pairs of rows of the matrix $R_{mn}$ in a way that if $s$ is the number of sub-matrix the row $r$ belongs to, and $t$ is the sequential number of this row in $D_{(m-s)\, n}$, then $r$ have the property:

- The "1" value in a coordinate of $r$ corresponds to the "difference" in the row pair $(s, t)$ of $R_{mn}$ in that coordinate, and
- The norm of $r$ represents the Hamming distance of rows $s$ and $t$.

These notes may have one more important interpretation:

- $R_{mn}$ consists of all different pairs of rows if and only if $D_{C_m^2 n}$ does not include the row with all $0$ coordinates.

The notes are reducing the problems with conditions of "difference" of all pairs of rows - to a specific set cover problems, with cover sub-sets composed from the rows of matrix $D_{C_m^2 n}$. In terms of $R_{mn}$ "differences" must cover all pairs of rows. If to recall that the columns of $R_{mn}$ are weighted, then the "differences" introduced by an individual column compose a bipartite graph, so that in fact the appearing set cover interpretation is very much specific and it works with a cover by a set of $n$ bipartite graphs. Also it is to mention that when $R_{mn}$ is a random matrix constructed directly, then its $D_{C_m^2 n}$ is a secondary construction, it is not given and not visible, so there is no direct way to check if it contains the all $0$ row or not. Two frames are used to estimate the probabilities of matrices that have no repeated rows.

- ***Different Coordinates.*** Firstly, we prove that in the considered random generation columns of $D_{C_m^2 n}$ and its sub-matrices homogeneously appear with high weights (that represent row differences). Then we use the greedy estimation of the columns that are able to cover the rows of $D_{C_m^2 n}$ – in this way they cover all pairs of initial rows and the rows appear different.
- ***Different Vectors.*** In a second approach we study the probabilities of pairs of rows to be different. Extending this property to all pairs we get a lower estimate of the probability that $m$ random rows are all different. This is valid/acceptable for some constraints over the $m$ and $n$. And of course we follow with combining the postulations of this section with the ones about the column weights to get the proper estimations for the discrete tomography problem.

***Different Coordinates***

Next random variable of our consideration is the number of different coordinates in the pairs of rows. The expected number of different coordinates in the pairs of rows in the one-column model (having only one coordinate) can be calculated as:

$$\sum_{t=0}^{m} t(m-t)C_m^t p_j^t\, q_j^{m-t} = \sum_{t=1}^{m-1} t(m-t)C_m^t p_j^t\, q_j^{m-t} = m(m-1)p_j q_j \sum_{t=0}^{m-2} C_{m-2}^t p_j^{t-1}\, q_j^{m-2-t}$$
$$= m(m-1)p_j q_j.$$

Probabilities of random difference of coordinates are computed and estimated in a regular way, and their use is tightly correlated to the matrix $D_{C_m^2 n}$. The idea here is to follow the property of $D_{C_m^2 n}$ to have 1's in rows having large values of the coordinate differences. Details of this part partially repeat the above narration and preferred to be a subject of a separate publication.

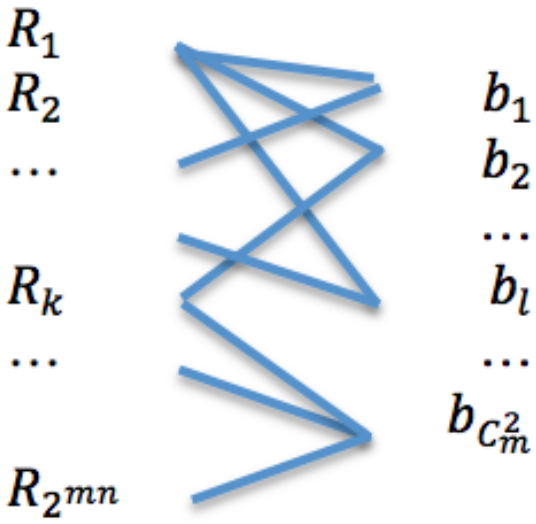
***Different Vectors***

Let us consider the random variable $\Delta_{mn}$, -- the number of different pairs of rows in the matrices at the model $\mathfrak{M}_{\mathrm{pq}}$.

Let $R_1, \dots, R_k, \dots, R_{2^{mn}}$ are all $m \times n$ matrices that may appear randomly at the model $\mathfrak{M}_{\mathrm{pq}}$. Let $p\ (\mathrm{R}_k)$ is the probability of $\mathrm{R}_k$, and let $\Delta_k$ is the number of "different" pairs of rows in $\mathrm{R}_k$. Then the main value of the number of "different" pairs of rows at the random outcome of $\mathfrak{M}_{\mathrm{pq}}$ can be presented as:

$$M(\Delta_{mn}) = \sum_{\mathrm{R}_k \in \mathfrak{M}_{\mathrm{pq}}} p(\mathrm{R}_k) \cdot \Delta_k.$$

It is imperceptible how $p(\mathrm{R}_k)$ and $\Delta_k$ can be brought to a concise computable form. To simplify the formula we consider the following standard [5] "bipartite" scheme:




Right side vertices $c_l$ represent all pairs of rows in the $m \times n$ matrices, listed in some fixed order. Edges, in this bipartite graph scheme connect $R_k$ and $b_l$ iff the pair $b_l$ in $R_k$ is "different" (consists of different rows). Vertex degrees at all $b_l$ constantly are equal to $2^n(2^n - 1)$. But edges are weighted, and the weight of link between $R_k$ and $b_l$ represents the probability that $b_l$ as a "different" pair appears in $R_k$.

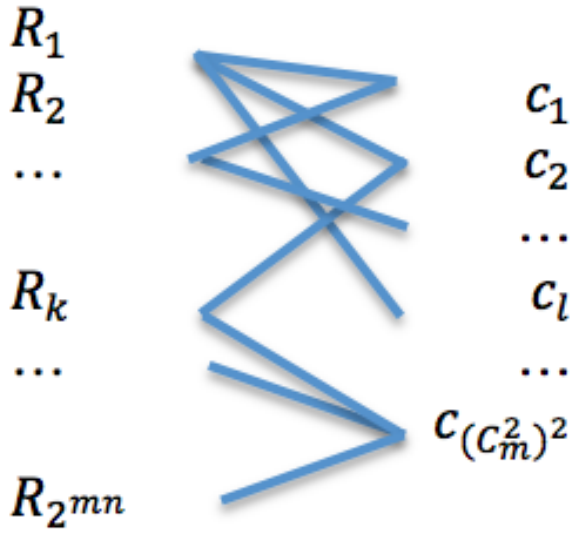
Consider an arbitrary pair of random rows. The probability that for a particular $j$-th coordinates on the considered pair of rows are identical is evidently $p_j^2 + q_j^2$. Due to $1 = p_j + q_j$ it is true that $p_j^2 + q_j^2 = 1 - 2p_jq_j$ so that this forms will be equivalently exchanged in need in our narration. The probability that the entire rows are identical (all coordinates) equals to $\prod_{j=1}^{n}(1 - 2p_jq_j)$ and the complementary probability that this rows are different will be some $\alpha = 1 - \prod_{j=1}^{n}(1 - 2p_jq_j)$ (for example when $p_j = q_j = 1/2$ then we receive the well known $\alpha = 1 - \frac{1}{2^n}$). We obtained that the probability that arbitrary $b_l$ to be "different" is equal to $1 - \prod_{j=1}^{n}(1 - 2p_jq_j)$ and finally, for the average number of "different" pairs of rows we obtain the following concise formula:

$$M(\Delta_{mn}) = \sum_{\mathrm{R}_k \in \mathfrak{M}_{\mathrm{pq}}} p(\mathrm{R}_k) \cdot \Delta_k = C_m^2 \left(1 - \prod_{j=1}^{n}(1 - 2p_jq_j)\right).$$

In a similar way we may obtain the formula for the variance of $\Delta_{mn}$. By the definition

$$D(\Delta_{mn}) = \sum_{\mathrm{R}_k \in \mathfrak{M}_{\mathrm{pq}}} p(\mathrm{R}_k) \cdot {\Delta_k}^2.$$

We apply again the formula $D(\Delta_{mn}) = M(\Delta_{mn}^2) - \left(M(\Delta_{mn})\right)^2$. And consider an analog of the bipartite scheme with random matrices. Matrices $R_1, \dots, R_k, \dots, R_{2^{mn}}$ appear by the scheme $\mathfrak{M}_{\mathrm{pq}}$ with the probabilities $p(\mathrm{R}_k)$.




Right part vertices $c_l$ correspond to all pairs of pairs of vertices of matrices. First to note is the formula

$$M(\Delta_{mn}^2) = \sum_{c_{l'}, c_{l''}} p(c_{l'}, c_{l''}).$$

Split the set of all pairs $b_{l'}, b_{l''}$ into the classes:

- At first – consider the class of the pairs (pairs of pairs of rows) of the type $b_l, b_l$. Number of this pairs is $C_m^2$ and they all have the probability of "difference" $1-\prod_{j=1}^{n}\left(1-2p_jq_j\right)$.
- The largest group consists of pairs $b_{l'}$ and $b_{l''}$ that have no common row. Number of this pairs is $C_m^2 C_{m-2}^2$ and they have probability of difference: $1-\prod_{j=1}^{n}\left(1-2p_jq_j\right)^2$.
- The last group of pairs includes the 2 pairs of rows with one common row. Number of these fragments is equal to $C_m^2 2(m-2) = m\,(m-1)(m-2)$. And the probability of difference is: $1-2\prod_{j=1}^{n}\left(1-2p_jq_j\right)+\prod_{j=1}^{n}\left({p_j}^3+{q_j}^3\right)$.

Combining into the general formula we obtain

$$M(\Delta_{mn}^2) = C_m^2\left(1-\prod_{j=1}^{n}(1-2p_jq_j)\right)+C_m^2C_{m-2}^2\left(1-\prod_{j=1}^{n}(1-2p_jq_j)^2\right) \quad (3.2)$$
$$+C_m^2 2(m-2)\left(1-2\prod_{j=1}^{n}(1-2p_jq_j)+\prod_{j=1}^{n}({p_j}^3+{q_j}^3)\right)$$

We aim to apply the Chebyshev inequality. To obtain a "for almost all" type result it is to prove that with $m,n\to\infty$ it is true that $\frac{D}{\varepsilon^2M^2}\to 0$. In this case we check if $\frac{D}{M^2}\to 0$ and then it is simple to take a proper $\varepsilon,\varepsilon\to 0$ and this proves the required "almost all" type result. Consider the proposition $\frac{M(\Delta_{mn}^2)}{(M(\Delta_{mn}))^2}\to 1$ that is equivalent to $\frac{D}{M^2}\to 0$. For $p_j=q_j=1/2$ the sub-formula $\prod_{j=1}^{n}(1-2p_jq_j)=2^{-n}$ so that it tends to 0. In our case of arbitrary probabilities $p_j$, sub-formula $\prod_{j=1}^{n}(1-2p_jq_j)$ exceeds $2^{-n}$ but it is acceptable to seem that $\prod_{j=1}^{n}(1-2p_jq_j)\to 0$. In this case it is easy to be convinced that the first and the last summands of $M(\Delta_{mn}^2)$ are $o\,(M^2)$. For the analysis of the midterm, let us note that

$$\frac{1-\prod_{j=1}^{n}(1-2p_jq_j)^2}{\left(1-\prod_{j=1}^{n}(1-2p_jq_j)\right)^2}=\frac{1+\prod_{j=1}^{n}(1-2p_jq_j)}{1-\prod_{j=1}^{n}(1-2p_jq_j)}=1+\frac{2\prod_{j=1}^{n}(1-2p_jq_j)}{1-\prod_{j=1}^{n}(1-2p_jq_j)}.$$

Having this and $\prod_{j=1}^{n}(1-2p_jq_j)\to 0$ we conclude that the request $\frac{D}{M^2}\to 0$ is valid and that the Chebyshev inequality lets us to obtain the required "almost all" type postulation for the random number of the "different" rows at $\mathfrak{M}_{\mathrm{pq}}$. Denote

$$\chi=\prod_{j=1}^{n}(1-2p_jq_j).$$

**Note 1.**: $M(\Delta_{mn})$ have the form $C_m^2(1-\chi)$ with $\chi\to 0$. Consider the case when $C_m^2\chi\to 0$. As this is the average number, there must be an outcome of trial, -- the matrix $R_k$, so that the offset number of different rows is $\le C_m^2\chi$. As $C_m^2\chi$ becomes $<1$ this means a trivial thing – existence of an $m$-sub-set of the cube. Markov inequality does not help as well.

**Claim 1.**: Now we involve the variance into the game. By the Chebyshev inequality we have

$$P\left(|\Delta_{mn} - M(\Delta_{mn})| > \varepsilon M(\Delta_{mn})\right) \leq \frac{D(\Delta_{mn})}{\varepsilon^2\left(M(\Delta_{mn})\right)^2}. \tag{3.3}$$

Consider the complementary to the $|\Delta_{mn} - M(\Delta_{mn})| \geq \varepsilon M(\Delta_{mn})$ event, and rewrite the inequality (3.3) in the form:

$$P\left((1-\varepsilon)M(\Delta_{mn}) < \Delta_{mn} < (1+\varepsilon)\, M(\Delta_{mn})\right) \geq 1 - D(\Delta_{mn})/\varepsilon^2\left(M(\Delta_{mn})\right)^2$$

and use its extension:

$$P\left((1-\varepsilon)M(\Delta_{mn}) < \Delta_{mn}\right) \geq 1 - \frac{D(\Delta_{mn})}{\varepsilon^2\left(M(\Delta_{mn})\right)^2}. \tag{3.4}$$

In a way similar to the considerations of Note 1. we obtain that in proper selection of $\varepsilon$, and applying reasonable constraints on $\chi$ (this means – constraints on probabilities $p_1, p_2, \dots, p_n$), we may ensure that $(1-\varepsilon)M(\Delta_{mn}) > C_m^2 - 1$. This implies that (3.4) estimates the probability of $\Delta_{mn} = C_m^2$, i.e. the probability that the random $R_{mn}$ will have all-different rows.

**Note 2.** We conclude in 2 steps. First – we further estimate the formula in (3.4). Then we consider the probability estimates of $R_{mn}$ with weights $s_1, s_2, \dots, s_n$. We sum probabilities of these two events. When the sum becomes $> 1$ this implies that the two events are intersecting. Intersection means existence of a random outcome of trial $\mathfrak{M}_{\mathrm{pq}}$ with weights $s_1, s_2, \dots, s_n$ and with all-different rows. Ignoring the probability of this integrative event and just requiring that it be positive, we obtain the statement on existence of constructions with weights $s_1, s_2, \dots, s_n$ and with different rows.

Evaluate the right side formula of (3.4). We intend to add this formula to (2.4) finding out the conditions for this sum to be $> 1$. Then wee simply look for conditions of

$$\prod_{j=1}^{n} C_m^{s_j} p_j^{s_j} q_j^{m-s_j} > \frac{D(\Delta_{mn})}{\varepsilon^2 \left(M(\Delta_{mn})\right)^2}. \tag{3.5}$$

Our analysis is asymptotical, in $n, m \to \infty$. At this point we suppose that $\frac{D}{M^2} \to 0$ and in this condition we choose $\varepsilon$ in a way that $\frac{D}{\varepsilon^2 M^2} \to 0$. We already checked these conditions for $p_j = 1/2$. Now let us consider the right side of (3.5).

Apply $\prod_{j=1}^{n}\left({p_j}^2 + {q_j}^2\right) \geq \prod_{j=1}^{n}\left({p_j}^3 + {q_j}^3\right)$ on the last term of (3.2), and combining the first ad last terms of this formula we obtain that this sum is:

$$\leq C_m^2(2m-3)\left(1 - \prod_{j=1}^{n}\left(1 - 2p_j q_j\right)\right) = a.$$

Compose the following difference with the midterm of (3.2)

$$(C_m^2)^2\left(1 - \prod_{j=1}^{n}\left(1 - 2p_j q_j\right)^2\right) - C_m^2 C_{m-2}^2\left(1 - \prod_{j=1}^{n}\left(1 - 2p_j q_j\right)^2\right)$$

$$= C_m^2(2m-3)\left(1 - \prod_{j=1}^{n}\left(1 - 2p_j q_j\right)^2\right) = b.$$

Additionally, denote

$$C_m^2 C_{m-2}^2\left(1 - \prod_{j=1}^{n}\left(1 - 2p_j q_j\right)^2\right) = c.$$

In (3.2) we want to delete the minor term $a$ keeping only the major term $c$. The objective is in keeping (3.2) increasing or, -- in the same order. The mentioned change is possible due to $a \ll c$. It is also challenging replacing $c$ by the term $(C_m^2)^2\left(1 - \prod_{j=1}^{n}\left(1 - 2p_j q_j\right)^2\right)$ but this is not acceptable due to $a < b$.

This analysis helps to correctly use the (3.5) for particular $p_1, p_2, \ldots, p_n$. The concise estimation of (3.5) can be done for some regular examples of $p_1, p_2, \ldots, p_n$. Two things are to be elaborated: does the probabilistic method bring some knowledge on tomography, and how accurate is this result. To get an

idea on the last point let us consider the example of $p_j = 1/2$ treating the case $s_j = m/2$. Above, for this case we accepted the constraint $m^2 \ll 2^n$. That is, $2\log m \ll n$. It is evident that for $s_j = m/2$ and $\log m \leq n$ the required tomography matrix exist. So if successful the probabilistic method requires al least 2 times more columns to differentiate the rows. The hope is that for nontrivial $p_1, p_2, \ldots, p_n$ this, even in approximation, may bring an additional knowledge about the tomographic property.

*L.* **Sperner families.**

In a short note consider case of Sperner families. Many of the existence issues about the Sperner families are already resolved – the maximal Sperner family, almost all Sperner families, weighted Sperner families (by recursive apply of Kruskal-Katona theorem [60]). To understand the relation between the random sets and the Sperner property consider the bipartite graph with left side, including all $m$-subsets, and with right side, that consists of all pairs of comparable vertices of $E^n$. Compute the average number of comparable pairs of vertices in all Sperner families:

$$\varepsilon = \frac{\sum_{i=0}^{n} C_n^i \left(2^i - 1\right) C_{2^n-2}^{m-2}}{C_{2^n}^m} = \frac{m(m-1)}{2^n(2^n-1)}\left(3^n - 2^n\right)$$

Let $m^2 = o(1.333\ldots^n)$, $\mathrm{n} \longrightarrow \infty$ then $\varepsilon$ is nearly zero value that indicates that there exist a Sperner family of size $m$, or in more precise, that the random subset of this size is a Sperner family.

## Conclusion

Considerations above intend to get an additional knowledge about the row-different matrices of the discrete tomography problem, using the probabilistic theory of combinatorics [1-21]. The objective is reasonable because the pure combinatorial approaches [22-77] are not able at the moment to give an efficient description of the column weighted row-different matrices. The probabilistic method gives knowledge on random subsets, which might be useful as a complementary knowledge about a different object or a situation concerned to discrete tomography.

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# SENSITIVITY ANALYSIS OF A DECISION-MAKING PROBLEM USING THE ANALYTIC HIERARCHY PROCESS

**Nataliya D. Pankratova, Nadezhda I. Nedashkovskaya**

***Abstract***: *The paper deals with the methodology of complex sensitivity analysis of solution given by one of the popular multiple-criteria decision-making methods, namely the Analytic Hierarchy Process. This methodology includes evaluation of sensitivity of hierarchy elements local ranking to changes in an expert pairwise comparison judgments and evaluation of sensitivity of global ranking of decision alternatives to changes in weights of hierarchy elements. The sensitivity analysis is illustrated on a problem of evaluation of renewable energy technologies for an eco-house in Ukraine.*

***Keywords***: *the analytic hierarchy process, uncertainty of expert pairwise comparison judgments, sensitivity analysis, stability intervals, critical expert pairwise comparison judgments, critical elements of a hierarchy.*

***ACM Classification Keywords***: *H.4.2. INFORMATION SYSTEM APPLICATION: type of system strategy*

## Introduction

In the paper one of the multiple-criteria decision-making methods – the analytic hierarchy process (AHP) – is investigated. The AHP results in relative weights or priorities of decision alternatives, is based on a hierarchical model of decision factors, criteria, goals and uses expert judgments of pairwise comparison of elements of a hierarchy as initial information [1, 2]. This method is applied in many areas, such as economy, industry, social sphere, ecology, politics, military science while solving such problems as: choice and evaluation of decision alternatives and decision factors, resource allocation, analysis of benefits-costs-opportunities-risks, forecasting, analytical planning, construction and evaluation of scenarios of development and other [1 - 5].

Expert pairwise comparison judgments contain uncertainty. Therefore the question of reliability of results given by the AHP arises. To evaluate reliability of obtained results it is reasonable to find dependency between results of the AHP and inaccuracies of initial data – expert judgments. In practice a sensitivity analysis of solutions obtained by the AHP, is often carried out using graphical methods, which are proposed by T.L. Saaty and implemented in the decision-support system SuperDecisions [7]. These methods are also implemented in decision-support systems Decision Lens [8], MakeItRational [9] and

LogicalDecisions [10]. In the graphical methods a decision-maker or analyst changes a local weight of some element of a hierarchy and observes changes in global weights of decision alternatives.

The AHP is successfully used while solving different decision-making problems [1–5]. In repetitive problems the graphical methods, which are implemented in the decision-support system SuperDecisions [7], are enough to use. However, a more complete, complex sensitivity analysis has to be done while solving such decision problems as evaluation and choosing of scenarios of development and of decision alternatives on a level of big companies, branches of industry and a country as a whole, resource allocation problems and planning complex target-oriented programs, and also when making decisions concerning innovation development. While solving these problems a complex sensitivity analysis has to be integrated in each stage of decision-making, included in continuous cyclic process of problem solution.

One of the approaches to a complex sensitivity analysis in the AHP is to investigate changes of calculated global ranking of decision alternatives while varying weights of hierarchy elements and changing a hierarchical model structure [11].

Change of global ranking of decision alternatives when adding or removing an alternative, so called rank reversal, was studied in [1, 12, 13]. In these papers it was shown that rank reversal may occur in different aggregation rules of the AHP, namely in the distributive, ideal, multiplicative, max-min rules and in the rule of group consideration of binary preferences of the alternatives. Probabilities of appearance of several types of rank reversals in the aggregation rules were estimated [12, 13]. Thus, the AHP is sensitive to changes of a hierarchical model structure.

In this paper a complex methodology of sensitivity analysis of results obtained by the AHP is proposed. This methodology includes:

- evaluation of sensitivity of a local ranking of decision alternatives to changes in expert pairwise comparison judgments,
- evaluation of sensitivity of a global ranking of decision alternatives to changes in weights of hierarchy elements.

As a result, stability intervals are defined which allow to find so called critical elements of the decision-making problem. These are critical expert pairwise comparison judgments that are sensitive to changes of a local ranking of decision alternatives, and critical hierarchy elements (decision criteria, goals) – elements that are characterized by the least changes of their weights that lead to changes of a global ranking of decision alternatives.

## Problem statement

Let H be an analytic hierarchy with $p+1$ levels. Level $L_0$ of the hierarchy has one element — the main goal of decision-making, the last level $L_p$ contains decision alternatives. Hierarchy levels, that are between $L_0$ and $L_p$, contain possible factors (criteria, goals) that influence the decision. Denote number of elements on a $L_k$-th level as $N_{L_k}$, $L_k \in \lfloor L_0; L_p \rfloor$.

$\hat{A}_r^{L_k L_{k-1}}$ is a pairwise comparison matrix (PCM) of elements of $L_k$-th level in terms of $r$-th element of $L_{k-1}$-th level, $r \in \lfloor 1; N_{L_{k-1}} \rfloor$, constructed on the basis of expert judgments.

$\hat{w}_{lr}^{L_k L_{k-1}}$ is a local weight of $l$-th element of $L_k$-th level in terms of $r$-th element of $L_{k-1}$-th level, $l \in \lfloor 1; N_{L_k} \rfloor$, $r \in \lfloor 1; N_{L_{k-1}} \rfloor$. Weight vector $\hat{w}_r^{L_k L_{k-1}} = \left\{ \hat{w}_{lr}^{L_k L_{k-1}} \mid l \in \left[ 1; N_{L_k} \right] \right\}$ calculates on the basis of the PCM $\hat{A}_r^{L_k L_{k-1}}$ using the eigenvector method, the row geometric mean method and others [1, 2].

$\hat{w}_l^{L_k}$ is a global weight of $l$-th element of $L_k$-th level, $l \in \lfloor 1; N_{L_k} \rfloor$. In the analytic hierarchy process vector of global weights can be calculated using the distributive or multiplicative aggregation rules [1, 2]. Vector of global weights of decision alternatives $\hat{w}^{L_p} = \left\{ \hat{w}_i^{L_p} \mid i \in \left[ 1; N_{L_p} \right] \right\}$ is a result of the analytic hierarchy process.

It is necessary to provide a complex sensitivity analysis of rankings obtained using the AHP to inaccuracy and subjectivity of expert judgments:

- to evaluate sensitivity of a local ranking of decision alternatives to changes in expert pairwise comparison judgments (elements of a PCM $\hat{A}_r^{L_k L_{k-1}}$);
- to evaluate sensitivity of a global ranking of decision alternatives to changes in weights of hierarchy elements;
- to find critical and stable expert pairwise comparison judgments;
- to find critical and stable elements of $L_k$-th hierarchy level, $L_k \in \lfloor L_1; L_{p-1} \rfloor$.

## The problem solving. Sensitivity analysis of a local ranking of decision alternatives when changing expert pairwise comparison judgments

Let us consider calculation of local weights of hierarchy elements, for example, decision alternatives $a_1, a_2, ..., a_n$ in terms of their common feature (an element of a parent hierarchy level). Suppose $D = \{(d_{ij}) \mid i, j = 1, ..., n\}$ is a PCM constructed on the basis of expert pairwise comparison judgments.

Using the Row Geometric Mean Method (RGMM), nonnormalized local weights $v_1, v_2, ..., v_n$ of decision alternatives are calculated as follows:

$$v_i = (\prod_{l=1}^{n} d_{il})^{1/n},\ i = 1, ..., n. \quad (1)$$

We are interested in how much ranking of decision alternatives, built on the basis of calculated local weights is insensitive to changes of expert judgments (PCM elements). Let us investigate two cases:

1. whether the best alternative remains unchanged,
2. whether an overall ranking of alternatives remains unchanged.

**A stability interval of expert pairwise comparison judgments** concerning change of ranking of decision alternatives is an interval within the bounds of which an expert judgment may be changed so that a local ranking of alternatives remains unchanged.

Denote $[\underline{d_{ij}}, \overline{d_{ij}}]$ a stability interval for an expert judgment $d_{ij}$.

Without loss of generality suppose that decision alternatives are renumbered in order of importance decreasing, that is the ranking of alternatives is

$$a_1 \succ a_2 \succ ... \succ a_n, \quad (2)$$

where $a_1$ and $a_n$ are the best (the most important) and the worst decision alternatives, respectively.

In terms of weights (2) means that $v_i > v_j$ for $i < j$.

Let us find for each expert judgment a stability interval concerning change of alternatives ranking, when the RGMM is used for weights calculation.

**A case when the best alternative remains unchanged**. At first consider a case when change of a PCM element does not lead to change of the best decision alternative $a_1$.

Suppose the PCM element $d_{1j}$, $j \neq 1$ is changed within the bounds of interval $[\underline{d_{1j}}, \overline{d_{1j}}]$. Then in accordance with the RGMM (1), weights of decision alternatives $a_1$ and $a_j$ are changed. Denote these new weights $v_1' = [\underline{v_1}, \overline{v_1}]$ and $v_j' = [\underline{v_j}, \overline{v_j}]$, where

$$\underline{v_1} = (\frac{d_{1j}}{\underline{d_{1j}}})^{1/n} \cdot v_1 \text{ and } \overline{v_1} = (\frac{\overline{d_{1j}}}{d_{1j}})^{1/n} \cdot v_1, \tag{3}$$

$$\underline{v_j} = (\frac{d_{j1}}{\underline{d_{j1}}})^{1/n} \cdot v_j = (\frac{d_{1j}}{\underline{d_{1j}}})^{1/n} \cdot v_j \text{ and } \overline{v_j} = (\frac{\overline{d_{j1}}}{d_{j1}})^{1/n} \cdot v_j = (\frac{d_{1j}}{\underline{d_{1j}}})^{1/n} \cdot v_j. \tag{4}$$

We want to find an interval $[\underline{d_{1j}}, \overline{d_{1j}}]$, such that the best decision alternative does not change, i.e. $v_1' > v_j'$, $j \neq 1$ and $v_1' > v_k'$, $k \neq j \neq 1$. This is equivalent to implementation of the following two conditions:

$$\underline{v_1} > \overline{v_j}, \ \underline{v_1} > v_k, \tag{5}$$

where $k \neq j \neq 1$.

Substitute expressions (3) and (4) in (5) and find the following constraints for the left bound of a stability interval:

$$\underline{d_{1j}} > d_{1j} \cdot (\frac{v_j}{v_1})^{n/2} \text{ and } \underline{d_{1j}} > d_{1j} \cdot (\frac{v_k}{v_1})^n \tag{6}$$

$$\text{or } \underline{d_{1j}} > d_{1j} \cdot \left( \frac{\prod_{l=1}^{n} d_{jl}}{\prod_{l=1}^{n} d_{1l}} \right)^{1/2} \text{ and } \underline{d_{1j}} > d_{1j} \cdot \frac{\prod_{l=1}^{n} d_{kl}}{\prod_{l=1}^{n} d_{1l}}, \text{ where } k \neq j \neq 1.$$

There are no constrains on the right bound of a stability interval, so let us assign it the maximum permissible value, namely, the largest value in the Saaty scale used by an expert when making an assessment: $\overline{d_{1j}} = 9$. Comparing right parts of inequality (6) one can formulate the following statement for a stability interval calculation.

**Statement 1:** A stability interval $[\underline{d_{1j}}, \overline{d_{1j}}]$ for an expert judgment $d_{1j}$, $j \neq 1$, such that the best decision alternative $a_1$ remains unchanged, when the RGMM is used for weights calculation, satisfies the conditions:

$\underline{d_{1j}} > d_{1j} \cdot (\frac{v_j}{v_1})^{n/2}$, if $v_1 \cdot v_j \geq (v_k)^2$, $k \neq j \neq 1$,

$\underline{d_{1j}} > d_{1j} \cdot (\frac{v_k}{v_1})^{n}$, if $v_1 \cdot v_j < (v_k)^2$, $k \neq j \neq 1$,

$\overline{d_{1j}} = 9$.

In practice the $\underline{d_{1j}}$ value is the nearest value of the Saaty scale that satisfies inequalities of the Statement 1.

Consider a case when any PCM element $d_{kj}$, $k \neq j \neq 1$ is changed. It is necessary to find a stability interval $[\underline{d_{kj}}, \overline{d_{kj}}]$ for this element. According to the RGMM (1), change of $d_{kj}$ leads to change of weights of decision alternatives $a_k$ and $a_j$. Denote these new weights $v'_k = [\underline{v_k}, \overline{v_k}]$, $v'_j = [\underline{v_j}, \overline{v_j}]$ and calculate their left and right bounds using the RGMM:

$$\underline{v_k} = (\frac{d_{kj}}{\overline{d_{kj}}})^{1/n} \cdot v_k \text{ and } \overline{v_k} = (\frac{\overline{d_{kj}}}{d_{kj}})^{1/n} \cdot v_k, \tag{7}$$

$$\underline{v_j} = (\frac{d_{jk}}{\overline{d_{jk}}})^{1/n} \cdot v_j = (\frac{d_{kj}}{\overline{d_{kj}}})^{1/n} \cdot v_j \text{ and } \overline{v_j} = (\frac{\overline{d_{jk}}}{d_{jk}})^{1/n} \cdot v_j = (\frac{d_{kj}}{\underline{d_{kj}}})^{1/n} \cdot v_j. \tag{8}$$

The best decision alternative does not change if inequalities $v'_1 > v'_j$, $j \neq 1$ and $v'_1 > v'_k$, $k \neq j \neq 1$ are satisfied. This is equivalent to implementation of the following conditions:

$$v_1 > \overline{v_j} \text{ and } v_1 > \overline{v_k}, \tag{9}$$

where $k \neq j \neq 1$.

Substitute expressions (7) and (8) in (9) and find the following constraints for the left and right bounds of a stability interval:

$$\underline{d_{kj}} > d_{kj} \cdot (\frac{v_j}{v_1})^n \text{ and } \overline{d_{kj}} < d_{kj} \cdot (\frac{v_1}{v_k})^n \quad (10)$$

or $\underline{d_{kj}} > d_{kj} \cdot \prod_{l=1}^{n} d_{jl} / \prod_{l=1}^{n} d_{1l}$ and $\overline{d_{kj}} < d_{kj} \cdot \prod_{l=1}^{n} d_{1l} / \prod_{l=1}^{n} d_{kl}$, where $k \neq j \neq 1$.

**Statement 2:** A stability interval $[\underline{d_{kj}}, \overline{d_{kj}}]$ for an expert judgment $d_{kj}$, $k \neq j \neq 1$, such that the best decision alternative $a_1$ remains unchanged, when the RGMM is used for weights calculation, satisfies the conditions:

$$\underline{d_{kj}} > d_{kj} \cdot (\frac{v_j}{v_1})^n, \ \overline{d_{kj}} < d_{kj} \cdot (\frac{v_1}{v_k})^n.$$

It should be noted that PCM elements take values from the Saaty scale (namely, values from the set $\{1/9,...,9\}$). Therefore in practice values $\underline{d_{kj}}$ and $\overline{d_{kj}}$ are the nearest values of this scale that satisfy the corresponding inequalities of the Statement 2.

**A case when an overall ranking of alternatives remains unchanged**. Now consider a case when change of a PCM element leads to steady overall ranking (2) of decision alternatives. Similarly to the previous case, it is necessary to analyze separately change of an element $d_{1j}$, $j \neq 1$ and an element $d_{kj}$, $k \neq j \neq 1$.

Consider change of $d_{1j}$, $j \neq 1$ within an interval $[\underline{d_{1j}}, \overline{d_{1j}}]$. To save the overall ranking (2), it is necessary to impose the following additional constraints besides mentioned above constraints $v_1' > v_j'$, $j \neq 1$ and $v_1' > v_k'$, $k \neq j \neq 1$:

$$v_j' > v_k' \text{ when } j < k \quad (11)$$

$$v_k' > v_j' \text{ when } k < j \quad (12)$$

where $k \neq j \neq 1$.

Inequalities (11) and (12) are equivalent to the following:

$$\underline{v_j} > v_k \text{ when } j < k$$

$$v_k > \overline{v_j} \text{ when } k < j$$

or using (3) and (4):

$$\overline{d_{1j}} < d_{1j} \cdot (\frac{v_j}{v_k})^n \text{ when } j < k \quad (13)$$

$$\underline{d_{1j}} > d_{1j} \cdot (\frac{v_j}{v_k})^n \text{ when } k < j \quad (14)$$

Taking into account conditions $v_1' > v_j'$, $j \neq 1$ and $v_1' > v_k'$, $k \neq j \neq 1$, that lead to (6), we obtain statement for a stability interval calculation.

**Statement 3:** A stability interval $[\underline{d_{1j}}, \overline{d_{1j}}]$ for an expert judgment $d_{1j}$, $j \neq 1$, such that the overall ranking $a_1 \succ a_2 \succ ... \succ a_n$ of decision alternatives remains unchanged, when the RGMM is used for weights calculation, satisfies the conditions:

3.1. $\underline{d_{1j}} > d_{1j} \cdot (\frac{v_j}{v_1})^{n/2}$, if $v_1 \cdot v_j \geq (v_k)^2, j \neq 1, j < k$

$\underline{d_{1j}} > d_{1j} \cdot (\frac{v_k}{v_1})^n$, if $v_1 \cdot v_j < (v_k)^2, j \neq 1, j < k$

$\overline{d_{1j}} < d_{1j} \cdot (\frac{v_j}{v_k})^n$, if $j \neq 1$, $j < k$.

3.2. $\underline{d_{1j}} > d_{1j} \cdot (\frac{v_j}{v_k})^n$, if $v_1 \cdot v_j \geq (v_k)^2, j \neq 1, j > k$

$\underline{d_{1j}} > d_{1j} \cdot (\frac{v_k}{v_1})^n$, if $v_1 \cdot v_j < (v_k)^2, j \neq 1, j > k$

$\overline{d_{1j}} = 9$, if $j \neq 1$, $j > k$.

Consider change of an element $d_{kj}$, $k \neq j \neq 1$ within an interval $[\underline{d_{kj}}, \overline{d_{kj}}]$. Similarly to previous case, to save the overall ranking (2), additional conditions (11) and (12) are added, which in this case take a form:

$$\underline{v_j} > \overline{v_k} \text{ when } j < k$$

$$\underline{v_k} > \overline{v_j} \text{ when } k < j$$

Using (7) and (8) we get:

$$\overline{d_{kj}} < d_{kj} \cdot (\frac{v_j}{v_k})^{n/2} \text{ when } j < k$$

$$\underline{d_{kj}} > d_{kj} \cdot (\frac{v_j}{v_k})^{n/2} \text{ when } k < j$$

Taking into account $v_1' > v_j'$, $j \neq 1$ and $v_1' > v_k'$, $k \neq j \neq 1$, which in this case lead to constraints (10), we obtain statement for calculation of a stability interval.

**Statement 4:** A stability interval $[\underline{d_{kj}}, \overline{d_{kj}}]$ for an expert judgment $d_{kj}$, $k \neq j \neq 1$, such that the overall ranking $a_1 \succ a_2 \succ ... \succ a_n$ of decision alternatives remains unchanged, when the RGMM is used for weights calculation, satisfies the inequalities:

4.1. $\underline{d_{kj}} > d_{kj} \cdot (\frac{v_j}{v_1})^n$ if $j \neq 1$, $j < k$

$\overline{d_{kj}} < d_{kj} \cdot (\frac{v_j}{v_k})^{n/2}$ if $(v_1)^2 \geq v_j v_k$ , $j \neq 1, j < k$

$\overline{d_{kj}} < d_{kj} \cdot (\frac{v_1}{v_k})^n$ if $(v_1)^2 < v_j v_k$ , $j \neq 1, j < k$

4.2. $\underline{d_{kj}} > d_{kj} \cdot (\frac{v_j}{v_k})^{n/2}$ if $(v_1)^2 \geq v_j v_k$ , $j > k, k \neq 1$

$\underline{d_{kj}} > d_{kj} \cdot (\frac{v_j}{v_1})^n$ if $(v_1)^2 < v_j v_k$ , $j > k, k \neq 1$

$\overline{d_{kj}} < d_{kj} \cdot (\frac{v_1}{v_k})^n$ if $j > k$, $k \neq 1$.

As was mentioned above, in practice the bounds of stability intervals $[\underline{d_{1j}}, \overline{d_{1j}}]$ and $[\underline{d_{kj}}, \overline{d_{kj}}]$ in the Statements 3 and 4 are the nearest values of the Saaty scale that satisfy the corresponding inequalities of the statements.

## Sensitivity analysis of a global ranking of decision alternatives when changing weights of elements of an hierarchy

Let us consider a multiple-criteria problem of calculation of global weights of decision alternatives on the basis on a hierarchy of criteria. A method of sensitivity analysis described in this section is a generalization of the method proposed in [6]. Without loss of generality suppose that decision alternatives are renumbered such that

$$\hat{w}_i^{L_p} \geq \hat{w}_j^{L_p},\ i \in [1; N_{L_p}],\ j \in [1; N_{L_p}] \text{ when } i < j.$$

Denote $\Delta_{i,j,l}^{L_k}$, $i \in [1; N_{L_p}]$, $j \in [1; N_{L_p}]$, $l \in [1; N_{L_k}]$, $L_k \in [L_1; L_{p-1}]$ value of an **absolute change of weight** $\hat{w}_l^{L_k}$ that leads to change of global ranking between $i$-th and $j$-th elements of $L_p$-th level ($i$-th and $j$-th decision alternatives). That is, a new weight of $l$-th element of $L_k$-th level equals $\hat{w}'^{L_k}_l = \hat{w}_l^{L_k} - \Delta_{i,j,l}^{L_k}$, $\hat{w}'^{L_k}_l > 0$, and $\hat{w}'^{L_p}_i < \hat{w}'^{L_p}_j$ holds when $i < j$, where $\hat{w}'^{L_p}_i$ is a new global weight of $i$-th element of $L_p$-th level.

Denote $\delta_{i,j,l}^{L_k}$, $i \in [1; N_{L_p}]$, $j \in [1; N_{L_p}]$, $l \in [1; N_{L_k}]$, $L_k \in [L_1; L_{p-1}]$ value of a **relative change of weight** $\hat{w}_l^{L_k}$ that leads to change of global ranking between $i$-th and $j$-th elements of $L_p$-th level ($i$-th and $j$-th decision alternatives). That is, a new weight equals $\hat{w}'^{L_k}_l = \hat{w}_l^{L_k} - \frac{\delta_{i,j,l}^{L_k} \hat{w}_l^{L_k}}{100}$, $\hat{w}'^{L_k}_l > 0$. The values of absolute and relative changes of weight $\hat{w}_l^{L_k}$ of $l$-th element in $L_k$-th level are in the following relation: $\delta_{i,j,l}^{L_k} = \frac{\Delta_{i,j,l}^{L_k}}{\hat{w}_l^{L_k}} 100\,\%$.

$l$-th element of $L_k$-th level **is stable** if any permissible changes of its weight do not lead to changes of global rank of any decision alternative.

**Degree of criticality** $C_l^{L_k}$ of $l$-th element of $L_k$-th level is a value of the least relative change of its weight $\hat{w}_l^{L_k}$ that leads to change of global ranking of decision alternatives: $C_l^{L_k} = \min\limits_{\substack{i,j\in[1;N_{L_p}],\\ i<j}} \left\{ | \delta_{i,j,l}^{L_k} | \right\}$.

**Sensitivity** $S_l^{L_k}$ of $l$-th element of $L_k$-th level is a reciprocal value to the degree of criticality of this element: $S_l^{L_k} = \dfrac{1}{C_l^{L_k}}$. $S_l^{L_k}$ is assigned a zero value if $l$-th element of $L_k$-th level is stable.

Less values of the degree of criticality $C_l^{L_k}$ mean that it is easier to change a ranking of decision alternatives. So less values of the degree of criticality $C_l^{L_k}$ indicate that less change of weight $\hat{w}_l^{L_k}$ is sufficient for a change of ranking of decision alternatives. Therefore "the easier" change of ranking of decision alternatives results in larger value of sensitivity $S_l^{L_k}$ of $l$-th element of $L_k$-th level.

**Critical** element of $L_k$-th level is an element of $L_k$-th level which has the least value $\left|\delta_{i,j,l}^{L_k}\right|$, that is $l_{crit}$-th element of $L_k$-th level is critical if $| \delta_{i,j,l_{crit}}^{L_k} | = \min\limits_{l\in[1;N_{L_k}]} \left\{ \delta_{i,j,l}^{L_k} | \right\}$, $i \in [1; N_{L_p}]$, $j \in [1; N_{L_p}]$.

Values of relative change $\delta_{i,j,l}^{L_k}$ when the distributive and multiplicative aggregation rules are used for calculation of global weights of hierarchy elements can be found using the following statements 7 and 8.

**Statement 7:** A value $\delta_{i,j,l}^{L_k}$ of relative change of weight $\hat{w}_l^{L_k}$ that is necessary for a change of global ranking between $i$-th and $j$-th elements of $L_p$-th level, $i \in [1; N_{L_p}]$, $j \in [1; N_{L_p}]$, $l \in [1; N_{L_k}]$, $L_k \in [L_1; L_{p-1}]$, when $i < j$ and the distributive aggregation rule is used for calculation of global weights satisfies the inequality [11]:

$$\delta_{i,j,l}^{L_k} < \delta_{i,j,l}^{L_k porog}, \text{ if } \hat{w}_{jl}^{L_p L_k} > \hat{w}_{il}^{L_p L_k},$$

$$\delta_{i,j,l}^{L_k} > \delta_{i,j,l}^{L_k porog}, \text{ if } \hat{w}_{jl}^{L_p L_k} < \hat{w}_{il}^{L_p L_k},$$

where the threshold value $\delta_{i,j,l}^{L_k porog}$ of $\delta_{i,j,l}^{L_k}$ is calculated as follows:

$$\delta_{i,j,l}^{L_k porog} = \Delta_{i,j,l}^{L_k porog} \frac{100}{\hat{w}_l^{L_k}}(\%), \tag{15}$$

$$\Delta_{i,j,l}^{L_k porog} = \frac{\hat{w}_j^{L_p} - \hat{w}_i^{L_p}}{\hat{w}_{jl}^{L_p L_k} - \hat{w}_{il}^{L_p L_k}} \tag{16}$$

under conditions:

1) $\hat{w}_i^{L_p} \geq \hat{w}_j^{L_p}$ when $i < j$;

2) $\hat{w}_l^{L_k} > \Delta_{i,j,l}^{L_k porog}$ (that is equivalent to $\delta_{i,j,l}^{L_k porog} < 100\%$).

**Corollary:** $l$-th element of $L_k$-th level, $l \in [1; N_{L_k}]$, is stable if $\hat{w}_l^{L_k} \leq \Delta_{i,j,l}^{L_k porog}$ holds when $i < j$ for all $i \in [1; N_{L_p}]$, $j \in [1; N_{L_p}]$, where threshold value $\Delta_{i,j,l}^{L_k porog}$ of absolute change $\Delta_{i,j,l}^{L_k}$ of weight $\hat{w}_l^{L_k}$ of $l$-th element in $L_k$-th level is calculated using the formula (16).

**Corollary:** If $\hat{w}_{jl}^{L_p L_k} \leq \hat{w}_{il}^{L_p L_k}$ holds for **all** $l \in [1; N_{L_k}]$, that is $j$-th element of $L_p$-th level does not dominate $i$-th element of $L_p$-th level in terms of **all** elements of $L_k$-th level, $L_k \in [L_1; L_{p-1}]$, then any changes of weights of $L_k$-th level elements do not lead to changes of global ranking between these elements of $L_p$-th level.

**Statement 8:** A value $\delta_{i,j,l}^{L_k}$ of relative change of weight $\hat{w}_l^{L_k}$ that is necessary for a change of global ranking between $i$-th and $j$-th elements of $L_p$-th level, $i, j = \overline{1, N_{L_p}}$, $l = \overline{1, N_{L_k}}$, $L_k = \overline{L_1, L_{p-1}}$, when $i < j$ and the multiplicative aggregation rule is used for calculation of global weights satisfies the inequality [1]:

$\delta_{i,j,l}^{L_k} < \delta_{i,j,l}^{L_k porog}$, if $\hat{w}_{jl}^{L_p L_k} > \hat{w}_{il}^{L_p L_k}$;

$\delta_{i,j,l}^{L_k} > \delta_{i,j,l}^{L_k porog}$, if $\hat{w}_{jl}^{L_p L_k} < \hat{w}_{il}^{L_p L_k}$,

where the threshold value $\delta_{i,j,l}^{L_k porog}$ of $\delta_{i,j,l}^{L_k}$ is calculated as follows:

$$\delta_{i,j,l}^{L_k porog} = \Delta_{i,j,l}^{L_k porog} \cdot \frac{100}{\hat{w}_l^{L_k}}(\%),$$

where $\Delta_{ijl}^{L_{p-k}порог} = \dfrac{\ln(w_i^{L_p} / w_j^{L_p})}{\ln\left( \prod_{j_1=1}^{N_{L_{p-1}}} \left(\frac{a_{ij_1}^{L_pL_{p-1}}}{a_{jj_1}^{L_pL_{p-1}}}\right) \prod_{j_2=1}^{N_{L_{p-2}}} (a_{j_1j_2}^{L_{p-1}L_{p-2}}) \cdots \prod_{j_{k-1}=1}^{N_{L_{p-k+1}}} (a_{j_{k-2}j_{k-1}}^{L_{p-k+2}L_{p-k+1}})^{a_{j_{k-1}l}^{L_{p-k+1}L_{p-k}}} \right)}$

under conditions:

1) $\hat{w}_i^{L_p} \geq \hat{w}_j^{L_p}$ when $i < j$;

2) $\hat{w}_l^{L_k} > \Delta_{i,j,l}^{L_k porog}$ (that is equivalent to $\delta_{i,j,l}^{L_k porog} < 100\%$).

**A case of change of local weights of decision alternatives**. In this subsection we will find an interval of changes of a local weight $\hat{w}_{ir}^{L_pL_{p-1}}$ of $i$-th decision alternative in terms of $r$-th element of a parent hierarchy level, that do not lead to changes of global ranking between $i$-th and $j$-th alternatives. This allows to define how critical (sensitive) every decision alternative is in terms of selected element of a parent hierarchy level, i.e. to find a value of the least change of a local weight of decision alternative that results in change of global ranking of decision alternatives.

Denote $\delta_{i,j,r}^{a}$, $i \in [1; N_{L_p}]$, $j \in [1; N_{L_p}]$, $r \in [1; N_{L_{p-1}}]$ value of a relative change of local weight $\hat{w}_{ir}^{L_pL_{p-1}}$ of $i$-th element of $L_p$-th level ($i$-th decision alternative) in terms of $r$-th element of a parent $L_{p-1}$-th level that leads to change of global ranking between $i$-th and $j$-th elements of $L_p$-th level ($i$-th and $j$-th decision alternatives). That is, a new weight of $i$-th element of $L_p$-th level in terms of $r$-th element of $L_{p-1}$-th level equals $\hat{w}'^{L_pL_{p-1}}_{ir} = \hat{w}_{ir}^{L_pL_{p-1}} - \dfrac{\delta_{i,j,r}^{a}\hat{w}_{ir}^{L_pL_{p-1}}}{100}$, $\hat{w}'^{L_pL_{p-1}}_{ir} > 0$, and $\hat{w}_i'^{L_p} < \hat{w}_j'^{L_p}$ holds when $i < j$, where $\hat{w}_i'^{L_p}$ is a new global weight of $i$-th element of $L_p$ - th level.

$i$-th element of $L_p$-th level **is stable** in terms of $r$-th element of a parent $L_{p-1}$-th level if any permissible changes of a local weight $\hat{w}_{ir}^{L_pL_{p-1}}$ of this element do not lead to changes of global rank of any decision alternative.

**Degree of criticality** $C_{ir}^{a}$ of $i$-th element of $L_p$-th level ( $i$-th decision alternative) in terms of $r$-th element of a parent $L_{p-1}$-th level is the minimum of values $\left|\delta_{i,j,r}^{a}\right|$ that lead to change of global rank of this decision alternative: $C_{ir}^{a} = \min\limits_{\substack{j\in[1;N_{L_p}]\\ j\neq i}} \left\{\left|\delta_{i,j,r}^{a}\right|\right\}$, $i\in[1;N_{L_p}]$, $r\in[1;N_{L_{p-1}}]$.

**Sensitivity** $S_{ir}^{a}$ of $i$-th element of $L_p$-th level ( $i$-th decision alternative) in terms of $r$-th element of a parent $L_{p-1}$-th level is a reciprocal value to the degree of criticality of this element: $S_{ir}^{a} = \dfrac{1}{C_{ir}^{a}}$, $i\in[1;N_{L_p}]$, $r\in[1;N_{L_{p-1}}]$. $S_{ir}^{a}$ is assigned a zero value if $i$-th element of $L_p$-th level is stable in terms of $r$-th element of $L_{p-1}$-th level.

**Critical** element of $L_p$-th level (**critical alternative***)* is an element of $L_p$-th level which has the least degree of criticality, that is $i_{crit}$-th element of $L_p$-th level is critical in terms of $r$-th element of a parent $L_{p-1}$-th level, if $C_{i_{crit}r}^{a} = \min\limits_{i\in[1;N_{L_p}]}\left\{\min\limits_{r_1\in[1;N_{L_{p-1}}]}\left\{C_{ir_1}^{a}\right\}\right\}$.

A value of relative change $\delta_{i,j,r}^{a}$ of local weight $\hat{w}_{ir}^{L_pL_{p-1}}$ of $i$-th alterative in terms of $r$-th element of $L_{p-1}$-th level can be found using the following statement 9.

**Statement 9:** A value $\delta_{i,j,r}^{a}$ of relative change of a local weight $\hat{w}_{ir}^{L_pL_{p-1}}$ that is necessary for a change of global ranking between $i$-th and $j$-th elements of $L_p$-th level ( $i$-th and $j$-th decision alternatives), $i\in[1;N_{L_p}]$, $j\in[1;N_{L_p}]$, $r\in[1;N_{L_{p-1}}]$, when the multiplicative aggregation rule is used for calculation of global weights satisfies the inequality:

$$\delta_{i,j,r}^{a} > \delta_{i,j,r}^{a\,porog}, \text{ if } i<j,$$

$$\delta_{i,j,r}^{a} < \delta_{i,j,r}^{a\,porog}, \text{ if } i>j,$$

where the threshold value $\delta_{i,j,r}^{a\,porog}$ of $\delta_{i,j,r}^{a}$ is calculated as follows:

$$\delta_{i,j,r}^{a\,porog} = \left(1-\left(\frac{\hat{w}_j^{L_p}}{\hat{w}_i^{L_p}}\right)^{1/\hat{w}_r^{L_{p-1}}}\right)\cdot 100\ (\%) \qquad (17)$$

under conditions:

1) $\hat{w}_i^{L_p} \geq \hat{w}_j^{L_p}$ when $i<j$;

2) $\delta_{i,j,r}^{a\ porog} < 100\%$.

**Corollary:** $i$-th element of $L_p$-th level is stable in terms of $r$-th element of $L_{p-1}$-th level if $\delta_{i,j,r}^{a\ porog} > 100\%$ holds for all $i \in [1; N_{L_p}]$, $j \in [1; N_{L_p}]$, where threshold value $\delta_{i,j,r}^{a\ porog}$ is calculated using the formula (17).

## Sensitivity analysis of a global ranking of decision alternatives in a problem of evaluation of renewable energy technologies for an eco-house in Ukraine

Let us consider a multiple-criteria decision-making problem of evaluation of renewable energy technologies for an eco-house and solve it using the analytic hierarchy process (AHP). Several technologies of renewable energy for an eco-house are selected for investigation by a decision-maker:

- geothermal thermal pump ($a_1$);
- biofuel production ($a_2$);
- solar plant ($a_3$).

To evaluate these technologies (alternatives) a decision-maker develops the following four criteria:

- accessibility ($c_1$);
- economic efficiency during the use of a technology ($c_2$);
- initial costs ($c_3$);
- costs during the use of a technology ($c_4$).

Criteria weights, local weights of decision alternatives in terms of each criterion and global weights of the alternatives using the distributive and multiplicative aggregation rules of the AHP are shown in the Table 1.

Table 1: Solution of the problem using the distributive and multiplicative aggregation rules of the AHP

| | $c_1$ (0.094) | $c_2$ (0.509) | $c_3$ (0.243) | $c_4$ (0.154) | Global weights | |
|---|---|---|---|---|---|---|
| | | | | | Distributive aggregation rule | Multiplicative aggregation rule |
| $a_1$ | 0.090 | 0.649 | 0.065 | 0.114 | 0.372 | 0.312 |
| $a_2$ | 0.455 | 0.279 | 0.361 | 0.405 | 0.335 | 0.436 |
| $a_3$ | 0.455 | 0.072 | 0.574 | 0.481 | 0.293 | 0.252 |

Sensitivity analysis was done separately for the global rankings of the decision alternatives using the distributive and multiplicative aggregation rules.

**A case of the distributive aggregation rule.** The global ranking of decision alternatives in this case is: $a_1 \succ a_2 \succ a_3$, and alternative $a_1$ is the optimal one. The criterion $c_2$ is the most important, its weight equals 0.509. Let us calculate a threshold value of relative change of this criterion weight that leads to changing of the global ranking, for example, between alternatives $a_1$ and $a_2$. This value is calculated as follows:

$$\delta_{1,2,2}^{porog} = \frac{0.335 - 0.372}{0.279 - 0.649} \cdot \frac{1}{0.509} = 0.198 .$$

A positive value of $\delta_{1,2,2}^{porog}$ means that the criterion $c_2$ weight has to be decreased to change the ranking between alternatives $a_1$ and $a_2$. The relative value of this decreasing equals 19.8%.

$\delta_{1,2,2} > \delta_{1,2,2}^{porog} = 0.198$, since $w_{22} < w_{12}$. Thus, an interval of relative change of the criterion $c_2$ weight that leads to changing of the global ranking between $a_1$ and $a_2$ is $\delta_{1,2,2} \in$ (0.198; 1.000).

For example, suppose that decision-maker preferences are changed and the criterion $c_2$ weight is decreased up to the value 0.407 (that is on 20%). The criteria weights after renormalization are $w_1^C = 0.105$, $w_2^C = 0.453$, $w_3^C = 0.271$ and $w_4^C = 0.171$. Then the global weights of the decision alternatives are: $w_1^{glob} = 0.341$, $w_2^{glob} = 0.341$, $w_3^{glob} = 0.318$, and the alternative $a_2$ becomes as important as the $a_1$.

Relative changes of all criteria weights that lead to changing of the global ranking between different pairs of alternatives are given in the Table 2.

According to the definition, a critical criterion for changing of optimal alternative defines as the minimum by absolute value in rows of the Table 2, that correspond to the optimal alternative $a_1$. This minimum value equals 19.8% and corresponds to the criterion $c_2$ and alternatives $a_1$ and $a_2$. Decreasing of the criterion $c_2$ weight on 19.8% leads to changing of the optimal alternative, and $a_2$ becomes optimal.

The criterion $c_2$ is the most sensitive in this problem, the next less sensitive criteria are $c_3$, $c_4$ and $c_1$ (Table 3).

Table 2: Threshold values $\delta_{i,j,l}^{porog}$ (case of the distributive aggregation rule)

| Pair of alternatives $(i,j)$ | $\delta_{i,j,l}^{porog}$, % | | | |
|---|---|---|---|---|
| | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
| (1,2) | -108.7* | 19.8 | -51.8 | -83.2 |
| (1,3) | -230.8 | 27.0 | -64.0 | -140.1 |
| (2,3) | - | 39.8 | -81.0 | -358.0 |

* Negative value of $\delta_{i,j,l}^{porog}$ means that the criterion $C_l$ weight has to be increased to change ranking between alternatives $a_i$ and $a_j$.

Table 3: Degrees of criticality $CritVal$ and sensitivity $SensVal$ for the criteria (case of the distributive aggregation rule)

| Criterion | $CritVal$, % | $SensVal$ |
|---|---|---|
| $c_1$ | 108.7 | 0.009 |
| $c_2$ | 19.8 | 0.051 |
| $c_3$ | 51.8 | 0.019 |
| $c_4$ | 83.2 | 0.012 |

**A case of the multiplicative aggregation rule.** The global weights of the decision alternatives in this case equal $w_1^{glob} = 0.312$, $w_2^{glob} = 0.436$, $w_3^{glob} = 0.252$ (see Table 1). So the global ranking of the alternatives is $a_2 \succ a_1 \succ a_3$, and $a_2$ is the best (optimal) one.

Relative changes of criteria weights that lead to changing of the global ranking are shown in the Table 4. For example, the value $\delta_{1,3,2}^{porog}$ of relative change of the criterion $c_2$ weight that leads to changing of the ranking between alternatives $a_1$ and $a_3$ is calculated as follows:

$$\delta_{1,3,2}^{porog} = \frac{\ln(0.312) - \ln(0.252)}{\ln(0.649) - \ln(0.072)} \cdot \frac{1}{0.509} = 0.193 \quad \Rightarrow \delta_{1,3,2} \in (19.3\%; 100\%).$$

Indeed the relative decrease of the criterion $c_2$ weight, for example, on 20%, results in a new weight $(w_2^C)' = 0.509 - 0.2 \cdot 0.509 = 0.407$. Then global weights of the alternatives are $w_1^{glob} = 0.277$, $w_2^{glob} = 0.443$, $w_3^{glob} = 0.280$, and alternative $a_3$ becomes more important than alternative $a_2$.

Table 4: Threshold values $\delta_{i,j,l}^{porog}$ (case of the multiplicative aggregation rule)

| Pair of alternatives $(i,j)$ | $\delta_{i,j,l}^{porog}$, % | | | |
|---|---|---|---|---|
| | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
| (1,2) | - | -77.8 | 80.3 | - |
| (1,3) | -141.7 | 19.3 | -40.8 | -97.3 |
| (2,3) | - | 79.8 | -488.3 | - |

A critical criterion for changing of optimal alternative defines as the minimum by absolute values in rows of the Table 4, that correspond to the optimal alternative $a_2$. This minimum value equals 77.8% and corresponds to the criterion $c_2$ and alternatives $a_1$ and $a_2$. Increase of the criterion $c_2$ weight more that on 77.8% results in changing of the optimal alternative, and $a_1$ becomes optimal. The criterion $c_2$ is also critical for changing the global ranking between any two considered alternatives: relative change of its weight that equal 19.3%, is enough for changing the global ranking between nonoptimal alternatives $a_1$ and $a_3$.

The criterion $c_2$ is the most sensitive in this problem, the next less sensitive criteria are $c_3$, $c_4$ and $c_1$ (Table 5).

Table 5: Degrees of criticality $CritVal$ and sensitivity $SensVal$ for the criteria (case of the multiplicative aggregation rule)

| Criterion | $CritVal$, % | $SensVal$ |
|---|---|---|
| $c_1$ | 141.7 | 0.007 |
| $c_2$ | 19.3 | 0.052 |
| $c_3$ | 40.8 | 0.025 |
| $c_4$ | 97.3 | 0.010 |

## Conclusion

The paper deals with the methods of complex sensitivity analysis of solution given by the Analytic Hierarchy Process. These methods include evaluation of sensitivity of a local ranking of hierarchy elements as to changes in an expert pairwise comparison judgments and evaluation of sensitivity of a global ranking of decision alternatives as to changes of weights of hierarchy elements.

Formulas for calculation of stability intervals of expert pairwise comparison judgments as to change of a local ranking are obtained. Within these intervals change of the expert judgments does not lead to change of the best decision alternative or an overall ranking of alternatives. The obtained formulas for the stability intervals calculation may be used when the Row Geometric Mean Method is applied to find local weights. The method of sensitivity analysis of multiple-criteria problem solution using the AHP is also considered. This method results in stability intervals of a global ranking of decision alternatives in terms of changing of hierarchy elements weights.

The stability intervals allow to find so called critical elements of a decision-making problem. Critical expert pairwise comparison judgments can be found that are sensitive to changes of a local ranking of decision alternatives. Also critical hierarchy elements, i.e. decision criteria, decision goals etc. can be determined – elements that are characterized by the least changes of their weights necessary for changes of a global ranking of decision alternatives.

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# SOME ASPECTS OF TEMPORAL REASONING INTEGRATION WITH SIMULATION MODELING FOR DYNAMIC INTEGRATED EXPERT SYSTEMS CONSTRUCTION USING AT-TECHNOLOGY WORKBENCH

**Galina Rybina, Victor Rybin**

***Abstract**: The scientific and technological problems of constructing dynamic integrated expert systems and the approaches to their solution are discussed. The new stages of the development of a problem-oriented methodology and AT-TECHNOLOGY workbench for constructing integrated expert systems in the context of solving dynamic integrated expert systems are considered. The theoretical and technical issues concerned with the representation and processing of temporal knowledge and simulation modeling are of special interest. The features of the implementation of temporal reasoning software and simulation modeling software for the dynamic version of the AT-TECHNOLOGY workbench are described.*

***Keywords**: dynamic integrated expert systems, problem-oriented methodology, AT-TECHNOLOGY workbench, time representation, interval Allen logic, temporal solver, integration, simulation modeling*

***ACM Classification Keywords**: software and its engineering, real-time systems software, software notations and tools, context specific languages, development frameworks and environments, software development techniques*

## Introduction

Dynamic integrated expert systems (IESs) and technologies are actively applied practically in all fields of social human activities [Рыбина, 2008; Рыбина, 2015]. As a whole, despite the lack of semantic unification of the terminology base, dynamic IESs classifications, and their separate classes as well as the circle of general scientific and technological problems have already accumulated [Рыбина и др., 2013; Рыбина и др., 2015]:

1. Difficulties in obtaining temporal knowledge (i.e., knowledge, where time is taken as the essence of the subject domain) using different sources (experts, texts, databases, etc.) for dynamic representation of the subject domain (SD).

2. The complexity of the development of formalisms for dynamic presentation of the subject domain, which is determined by the variable composition of its essences, a change of input data

coming from the external sources (the external world model) in time, and the need for structuring, storing, and analyzing data that vary in time.

3. The complexity of solving dynamic problems, which concerns the implementation of the concurrent temporal reasoning about several different asynchronous processes (tasks), limited resources (time and memory), and changes in the composition of knowledge and data during problem solution.

4. Problems of simulation modeling the external world (surroundings) and its different states in real time at all stages of design and development of the system up to the startup.

5. The high cost of foreign software to support the development and maintenance of dynamic IESs of different classes, as well as the practical absence of domestic facilities.

6. The necessity for special software and hardware facilities to connect to the external world (sensors, controllers, etc.).

Moreover, there is no universal complex method for solving the described problems (or a part of them) that implies the development of an integrated integral methodology and technology for creating such complicated systems at all lifecycle stages. Modern commercial software tools to support the construction of most dynamic intelligent systems (G2, Rtworks, RTXPS, etc.) despite its power and versatility, is not able to solve the above problems in terms of integrated methodology fully [Рыбина, 2015; Рыбина и др., 2015; Rybina et al., 2014a].

A considerable step towards generating such methodology can be a new stage of developing the theory and technology of IESs construction based on the problem-oriented methodology, whose main statements were offered by G.V. Rybina in the 1990s [Рыбина, 2008]. Today, this is the basis that is used to create the intelligent programs and automated workstation of a knowledge engineer, namely, the AT-TECHNOLOGY workbench, on whose basis several tens of applied IESs have been created, where a wide spectrum of models and methods of solving different unformalized and formalized problems is used in terms of integrated IES architecture [Рыбина, 2008; Рыбина, 2015].

The analysis of the above problems of constructing dynamic IESs that are central in the present paper shows that the closely related methods to obtain, present and process the temporal knowledge with simulation modeling the external world are least studied in terms of the system approach and the integrated methodology development. The ability to present the temporal relationships between the events that occur in a system and their use during the search for a problem solution allow one to reduce the search space considerably, which influences the functioning speed of dynamic IESs as a whole. In the above-mentioned commercial tools (G2, Rtworks, RTXPS, etc.), the approaches to time reflection, much less temporal knowledge, are rather simple. They are practically not used in the solution of dynamic problems [Рыбина и др., 2015].

Thus, there is a need to create the models, methods, and software tools that execute procedures for temporal inference and simulation modeling in dynamic IESs. These models, methods, and software tools must be integrated in terms of integrated methodology and technology, roles that the problem-oriented methodology of constructing the IESs and the supporting AT-TECHNOLOGY workbench play. Moreover, the problems of integration of simulation modeling technology with dynamic IESs got the most complete development under problem-oriented methodology and supporting software tools (AT-TECHNOLOGY workbench) [Рыбина, 2015; Рыбина и др., 2014].

The focus of this article which is a continuation of the researches described in [Рыбина и др., 2015; Rybina et al., 2014a; Рыбина и др., 2014; Rybina et al., 2015] is technological and applied aspects connected with the expansion of the architecture of the dynamic version of the AT-TECHNOLOGY workbench by integrating the subsystem simulation modeling of the external world and the combined functioning of this subsystem with temporal solver and other basic workbench components in the development of dynamic IESs prototypes.

## Some aspects of temporal inference in dynamic IESs

The representation of temporal relationships and analysis of different models and methods of time presentation is of special interest to modern research in the dynamic intelligent systems area. Papers [Shoham, 1987; Shoham et al., 1988; Еремеев и др., 2004; Spranger, 2002; Allen, 1991], as well as time control papers [Осипов, 2008; Осипов, 2011], point linear time model papers [Еремеев и др., 2004; Еремеев и др., 2009], interval temporal logic papers [Еремеев и др., 2012; Allen, 1983; Плесневич, 1999], branching time papers [Еремеев, 2006; Ladkin et al., 1990], and [Рыбина, 2015; Рыбина и др., 2013], where arguments justify the use of the Allen interval logic [Allen, 1983] and time management [Осипов, 2008] in the dynamic version of the AT-TECHNOLOGY workbench, were devoted to this question.

As mentioned above, the modified Allen logic based on the classical logic [Allen, 1983] and the time control logic [Осипов, 2008] were selected for the dynamic presentation of the subject domains (SDs) in terms of analysis and experimental investigation. Let us explain the idea of the proposed method. To represent temporal knowledge in terms of the basic knowledge representation language (KRL) of the AT-TECHNOLOGY workbench [Рыбина, 2008], a generalized KRL (GKRL) [Рыбина и др., 2015] for dynamic IESs has been developed. It allows one to present temporal knowledge based on the modified Allen logic and the timecontrol logic together with basic knowledge, including knowledge with vagueness, inaccuracy, and carelessness. The main elements of the basic KPL are objects and rules.

The objects correspond to SD essences, which are described as follows [Рыбина, 2008]: an object (IO, NameO, L), where IO is the sequence number of the object; NameO is the object name; L is the list of attributes of the following type: Attribute (IA, NameA, Type), where IA is the sequence number of an

attribute; NameA is the attribute name; Type is the type of the form: Type (IT, NameT, U), where IT is the number of the attribute type; NameT is the name of the attribute type; U is the set of possible values of the attribute (the list of certain values of the attribute, or the range of the values in which the maximum and minimum of the attribute or the membership function are specified).

In the GKPL, the event appears as an object of the SD in the form of one with the main attribute, namely, the origin condition. The type of the given attribute is a logical expression, whose value is an ordinary logical expression that connects the attributes of other SD objects that are not temporal primitives (this is a new type of element of the U set).

The validity of the value of the attribute origin condition at time T shows that the given event occurred at the time instant T. Moreover, the event also has the attribute the number of origins, which is an integer and describes the number of SD observation times. In a similar manner, the temporal interval appears as an object with two main attributes, viz., beginning and end conditions. Both the attributes are of the type "logical expression." The truth of the attribute value of the beginning condition at the time instant T1 shows that the given interval started at T1, while the truth of the attribute value of the end condition at T2 shows that the given interval terminates at T2, i.e., the interpretation of the given interval on the temporal axis is the section [T1, T2], where T1 is always less than T2. The interval also has two more attributes, viz., the number of origins (like events) and the duration (an integer that describes the observation duration of the interval in the SD).

The rule in the basic KPL [Рыбина, 2008] is in the form of (IR, Ins, Cons), where IR is the sequence number of the rule; Ins, a rule parcel containing the list of the "attribute–value" pairs relating to each other by the logic relationships of conjunction and disjunction; Cons is a rule action, which contains the list of attributes with the assigned values. The parcel of the temporal production rule in the GKPL with such an approach involves (apart from ordinary components) a local model of the event development described by events, temporal intervals (taking durations into account), their origin frequencies, and the relationships between them. The application of the right part (signification) of the rule in this case can be executed only under the correspondence of this local model to the current event development in the SD. Thus, the use of the modified Allen logic allows one to describe the temporal dependences between the objects of the SD directly inside the rules; tracing the displays of these relationships results in solutions that take the current event development in the SD into account.

Let us now consider the features of time control, which requires introduction of new rules into the KPL. On the one hand, such rules should provide the rapid reaction on certain (usually, urgent) events of the SD ("rules–reactions"). On the other hand, there is a need to watch certain cycles in operation ("periodic rules"). As a whole, the "rules–reactions" corresponds to the modified Allen logic, being rules that contain the elementary formula conditions in the parcel. These formulas make up a single temporal object (event or interval). To present the "periodic rules," the new attribute TYPE, which is able to take

one of two values, viz., "Ordinary" and Periodic, is introduced in the KPL rule. The parcels Ins of periodic rules add an extra condition, which is the execution period.

## Temporal reasoning software (temporal solver)

As a result of the research work there was developed temporal reasoning software (temporal solver) deeply integrated with the all-purpose solver (AT-SOLVER) within the AT-TECHNOLOGY workbench [Рыбина, 2015; Рыбина и др., 2015; Rybina et al., 2014a]. Temporal solver is included into dynamic extension of the AT-TECHNOLOGY workbench. That allows dynamic IESs successfully operate both in static and dynamic problem domains. The temporal solver, which is one of the new components of the dynamic version of the AT-TECHNOLOGY workbench executes the solution of two problems, which are the construction of the interpretation of the event development model in the SD and the signification of the temporal part of the production rules during functioning according to the problem formulation of the temporal inference on rules and functional requirements.

Let us briefly consider the functions of the modules and blocks of the temporal solver [Рыбина и др., 2015; Rybina et al., 2014a]. *The module for the interpretation of the event development model* provides the construction of initial interpretation and its modification on each step and includes several blocks:

*The event and interval loading block* loads the events and intervals into the internal representation of event descriptions and intervals of the SD;

*The event and interval identification block* checks the correspondence between the data that enter at each operation step and the event and interval origin condition;

*The non-standard situation solution block* provides the solution of the conflicts between the input data and the current interpretation (e.g., observation of the end of an interval up to its beginning).

*The production-rule processing module* is intended for the loading and signification of the temporal parts of rules according to the current interpretation of the event development model.

*The block for integration with other* components allows the program interface to interact with the rest of the components of the AT-TECHNOLOGY workbench [Рыбина, 2014].

## Some aspects of the application of simulation modeling software in the construction of dynamic IESs

In the context of the use of the problem-oriented methodology for constructing IESs [Рыбина, 2008], the additional functionality of described tasks entails a significant change of the IES architecture as all basic components of static IES are practically modified, especially, knowledge base and reasoning tools, and two new subsystems are added—subsystem modeling the external world (environment) and subsystem interfacing with the physical equipment, as well as the technology of constructing dynamic IESs is

significantly changed. The subsystem interfacing with the external environment is necessary to obtain a constant data stream from external equipment and sensors, and the subsystem modeling the external world (environment) is intended to simulate the data stream at all stages of the life cycle of dynamic IES development [Рыбина, 2015; Рыбина и др., 2014].

In the context of this work, the subject of discussion is the subsystem modeling the external world, because the data that is transferred to working memory by the subsystem uses temporal and all-purpose solvers of the AT- TECHNOLOGY workbench [[Рыбина, 2008] Рыбина Г.В. Теория и технология построения интегрированных экспертных систем. Монография. – М.: Научтехлитиздат, 2008. − 482с.; Рыбина, 2014; Рыбина, 2015] to realize a deduction and to obtain recommendations. Basing on these objectives for computer simulation of the complex engineering systems (CES) and complex engineering and organizational systems (CEOS) behavior in time the simulation modeling concept using RAO-approach [Емельянов и др., 1998], which implements the process-oriented approach to construct simulation models (SM) achieved the most development and application. As the expansion experience of the AT-TECHNOLOGY workbench architecture by specialized tools in the form of the simulation modeling subsystem of the external world which is realized on the basis principles of the RAO-approach [Рыбина и др., 2014; Рыбина, 2014; Rybina et al., 2014b] has shown, this way was quite effective for deep integration of all components of the dynamic IES nucleus and combined functioning of the simulation modeling subsystem with the temporal solver, all-purpose AT-SOLVER and other basic components of the dynamic version of AT-TECHNOLOGY workbench.

The basic principles of the simulation modeling subsystem implementation based on RAO-approach and task-oriented methodology requirements are considered in more detail. In the architecture of the simulation modeling subsystem, the functionality of the developed tools is divided between two global modules [Рыбина, 2015; Рыбина и др., 2014; Rybina et al., 2015] – the "SM development module" whose tasks are to support the development process and debugging of SM and other functions requiring the visual interface, and the "SM computation module" ensuring the computation of the conditions of SM in each time step (cycle) of the functioning process of the dynamic IES.

The development of a powerful full-featured high-level language to describe the SM and the creation of a corresponding compiler for that language are the unifying conceptual framework for the two basic modules. To implement this approach, at the first stage of the researches, formalism RAO [Емельянов и др., 1998] is used as a language to describe the SM, the basic version of which is given in [37]. In the future, based on the analysis of current requirements to design models of CES / CEOS to create dynamic IESs developed a special language "RAOAT" including new conceptual changes associated with object -oriented language and significant technological expansions due to the addition of new instructions and data storage structures [Рыбина, 2015; Рыбина и др., 2014; Rybina et al., 2015; Рыбина, 2014].

Composition and structure of the "SM development module" and the "SM computation module" functioning in the structure of the current version of the simulation modeling subsystem (dynamic version of the AT-TECHNOLOGY workbench) are briefly considered bellow. General architecture, composition and structure are of the basic components of the current version of simulation subsystem, detailed description of which is given below [Рыбина, 2015; Рыбина и др., 2014; Rybina et al., 2015; Рыбина, 2014].

**Visual Objects Editor**: The component "Visual objects editor" allows you to create objects, setting their properties and attributes, as well as establish relationships between the model objects in graphical mode. Knowledge engineer and / or specialist on simulation modeling, operating with the models editor, designs SM on graphic canvas containing a predetermined number of various objects, at that the ability to create, delete, copy objects and set relationships between them is ensured. The values of a particular set of properties are able to change for each object. The created visual representation of SM and properties of all objects are stored in the memory of the editor, as well as in a separate text file, which is further processed by the RAOAT language compiler. This visual tool allows you to load a saved SM to update the visual presentation and insert the changes manually to obtained code which is described in RAOAT language.

**Models Synthesis Component**: The models synthesis component, interacting with the visual objects editor by processing the stored collections of objects, generates a description of SM in RAOAT language in XML format which is passed to the RAOAT language compiler.

**Component of Visualizing SM**: Knowledge engineer, if necessary, with the support of the "Animation frames and displayed rules editor" selects the description of the model in RAOAT language and makes animation frames for corresponding objects, and the tool "Visualizer" based on the values of resource parameters, descriptions of animation frames and displayed rules ensures drawing the animation frames.

**RAOAT Language Compiler**: The obtained description of SM in RAOAT language is passed to the input of the "SM computation module" where the compilation from RAOAT language to C# language occurs and the further interpretation and run the developed model. The kernel of the "SM computation module" is the "RAOAT language compiler" which structure has a standard form for the syntax-driven three-pass compiler. This compiler consists of an analyzer which includes components of the lexical, syntactic and semantic analysis, and synthesis component including a generation component of output code. It should be noted that the availability of such objects in the RAOAT language, as irregular events and temporary resources, requires the time coordination with each object of the model. Each object has its own internal timer showing within the modeling time scale, how much time obtains an object. In addition, this timer is associated with the total time of an activity of the SM for a corresponding pause during transferring and obtaining the data from the working memory of the temporal solver.

**Supporting Component of Computing the SM States**: The "Supporting component of computing the SM states" ensures the generation of a discrete modeling time, as well as the generation of the control

actions used to start or stop the activity of SM in the form of messages on each discrete time step. Computation of the new state of SM is based on the state at the previous time step and an allowance of executed operations.

## Some aspects of the interactions between the temporal solver, all-purpose AT-SOLVER and the simulation modeling subsystem

The important feature of the temporal solver is the close interaction with the all-purpose AT-SOLVER and the subsystem of the simulation modeling of the external environment (external world), which is an obligatory component of any dynamic IES. The temporal solver, as well as the subsystem of simulation modeling, acts on the times and process of the interaction between the temporal solver, while the subsystem of simulation modeling is carried out by data and command exchange in the asynchronous mode.

Figure 1 shows the chart of the interactions between the temporal solver, all-purpose AT-SOLVER, and the simulation modeling subsystem. These interactions are provided by joint functioning support facilities. Moreover, the components interact with the total working memory. It should be noted that the interaction is carried out in two modes: the development of the applied dynamic IESs (including the adjustment of a series of IES prototypes) and functioning of the final prototype of the dynamic IESs. The first mode that is needed for dynamic IES construction is the first that was considered in the present paper.

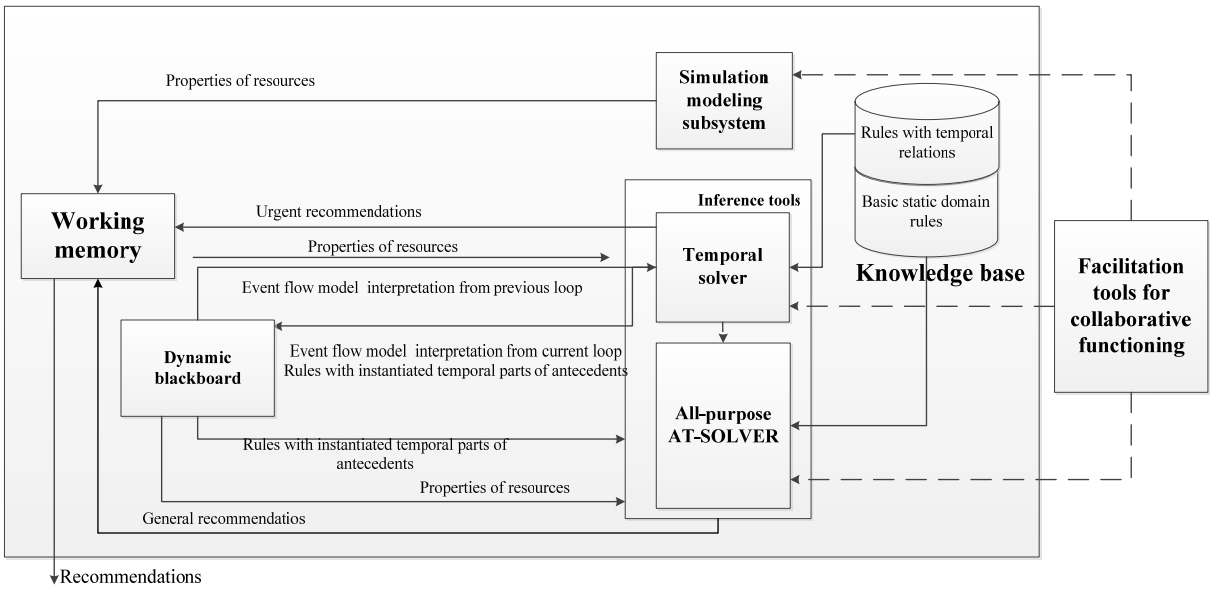


Fig. 1. Interaction diagram of temporal solver, all-purpose solver, and simulation modeling subsystem

The interaction between the components begins after the support facilities receive a message about the start of the adjustment of the developed prototype of the dynamic IES. At the initial stage, this is the configuration of components, including the setting of the duration of the operation cycle for the simulation modeling subsystem, indicating the database for the temporal solver and the AT-SOLVER.

The joint functioning support facilities provide the synchronization of component operation by sending messages with start or stop commands. The selected objects of the SD, whose collection of attributes describes the system state, are presented in the working memory. The KB contains the temporal rules that are necessary to solve the formulated problem, as well as to describe the events and intervals. As a result of temporal inference on rules, the system state changes, i.e., the attributes of the working memory objects vary according to the solved problem. A synchronous interaction would mean that the simulation modeling subsystem goes to the standby mode up to the inference completion by the temporal solver following data transmission (from the subsystem to the solver). Otherwise, the asynchronous interaction is said to be the ability to continue the operation of the subsystem without waiting for the temporal inference. Asynchronous interaction allows higher productivity due to the use of the time of the processing of general situations by the temporal solver to execute the next modeling cycle. Note that similar asynchronous interactions are applied in real practice, when it is impossible to react to an event immediately. This is the reason that the temporal solver and subsystem are synchronous.

The functioning of the simulation modeling subsystem and the inference facilities is an asynchronous process that is executed in parallel. The functioning of the temporal solver and ATSOLVER is an synchronous process that is executed sequentially. The interaction between the components of the AT-TECHNOLOGY complex is a very difficult process that requires the development of models, methods, and software facilities to support interactions. The functions of the modules and blocks are the following:

The configuration block carries out the component configuration. It sets the duration of the cycle of the discrete model time and the assignment of the names for the simulation modeling subsystem objects, temporal solver, and AT-SOLVER.

The model time generation block counts the cycles of the discrete model time according to the cycle duration specified by the configuration block. The working memory scanning block observes the changes in the working memory.

The control effect calculation block implements the target function of the interaction model. As a result of the block operation, the target component and the control effect, which should be set, are defined.

The control effect generation block forms the control effect as the message to the certain component. The interface module of the message exchange with the components processes the input messages and sends control effects. For joint functioning support facilities, the special adjustment tools allow one

to emulate the combined work of the components of AT-TECHNOLOGY in both the stepbystep and realtime modes.

The use of these tools allows the study of the operation of the main components of the dynamic version of AT-TECHNOLOGY in the fullest manner.

## Conclusion

These experimental investigations showed the advantages of the developed software tools compared to similar ones according to such criteria as the KPL power, operation speed, and reduction of the lead time of dynamic IESs. Verification of the performance and efficiency of these tools was done by developing a set of basic components, which is the minimum that is needed for dynamic IES operation.

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# ASSEMBLING DECISION RULE ON THE BASE OF GENERALIZED PRECEDENTS

**Vladimir Ryazanov, Alexander Vinogradov, Yuryi Laptin**

***Abstract**: A new approach to analysis of structure of the training sample based on identification and parameterization clusters of local regularities that are considered as generalized precedents of manifestation of partial interrelations in data is investigated. Substantive treatment of non-uniformity in images of empirical distributions in parametric spaces is proposed, and possibilities of use of secondary cluster structure for reduction of complexity of decisions and increase of processing speed, identification and verification of regularities, are studied.*

***Keywords**: local dependency, generalized precedent, parametric space, cluster, hyper-parallelepiped, logical regularity, hypercube bitmap, derivative distribution, decision rule*

## Introduction

Higher dimensions are the main obstacle at creation analogs of the Hough Transform for abstract feature spaces which would be as effective as it takes place in case of Image Processing and Scene Analysis [de Berg, 2000], [Shapiro, 2001]. We consider here the Hough Transform in some expanded sense as the instrument of coding of statistically reliable local correlations of parameters. In this paper, some broad analogs of the Hough Transform are proposed to be used in problem of choice and optimization of the Decision Rule (DR) in feature spaces of a general kind, on the basis of detection local partial regularities in small-sized subspaces.

It is known that complexity of a problem of detection statistically significant dependences between parameters quickly increases with growth of number of the last. So, development of efficient methods of search and verification of partial dependences binding together limited subsets of parameters becomes now an important problem in the field of Data Analysis. One of the promising areas here consists in search of typical partial dependencies within a pre-selected family. Typicality here means that within the training sample a representative accumulation of objective statistics for dependencies of selected kind can be fulfilled, and estimates created of their statistical significance and their potential contribution to the DR.

The paper investigates the problem of detecting local dependencies in data that appear as features of the training sample geometry. The advantages achieved due representation of geometric forms of clusters, its spatial arrangement and filling in parametric spaces of various kinds, are shown further.

Efficient methods of coding some basic forms of clusters in the feature space $R^n$, in particular, in the form of a hypercube bitmap, hyper-parallelepiped of elementary logical regularity (ELR), component of Gaussian mixture, and others, are proposed in the framework of this approach [Ryazanov, 2007], [Kovshov, 2008], [Vinogradov, 2010]. Each combined object is considered as a single precedent implementing some local regularity in data - Generalized Precedent (GP) - and is represented by a point of appropriate parametric space. Important information about the presence and severity in the training sample regularities of selected kind becomes represented in the derivative distribution of GPs. Along with selection a code for regularities, a priori and indirect information of various origins can be involved to making the search of GPs more targeted. This approach based on GPs is pioneering, and results of preliminary studies showed significant benefits which arise from its use in problems of finding regularities in data, recognition and prediction [Vinogradov, 2015], [Ryazanov, 2015].

## Generalized Precedents

In case of large amounts the problem of analyzing numerous data highlights the priority of processing speed. Under the new conditions simple and well-researched approaches, in particular linear, get rebirth [Berman, 2003]. The most quick are methods in which all calculations can be reduced to comparisons on special linear scales of a particular type. In this series, one of the highly successful approaches turned out to be the one based on the use of Logical Regularities (LR) [Zhuravlev, 2006], [Ryazanov, 2007]. This approach uses data clusters in the form of hyper-parallelepipeds in $R^N$, each cluster is described by the conjunction of the form $L = \&R_n,\ R_n=(A_n<x_n<B_n)$, and substantially interpreted as recurring joint manifestation of the feature quantities $x=(x_1,x_2,\ldots,x_N)$ on intervals $(A_n<x_n<B_n)$. The principle of proximity precedents of the same phenomenon to each other here is embodied in the requirement of filling the interior of the cluster by objects of the same class. Same time, the geometric shape of the cluster represented by the parameters $A_n, B_n$, becomes of particular importance. Multiple joint appearances of feature values inside this shape are regarded as substantive independent phenomenon that is called Elementary Logical Regularity (ELR). Thus we get an example of GP in the form of a parametric code $\{A_n<x_n<B_n\},\ n=1,2,..,N,$ for each hyper-parallelepiped of ELR.

A special case of ELR is hypercube of Positional Representation of Data (PRD) [Aleksandrov, 1983]. In this representation, data conversion is executed on bit layers. To perform comparisons $A_n<x_n<B_n$ , linearly ordered bit scale of increased depth is used, and, instead of comparison numbers as whole, serial comparisons of descending bits are performed. The PRD in $R^N$ is defined by a bit grid $D^N \subset R^N$ where $|D| = 2^d$ for some integer $d$. Each grid point $x=(x_1, x_2, ..., x_N)$ corresponds to effectively performed transformation on bit slices in $D^N$, when the $m$-th bit in binary representation $x_n \in D$ of $n$-th coordinate of $x$ becomes $p(n)$-bit of binary representation of the $m$-th digit of $2^N$-ary number that represents vector $x$ as whole. It's supposed here $0<m\le d$, and function $p(n)$ defines a permutation on $\{1,2, .., N\}$, $p \in S_N$. The

result is a linearly ordered scale *S* of length $2^{dN}$, representing one-to-one all the points of the grid in the form of a curve that fills the space $D^N$ densely. For chosen grid $D^N$ an exact solution of the problem of recognition with *K* classes results in *K*-valued function *f*, defined on the scale *S*. As known, *m*-digit in $2^N$-ary positional representation corresponds to *n*-dimensional cube of volume $2^{N(m-1)}$. It's called *m*-point. For each *m* the entire set of *m*-points is called *m*-slice. There are just one *d*-point, $2^N$ ones of *(d-1)*-points, and $2^{dN}$ ones of *1*-points on the scale *S*. Each of *m*-points, $0<m<d$, can be regarded as separate cluster in $D^N$. If it's non-empty and filled with data of certain class only, we have got GP [Vinogradov, 2015].

Recently, intensive studies develop related to the generalization of the local parameterization ideas to a wider range of partial dependencies in data. An important case is, for example, normal mixture with constant covariance matrix $\boldsymbol{\sigma}$

$$\sum_{\mathrm{i}}^{\mathrm{I}} \mu_{\mathrm{i}} \exp\left(-\frac{1}{2}(\mathrm{x_i}-\mathrm{x})^{\mathrm{T}}\boldsymbol{\sigma}^{-1}(\mathrm{x_i}-\mathrm{x})\right)$$

where the geometry of components as individual clusters is also very specific and described with small number of parameters. As we have seen above, in case of PRD each cluster can be coded with one integer and one real parameter $(q^i, \mu^i)$, for homogeneous normal mixture pairs of the form $(x^i,\mu^i)$ are sufficient, in the case of ELR - sets of *2N* border marks on the main axes $A^i_n$, $B^i_n$ , as well as the weight $\mu^i$ of the ELR $L^i$. In many other approaches, to define the basic clusters, their spatial layout and filling, a limited number of parameters also could be enough. Below we present variants of using such parameterizations.

## Examples of use GPs in practical tasks

In the first example we mean as GPs certain representations of the training data in the form of samples of new ordinary precedents that fit both the original precedents and classes, as well as results of analysis of the initial training sample. As additional information for each class $K_\lambda, \lambda = 1,2,...,l$ , sets of LRs of classes $\mathrm{P}_\lambda = \{P_t(\mathbf{x})\}$ are used [Ryazanov, 2007], [Kovshov, 2008], ie, the predicates of the form $P^{\Omega_1,\mathbf{c}^1,\Omega_2,\mathbf{c}^2}(\mathbf{x}) = \underset{j\in\Omega_1}{\&}(c_j^1 \le x_j)\underset{j\in\Omega_2}{\&}(x_j \le c_j^2)$ , $\Omega_1,\Omega_2 \subseteq \{1,2,...,n\}, \mathbf{c}^1,\mathbf{c}^2 \in R^n$ , where

1) $\exists \mathbf{x}_t \in \tilde{K}_\lambda : P^{\Omega_1,\mathbf{c}^1,\Omega_2,\mathbf{c}^2}(\mathbf{x}_t) = 1,$

2) $\forall \mathbf{x}_t \notin \tilde{K}_\lambda : P^{\Omega_1,\mathbf{c}^1,\Omega_2,\mathbf{c}^2}(\mathbf{x}_t) = 0,$

3) $P^{\Omega_1,\mathbf{c}^1,\Omega_2,\mathbf{c}^2}(\mathbf{x})$ is locally optimal for the standard predicate quality criterion.

Here, training sample objects from the class $K_\lambda$ are designated $\tilde{K}_\lambda$. We considered two schemes of definition of GPs. In the first scheme, sets of objects that satisfy the predicates of $\mathrm{P}_\lambda$ are put in

correspondence to the set of precedents $\tilde{K}_\lambda$. We considered an analog of the "nearest neighbor" algorithm.

The object $\mathbf{x}$ belonged in the class, LR of which was considered the nearest, the "distance" $\mathbf{x}$ to this LR was calculated according to the formula

$$d_\alpha(\mathbf{x}) = \frac{\sum\limits_{\mathbf{x}_t : P_\alpha^{\Omega_1,\mathbf{c}^1,\Omega_2,\mathbf{c}^2}(\mathbf{x}_t)=1} \rho(\mathbf{x},\mathbf{x}_t)}{\left|\{\mathbf{x}_t : P_\alpha^{\Omega_1,\mathbf{c}^1,\Omega_2,\mathbf{c}^2}(\mathbf{x}_t) = 1\}\right|}$$, where $\rho$ is Euclidean metrics in $R^n$.

Comparison was held on data of credit scoring (2 classes, 15 features, 348 test objects) [Bache, 2013]. The accuracy of standard and modified method of "nearest neighbor" on the test data was, respectively, 75.6% and 77.5% of correct answers.

In the second scheme, we regard as GP the values of all ELR of the object and disjunction of their negations, values of all ELRs of another class and disjunction of their negations (in the classification with 3 classes and more we used the scheme "one against all"). Thus, each object corresponds to a vector of digits $\{0,1\}$. Here, GP is simply a description of the single source object in the new feature space.

Figures 1 and 2 show examples of imaging the original training sample and the test sample in the breast cancer detection task [Mangasarian, 1990]. Objects of different classes are shown in gray and black. Generalized precedents for the test training sample turned out to be linearly separable.

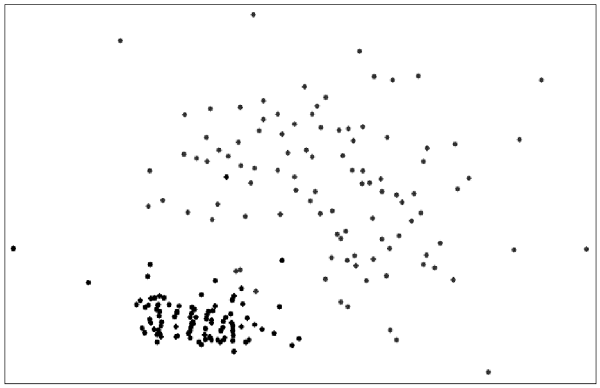

Fig.1. Visualization of the training sample

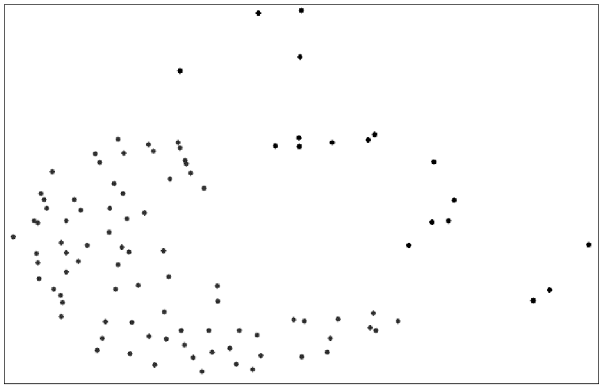

Fig.2. Visualization of the sample of GPs

Results of the comparison of this two test data pattern recognition methods in various problems are presented in the table below. Version of SVM implemented in [Zhuravlev, 2006] was used as classification method.

| Task | K | N | Number of training objects | Number of control objects | Exactness on test sample | Exactness on GPs |
|---|---|---|---|---|---|---|
| «breast» | 2 | 9 | 344 | 355 | 94.6 (0.8) | 96.1 |
| «credit» | 2 | 15 | 342 | 348 | 80.5 (4.3) | 84.5 |
| «image» | 7 | 16 | 210 | 2100 | 68.8 (27.7) | 92.0 (0.6) |

## GPs and typical local dependencies

We will explore those variants of local parameterization, in which typicality of selected geometric specialty results in representativeness of corresponding cluster in the parametric space. In this approach, information about the presence of such clusters is used to optimize the DR starting from analysis of derivative distributions to realization DR in original feature space $R^n$.

Further we use the coding techniques for lineaments to illustrate applications of the approach in general. In Figure 1-2 it is easy to notice that in two imaged dimensions many objects are grouped in expressed elongated formations - lineaments. Partial dependence between parameters of objects corresponding to features of this type may reflect, for example, the lability of strongly correlated parameters, repeating bias in data logging, etc. In any case, objective specialties of this type must be taken into account at development of improved solutions. We present below the calculation scheme in which local geometrical features of this kind are used for reduction of data volume and simplifying the DR. The goal is to find among these local features the most typical:

a) at the first stage, we construct the set $L$ by finding all ELRs of 2nd kind [Ryazanov, 2007] with rectangular hyper-parallelepipedal boundaries;

b) it is chosen a limited number of parameters characterizing, in the description of each obtained ELR-2, its length and position relatively to the axes. These may be size of the maximum $R_l$ and minimum $r_l$ edges of ELR-2 $l \in L$, or maximum length of edges $R_l$ and guide angles $\alpha_n$, etc .;

c) one-to-one mapping $f: L \to \boldsymbol{C}$ of the set $L$ into selected parametric space $\boldsymbol{C}$ is constructed, where clustering is performed including the search for the set of expressed compact clusters $\boldsymbol{C}^T=\{\boldsymbol{c}^t\}$, $t \in T$. Each cluster $\boldsymbol{c}^t$, $t \in T$, represents separate kind of typical lineament.

Thereafter, the reverse assembly of the DR can be done on the base of detected GPs, when typical ELRs-2 are reconstructed in $R^n$ from points $c^t$ of each cluster $\boldsymbol{c}^t$, $t \in T$, by inversing $f^{-1}: \boldsymbol{C} \to L$. When this, ranked priorities may be given to different elements of $\boldsymbol{C}^T=\{\boldsymbol{c}^t\}$, $t \in T$, depending on the nature of the data, requirements for the solution, etc. For instance, in the first place those ELRs may be used which

correspond to lineaments with the highest internal density of objects. Thus, in Fig. 3 a model example of sample with two classes is represented, where systematic error of kind "smear" appears in the data, and the observed distortions are different for two classes. In such a problem it is important to reveal exact boundaries of parameters' spread, and the preferred solution may include, in particular, the replacement of every smear lineament by point (i.e., by an average estimate of the true feature vector).

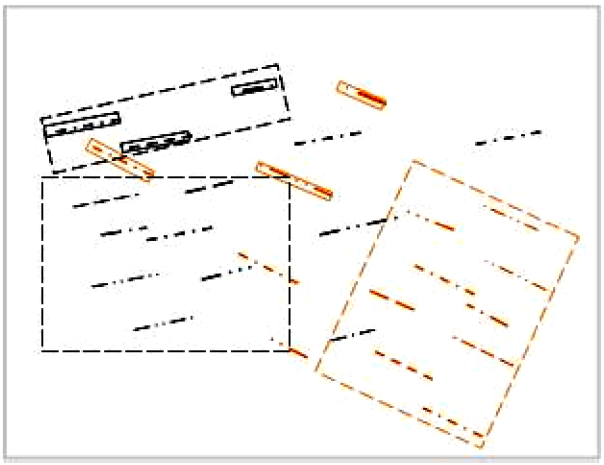

Fig.3. Modeled training sample, $K$=2, dotted borders show some of found ELRs-2

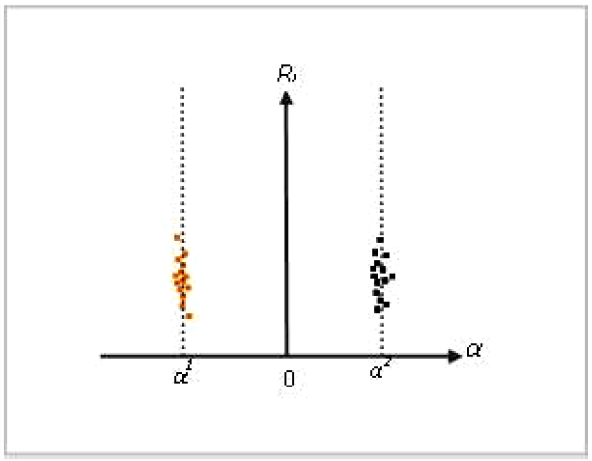


Fig.4. Two essential clusters of ELRs-2 revealed in GP space $\boldsymbol{C}$ for parameters $\alpha$, $R_l$

Fig.4 shows two major compact clusters $\boldsymbol{c}^1,\boldsymbol{c}^2 \subset \boldsymbol{C}$ in parametric space $\boldsymbol{C}$ for parameters $\alpha$, $R_l$, which represent some of constructed in step a) ELRs-2 (small rectangles densely filled by objects of the own class $k$ only, $k = 1, 2$). Some of the built at step a) ELRs contain objects of different classes $k$, $k = 1, 2$, which could result in their exclusion from consideration (large rectangles in Fig.3 are not mapped into the space $\boldsymbol{C}$ in Fig.4). The screening is similar to that of used in the case of conventional two-dimensional Hough Transform for lines: only significant clusters in parametric space are considered, each of which correspond to elements of the same line, and all the other details of the image are ignored. Likewise, in discussed case the essential clusters $\boldsymbol{c}^1,\boldsymbol{c}^2 \subset \boldsymbol{C}$ unite images $c=f(l) \in \boldsymbol{C}$, $l \in L$, of the small densely filled ELRs-2 of Fig.3 that are selected not on the basis of intended revealing these properties, but only because of their typicality and multiple occurrence in parametric space $\boldsymbol{C}$. Similarly, it is the case for any version of Hough Transform, including ones for complicated shapes in higher dimensions [Shapiro, 2001]. Of course, the presence and dominance of lineaments in the empirical distribution - this is just one of many kinds of local geometric specialties which could manifest itself in the training sample. We chose the example with lineaments because of its visibility.

**Reversed assembly of the Decision Rule in $R^n$**

Notice in Fig.4 that images of classes in parametric space $\boldsymbol{C}$ become also linearly separable as that of in Fig.2 due to selection the most typical GPs only. In initial feature space $R^n$ the same sample has more complicated geometry, and on the linear separability of classes there can be no question. We want, if possible, to keep and use these and other potential benefits when returning to the original feature space $R^n$ and constructing DR in it. In simple cases, the reverse transition from the space of GPs $\boldsymbol{C}$ in the feature space $R^n$ is not a problem. Point $c \in \boldsymbol{c}^t$ of some cluster $\boldsymbol{c}^t$, $t \in T$, selected as essential and typical, returns ELR-2 $I = f^{-1}(c)$ that correspond to cluster in $R^n$ of known form (in other tasks among the latter could be as hyper-parallelepipeds of ELR-2, as hypercubes of $m$-digits in $2^N$-ary PRD, Gaussian hats, etc.), which can directly participate in the formation of improved DR. For instance, in Fig.3 elements of only clusters $\boldsymbol{c}^1, \boldsymbol{c}^2$ when returning in $R^n$ describe in detail the two classes of training sample, and other ELRs may be also used in DR to avoid, perhaps, data overfitting, superfluous computational spending, or another shortcoming.

However, it's more difficult to maintain and to build in the DR information on those significant clusters in the parametric space, the presence of which provides additional opportunities for global optimization of the DR. We will show how this can be done in case of involvement in DR the property of linear separability of classes found in the space of GPs. We use the same example with lineaments.

Let $x$ be the new object. We assume that the plane in Fig.3 represents the maximum $R_l$ and minimum $r_l$ edge sizes of a lineament, and the condition $R_l >> r_l$ holds. The next criterion allows to use in DR the information on the values of parameters $R_l$, $r_l$, $\alpha^1$, $\alpha^2$ used in the description of clusters $\boldsymbol{c}^1, \boldsymbol{c}^2 \subset \boldsymbol{C}$.

For triple $(R_l, , r_l, \alpha^s)$, $s=1,2$, let's construct two sets $H^s=\{h^s\}$, $s=1,2$, containing all sorts of covering the object $x$ hyper-parallelepipeds of ELR-2 with parameters $R_l$, $r_l$, $\alpha^s$. If the object $x$ belongs to the class $s$, $s = 1,2$, then it could be found inside a virtual hyper-parallelepiped $h^s$ the more likely, the more typical parameters $R_l$, $r_l$, $\alpha^s$ are, and the more representative cluster $\boldsymbol{c}^s$ is. Let $c_i^1, c_j^2$ $(i=1,2,\ldots,I, j=1,2,\ldots,J)$ be lists of hyper-parallelepipeds of clusters $\boldsymbol{c}^1, \boldsymbol{c}^2$, having non-empty intersection with $h^s$, $h^s \cap c_i^1 \neq \emptyset$. We assume that both lists $I$, $J$ are non-empty, and denote $\rho_i^{s,1}$ the distance between all centers of the hyper-parallelepipeds $h^s$ and $c_i^1$ along the edge $r^s$, and similarly, $\rho_j^{s,2}$ – for non-empty intersections $h^s \cap c_j^2$. Then the index of smaller averaged distance among

$$\rho^1 = \frac{1}{I}\sum_{i=1}^{I} \rho_i^{s,1}, \quad \rho^2 = \frac{1}{J}\sum_{j=1}^{J} \rho_j^{s,2}$$

points to the more probable class for object $x$, because when $R_l >> r_l$ holds, the intersection of rectangular lineament $c_i^1$ or $c_j^2$ $(i=1,2,\ldots,I, j=1,2,\ldots,J)$ with hyper-parallelepiped $h^s$, $s = 1$ or 2, having the same direction of maximal edge, is possible only for smaller values of distances $\rho_i^{s,1}$, $\rho_j^{s,2}$.

Of course, creation and usage of such estimates and criteria imply additional extraordinary efforts which are justified only in case of significant prospects for improvement DR. In other cases, for instance, when sets $H^s=\{h^s\}$, $s=1, 2$, are empty and estimates $\rho^1$, $\rho^2$ don't function, conventional rules of organization of DR are used that refer to usual principles of proximity of the object $x$ to own class in $R^n$.

## Conclusion

Paper presents a new approach to data analysis tasks, including recognition, classification and prognosis, based on the use of concept Generalized Precedent. Approach focuses on application in cases where geometric structure of the sample has typical local details, different for different classes. It can be said that classes differ not only by its location in $R^n$ and the density of filling, but also by detectable "texture" of this filling. As a result of construction of the space of GPs, new "textural features" that may contain significant auxiliary information become added to $n$ basic features of $R^n$. Potential usefulness and targeted searches of one or another type of local geometric features of classes can be influenced by formulation of the problem, nature of data, direct or indirect demands for quality of solutions, computational cost, etc. Despite this diversity, unifying factor of the approach is the selection of suitable partial parameterization, covering sufficient variety of local dependencies. Essential compact clusters in secondary distribution on the parametric space highlight the most important forms of local dependency between basic features. It is shown that the derivative distribution may be of a complicated structure, but, adequate analysis of this structure allows revealing intrinsic regularities in data and thus achieving essential improvements of the quality of solutions in hard tasks of data analysis. Development of specialized research techniques for analysis of distribution on parametric space and methods of the use of additional information provided is doubtless the prospect of further development of the approach.

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# EXEMPLAR BASED INPAINTING USING DEPTH MAP INFORMATION

**Vahan Gevorgyan Vladimir, Aram Gevorgyan Vladimir, Gevorg Karapetyan Arakel**

***Abstract**: Image inpainting is the process of filling in missing or damaged parts. Exemplar based image inpainting algorithm fills in the missing parts using information from the known part of the image. Scanning the whole image for finding the most similar patch is a very time consuming task. Also the algorithm doesn't consider the depth information which harms the quality of the result. In this paper, we use depth information from the image depth map and this approach improves efficiency and quality of inpainted image. The paper includes experiment results and comparison with exemplar based inpainting algorithm.*

***Keywords**: stereo image, inpainting, exemplar based, depth map, image completion*

***ACM Classification Keywords**: I.4.4 Image Processing and Computer Vision - Restoration*

## 1. Introduction

The term inpainting comes from medieval art restoration, where pictures were restored by filling-in any gaps or scratches to bring them "up to date". In our application we mainly use inpainting for removing unwanted objects from an image. But there are also other applications for image inpainting such as repairing old photographs, text removal or improving quality of medical pictures. The goal of inpainting is to replace the missing region in such a way that it will be not observable for a viewer that there is something wrong with the image.

There are many known inpainting algorithms for 2D (ordinary) images, which trying to solve that problem. Those algorithms are usually classified into two main groups: diffusion based and texture based.

Diffusion based approach fills holes in images by propagating linear structures from known region into the missing region via process of diffusion. This problem is mainly modelled by Partial Differential Equation (PDE), so it is also called a PDE based approach. When the region to be inpaint is small, this approach provides good results, but it is not convenient to use it when the missing region is large, because some blur is noticeable due to diffusion process.

First texture based techniques [Igehy, 1997] produced good results if an image consisted of "pure" texture and were able to fill-in large regions. However, they were not able to propagate linear structures

into the hole. In 2003 Criminisi *et. al* introduced an exemplar based algorithm [Criminisi, 2004], which is also referred to the texture based approach. In contrast with prior texture based approaches, exemplar based technique reconstructs both texture and structure features of the image and also it deals with large regions. There are also many other known algorithms from that class, for example PatchMatch [Barnes, 2009].

3D technologies have become very popular and are rapidly developing. With the development of 3D develops stereo vision. Stereo vision is a technique that gets 3D information of the world using two or more cameras. Stereo image is an image pair taken form left and right cameras, which show image from position of the left and right eyes respectively. Such approach enables users to feel depth. If we apply even best 2D inpainting approaches for stereo images, we will get unnatural results, so the appropriate algorithms for stereo images start to develop [Wang, 2008].

In this paper, we present an approach, which generates depth map for the image and improves exemplar based algorithm [Criminisi, 2004] using values from that map. In our approach, after removing some object, we use only background of it as a supporting region, because foreground objects may ruin the result.

In section 2.1 we will speak about depth maps. Section 2.2 is a brief review of exemplar based inpainting algorithm and our extension of that algorithm, using depth map values, is presented in section 3. Finally, section 4 shows the experimental results and comparisons with exemplar based algorithm [Criminisi, 2004], and conclusion is in section 5.

### 2.1 Depth map

A depth map contains information relating to the distance and position of the surfaces of scene objects from a viewpoint. Depth map can be constructed with special depth map cameras such as Kinect and active methods that use for example: radar, ultrasound, laser pulse or laser line scan. Nevertheless, these methods need special equipment and are expensive. The other more practical way of getting depth map is passive methods that use such techniques as stereovision. Multi-view stereovision computes depth map from several images of the same object or scene, taken from different angles and positions. Binocular stereovision produces depth map from a pair of images taken from two cameras, this pair is called stereo image.

Getting depth map usually consists of the following steps: calibration, rectification, stereo correspondence and triangulation. Calibration is a process of getting cameras external and internal parameters. More about calibration is discussed at [Bradski, 2008] Chapter 11, [Zhang, 2000], [Mann, 2004].

If the two cameras are coplanar, the two images have the same image plane, so the corresponding points have the same row coordinates and stereo correspondence problem is reduced from 2D to 1D. However, in real world it is impossible to place two cameras ideally coplanar so rectification is needed. Rectification is a process of reprojecting image planes onto the common plane parallel to the line between optical centers.

Stereo correspondence is a problem of finding matching pixels in left and right images corresponding to the same points on the 3D surface. The output of stereo correspondence algorithms is disparity map. Disparity shows the difference of point location in corresponding left and right images. There are many stereo correspondence algorithms. State-of-the-art of stereo matching algorithms and their taxonomy is described at [Sharstein, 2002]. One of the most commonly used stereo correspondence algorithms is Stereo-Global Matching (SBM) [Hirschmuller, 2005] that is fast enough and gives good results.

Knowing disparity map and geometric arrangement of the cameras and their internal parameters, such as focal length, we can calculate depth map by triangulation [Marshall, 1997], [Bagga, 2013].

## 2.2 Exempla based algorithm

This algorithm was proposed by Criminisi *et al*. [Criminisi, 2004]. For input, it has an image $\mathrm{I}$. A user selects a target region $\Omega$, which should be removed and filled in and its contour denotes as $\delta\Omega$. This contour also called as "fill front" and it evolves inward during the algorithm. The supporting region from where the algorithm will take information, which is called source region and denotes as $\Phi$, should be specified. By default it is defined as whole image minus the target region $\Phi = \mathrm{I} / \Omega$ This algorithm is fragment based, which means that it fills the missing area by patches not by pixels. The default size of a patch that authors suggest is $9\times9$ pixels, but this parameter may be specified by the user, depending on the image size and color features. After setting all input parameters, the algorithm starts to inpaint the removed region. The steps of the algorithm can be briefly described as:

- Detection of fill front (boundary points) and calculation of priority values for that points,
- Selecting a target patch (patch which center is a point with highest priority value ),
- Finding a source patch (the most similar patch to the target patch),
- Filling values for unknown points from the target patch
- Update terms values for filled pixels which participate in priority function.

This steps are executing iteratively until all pixels are filled in. For every boundary point priority function is computed, which has the following representation:

$$P(p) = C(p)D(p), \forall p \in \delta\Omega,$$

where $C(p)$ is called confidence term and $D(p)$ – data term and they have following form:

$$C(p) = \frac{\sum_{q \in \Psi_p \cap (I/\Omega)} C(q)}{|\Psi_p|}, D(p) = \frac{|\nabla I_p^{\perp} \cdot n_p|}{\alpha},$$

where $|\Psi_p|$ is the area of a patch $\Psi_p$ centered at point $p$, $\alpha$ is a normalization factor (its value is 255 for a typical grayscale image), $n_p$ is a unit vector orthogonal to the front $\delta\Omega$ in the point $p$ and $\Delta I_p^{\perp}$ - the isophote (a line of equal intensity value) at $p$.

The confidence term shows how much is reliable the information surrounding the pixel $p$. The idea is to give preference to those patches, which have more of their pixels already filled and the earlier pixel filled (or it was never part of the target region) the more reliable it is. The function $C(p)$ initially is set to the following values:

$C(p) = 0, \forall p \in \Omega$ and $C(p) = 1, \forall p \in I / \Omega$.

The data term $D(p)$ shows the strength of an isophote hitting the front $\delta\Omega$ at point $p$. This term gives preference to those patches, which prolongs isophote direction. Due to it, linear structures are encouraged to be inpainted first and therefore propagate in a plausible way into the target region.

The target patch (patch which center is a point with highest priority) are fixed and the algorithm searches for the source patch calculating similarity between patches by the following formula:

$$\Psi_q = \arg\min_{\Psi_l \in \Phi} d(\Psi_l, \Psi_p),$$

where $\Psi_q$ is the source patch, $\Psi_p$ – the target patch, $d$ - a function of distance (similarity) between two patches. The distance function that algorithm uses is sum of squared difference (SSD) of known pixel color values from the target patch, with their corresponding pixels from the source patch.

Then for each unfilled pixel in $\Psi_p$ the algorithm copies pixel values from the corresponding position inside $\Psi_q$. Fig. 1 illustrates the inpainting process described above.

***Figure 1 :*** **Inpainting process**

Finally, the confidence values for the already filled pixels should be updated. The confidence term $c(p)$ is updated as follows: $C(t) = C(p), \forall t \in \Psi_p \cap \Omega$.

## 3. Modification of exemplar based algorithm using depth information

We take stereo pair (left and right image) and generate disparity map using Semi-Global Matching stereo correspondence algorithm [Hirschmuller, 2005] that was mentioned above. Then from disparity map by triangulation, we calculate depth value for each pixel and due to it a depth map is generated, which shows objects positions related to each other.

Now we have an image with its depth map. The main idea of our approach is the following, that after removing some object from a stereo image, the information for inpainting that region should be taken from its background.

After user selects the target region that should be removed and inpainted, we generate a mask for inpainting. In contrast with standard 2D inpainting, this mask is ternary not binary. First value stands for pixels from a region that should be inpainted (target region), second value is for pixels from supporting region from which the information will be taken (source region) and third value is for other points that will not participate in the inpainting process. While a user selects an object to remove, he can also touch neighborhood points. Because of it, we calculate the average depth of the target region, which is almost equal to the object depth if the object is homogeneous. As the source region, we consider pixels that have disparity great or equal than this average value minus some threshold. This threshold is for the case when removing object is the furthest object from the camera.

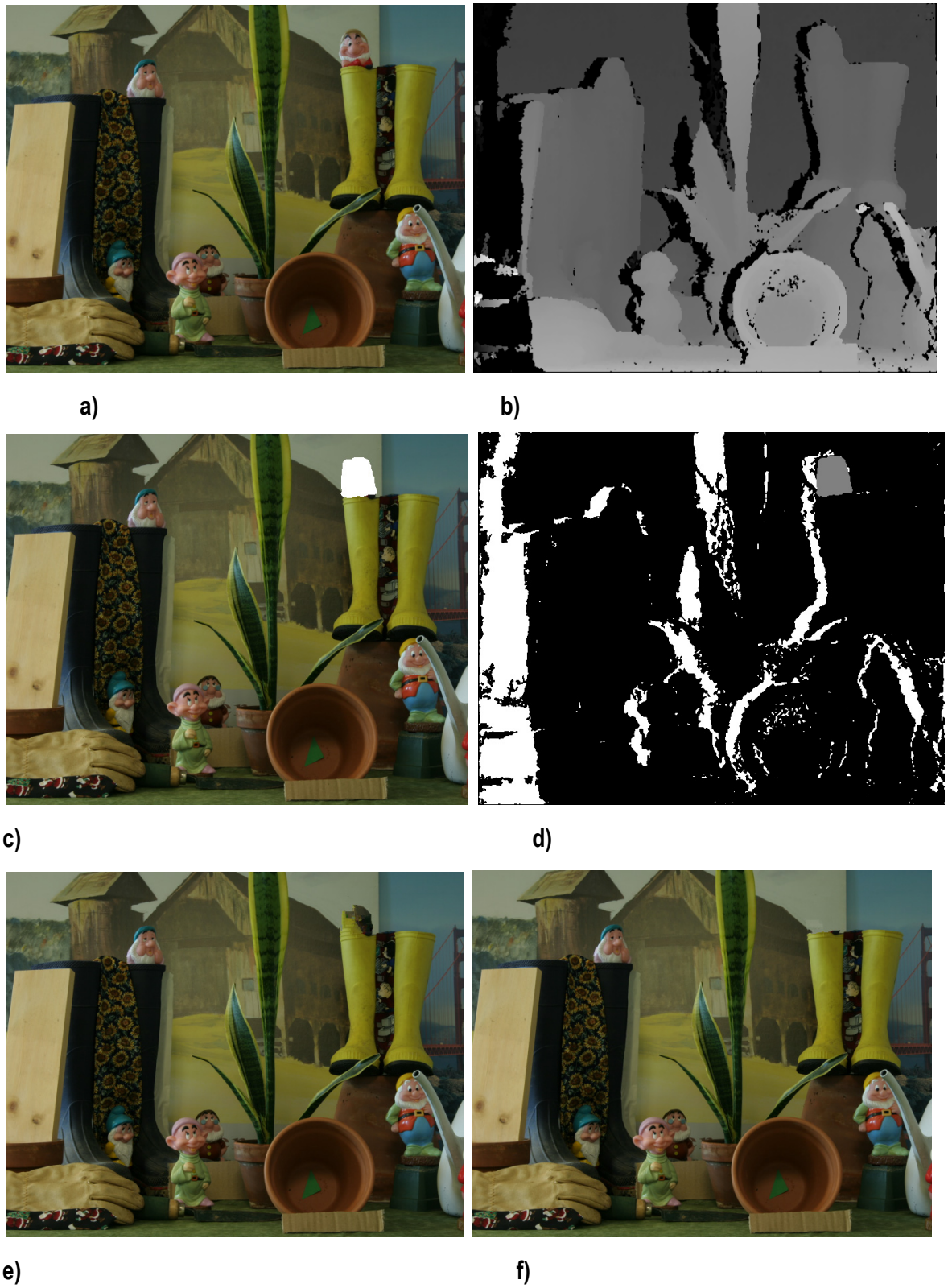
Our approach improves quality of the result because information from impeding foreground objects is not taken and after removing some object, only its background will remain. Execution time of the algorithm is also noticeably increased, because area of source region is limited only by its background, while searching for the most similar patch for every point.

## 4. Experimental results

We implement Criminisi's exemplar based algorithm and our approach on C++ using OpenCV library [Opencv]. In our experiments, we use Middlebury Stereo Datasets [Middlebury, 2014]. For comparing the original algorithm result with our modified result, we use **PSNR** (peak signal-to-noise ratio) [PSNR]**.** PSNR is an approximation to human perception of reconstruction quality. Generally, a higher PSNR value means that reconstruction result is of higher quality. We test algorithms on more than 100 examples and in average, our algorithm is 4-5 times faster than the original one and it gives about 3% increase of PSNR value. Fig. 2,3 show some of examples. We take a default patch size of 9x9 pixels. In inpainting mask from below examples, gray colored region means the region that should be inpainted, black – is the background of it and white – foreground.

In example 1, execution time of Criminisi's algorithm takes 1.01667, our modification – 0.266667 minutes. Average disparity of target region is 362.306 mm. PSNR-s are respectively 35.7465 and 38.4639

In example 2, execution time of Criminisi's algorithm takes 1.08333, our modification – 0.166667 minutes. Average disparity of target region is 34.247027 mm. PSNR-s are respectively 35.7465 and 38.4639.



a) b)

c) d)

e) f)

*Figure 2:* Example 1. Comparison of exemplar based inpainting and our approach. Image size is 695x555, a) the original image , b) depth map, c) inpainted image d) inpainting mask, e) result of Criminisi's algorithm, f) our result

a) b)

c) d)

e) f)

*Figure 3:* Example2. Comparison of exemplar based inpainting and our approach. Image size is 640x480, a) the original image , b) depth map, c) inpainted image d) inpainting mask, e) result of Criminisi's algorithm, f) our result

## 5. Conclusion

In this paper, we introduced a new approach for digital image inpainting, which extended exemplar based inpainting algorithm [Criminisi, 2004], using 3d features of images, such as depth map. We try to improve the case of removing unwanted object from an image. We reduce the source region area of the algorithm, taking in account only background points of removing object. This approach significantly improves execution time of the algorithm and gives better results in most of the cases.

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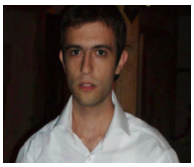

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# STORING INFORMATION VIA NATURAL LANGUAGE ADDRESSING – A STEP TOWARD MODELING HUMAN BRAIN MEMORY

**Krassimira Ivanova, Stefan Karastanev**

***Abstract***. *Our main goal in this paper is to propose a new approach for modeling human brain memory by storing information using only the names but not pointers. This approach gives us new point of view to the human brain processes. In other hand, it simplifies and speeds up the corresponded computer programs. It is called "Natural Language Addressing" (NLA). This approach is a possibility to access information using natural language words or phrases as direct addresses of the information in the computer memory. For this purpose the internal encoding of the letters is used to generate corresponded address co-ordinates.*

***Keywords***: *modeling human brain memory, ontologies, addressing, natural language addressing*

***ACM Classification Keywords***: *H.2 Database Management; H.2.8 Database Applications*

## Introduction

The definition of the concept "intelligence" was given in [Mitov et al, 2010]. It follows from the General Information Theory (GIT) [Markov et al, 2007] and especially from the "Theory of Infos" [Markov et al, 2009].

The intelligence is a synergetic combination of:

- ***(primary) activity for external interaction***. This characteristic is basic for all open systems. Activity for external interaction means possibility to reflect the influences from environment and to realize impact on the environment;
- ***information reflection and information memory***, i.e. possibility for collecting the information. It is clear; memory is basic characteristic of intelligence for "the ability to learn";
- ***information self-reflection***, i.e. possibility for generating "secondary information". The generalization (creating abstractions) is well known characteristic of intelligence. Sometimes, we concentrate our investigations only to this very important possibility, which is a base for learning and recognition. The same is pointed for the intelligent system: "To reach its objective it chooses an action based on its experiences. It can learn by generalizing the experiences it has stored in its memories";
- ***information expectation*** i.e. the (secondary) information activity for internal or external contact. This characteristic means that the prognostic knowledge needs to be generated in

advance and during the interaction with the environment the received information is collected and compared with one generated in advance. This not exists in usual definitions but it is the foundation-stone for definition of the concept "intelligence";

– ***resolving the information expectation***. This correspond to that the "intelligence is the ability to reach ones objectives". The target is a model of a future state (of the system) which needs to be achieved and corresponding to it prognostic knowledge needs to be "resolved" by incoming information.

*In summary, the intelligence is creating and resolving the information expectation* [Mitov et al, 2010].

The concept "intelligence" is a common approach for investigating the natural and artificial intelligent agents. It is clear; the reality is more complex than one definition.

Presented understanding of intelligence is important for realizations of the intelligent computer systems. The core element of such systems needs to be possibility for creating the information expectation as well as the one for resolving it. The variety of real implementations causes corresponded diversity in the software but the common principles will exist in all systems. Summarizing, the artificial system is intelligent if it has:

– Activity for external interaction;

– Information reflection and information memory;

– Possibility for generalization (creating abstractions);

– Information expectation;

– Resolving the information expectation.

Following the definition of the concept "intelligence" given in the General Information Theory (GIT) [Markov et al, 2007], the five main problems to be solved by the science "Artificial Intelligence" were defined in [Ivanova, 2013]. They are to develop more and more "smart":

– sensors and actuators - to realize external interaction;

– memory structures - to learn;

– generalization algorithms - to make abstractions;

– prognostic knowledge generation - to create information expectation;

– resolving the information expectation - to reach objectives.

In this paper we discuss the second problem – "smart" memory structures. Our hypothesis is that we may build model of storing information which is similar but not the same as human brain memory.

The structure of this paper is as follows. In the next chapter we will remember the features of Natural Language Addressing (NLA), than we will discuss storing ontological information by NLA, and finally some practical experiments and implementations will be outlined. Paper ends with main conclusions and propositions for future work.

## Natural Language Addressing

The main idea of Natural Language Addressing (NLA) was presented in [Ivanova et al, 2012; Ivanova et al, 2013a].

We usually make difference between human aspect of the concept "address" and its computer "understanding", where the basic element is an index couple: (name, address). In different sources the "name" is called "key", "concept", etc. The address usually is given by any "number", "pointer", etc.

The NLA approach is based on using human representation of the address (by natural language words) as computer memory address. NLA uses computer encoding of name (concept) letters as address of connected to it information. This way no indexes are needed and high speed direct access to information elements is available. It is similar to natural order addressing in a dictionary book where no explicit index is used but the concept by itself locates definition. For instance, let have the next concept and corresponded definition:

"London: The capital city of England and the United Kingdom, and the largest city, urban zone and metropolitan area in the United Kingdom, and the European Union by most measures."

In computer memory, for example, the definition may be stored at address "FF084920". The index couple is:

("London", "FF084920"),

i.e. at memory address "FF084920" the main text, "The capital … measures." will be stored. To read/write the main text, firstly we need to find name "London" in the index and after that to access memory address "FF084920" to read/write the definition. If we assume that name "London" in the computer memory is encoded by six numbers (letter codes), for instance by using ASCII encoding system London is encoded as (76, 111, 110, 100, 111, 110), than we may use these codes as direct address to memory, i.e. ("London", "76, 111, 110, 100, 111, 110").

One may remark that above we have written two times the same name and this is truth. Because of this we may omit this couple and index, and read/write directly to the address. For human this address will be shown as "London", but for the computer it will be "76, 111, 110, 100, 111, 110".

From other point of view, the array (76, 111, 110, 100, 111, 110) may be assumed as co-ordinates of point in multidimensional (in this case – six dimensional) information space and the definition can be stored in this point.

It is clear, the words have different lengths and, in addition, some phrases may be assumed as single concepts. This means that we need a tool for managing multidimensional information spaces with possibility to support all needed dimensions in one integrated structure.

The independence of dimensionality limitations is very important for developing new intelligent systems aimed to process high-dimensional data [Gladun, 2003]. To achieve this, one need information models and corresponding access methods to cross the boundary of the dimensional limitations and to obtain the possibility to work with information spaces with variable and practically unlimited number of dimensions. Such possibility is given by the Multi-Dimensional Information Model (MDIM) [Markov, 2004] and corresponded to it Multi-Dimensional Access Method (MDAM) [Markov, 1984]. Its advantages have been demonstrated in many practical realizations during more than thirty years. In recent years, this kind of memory organization has been implemented in the area of intelligent systems memory structuring for several data mining tasks and especially in the area of association rules mining [Markov et al, 2012].

The program realization of MDIM is called Multi-Domain Access Method (MDAM). For a long period, it has been used as a basis for organization of various information bases. There exist several realizations of MDAM for different hardware and/or software platforms [Markov, 2004].

## The brain and the computer memories from point of view of NL-Addressing

The idea of NL-addressing was born under the influence of the models of the natural and artificial neuron networks. In general, they may be modeled by graphs. Special kinds of such graphs are ontologies. They represent knowledge for given domain and maybe assumed as partial models of human brain memory. Because of this, storing ontologies is important problem to be solved effectively. A possible solution is using the NLA for storing graphs [Ivanova et al, 2012; Ivanova et al, 2013b] and, respectively, ontologies [Ivanova et al, 2013a; Ivanova et al, 2013d]. It is clear; the human brain does not create indexes. The information processing in the brain looks like our model for NL-addressing. It is very interesting to provide research in this area.

Series of experiments aimed to show the possibilities of NL-addressing to be used for NL-storing of small, middle-size and large datasets were provided.

The experiments began with analyzing the easiest case: NL-storing dictionaries, and after that, NL-storing of thesauruses was realized. An experiment with WordNet thesaurus was provided using the program WordArM based on NL-addressing [Ivanova et al, 2013c].

Special attention was given to NL-storing ontologies [Ivanova et al, 2013d]. Experiments were provided with (both real and artificial) middle-size and large RDF-datasets as well as so called BigData [Markov et al, 2014]. Estimation of the results from series of experiments had shown that storing time:

(1) *Depends* on number of elements in a dataset's instances;

(2) *Not depends* on number of instances in the dataset.

The second is very important for multi-processing because it means linear reverse dependence on number of processors. The same feature we may discover in human brain memory. It works in practically equal speed independently of the amount of stored information. What is important – the NL-addressing models only the possibilities of the left human brain hemisphere, i.e. – the linguistic storing and accessing the information.

The NLA storing time varies between 2.2 and 2.5 milliseconds.

The next graphic illustrates independence from size of the datasets. On Figure 1 the storing time of 100 millions data instances is illustrated. The regularity is the constant time for storing independently of the number of already stored instances. [Markov et al, 2015]

This time does not depend on the size of data sets, i.e. of the number of instances. For comparison with traditional indexed databases with logarithmic complexity, the graphic of function ***log n*** in comparison with graphic of NLA approach (RDFArM realization) is shown in Figure 2. [Markov et al, 2015]

The information processing in the human brain looks like our model for NL-addressing. Neurons react for less than 4 ms independently of the amount of stored information (Figure 3) [Forehand, 2009].

NL-ArM has similar to the neurons' individual time for reaction – 2.5 – 3 ms. In addition; it seems that the human brain does not create indexes. A very important feature of NLA is that the database can be updated dynamically without recompilation of the database as it looks like in the human brain.

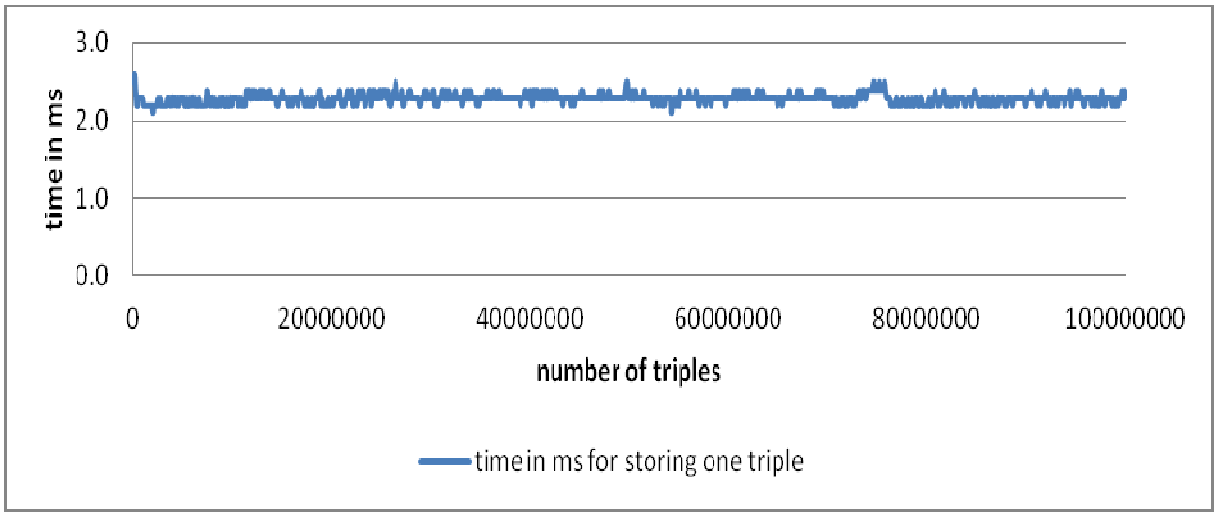


Figure 1. Storing time for 100 millions instances

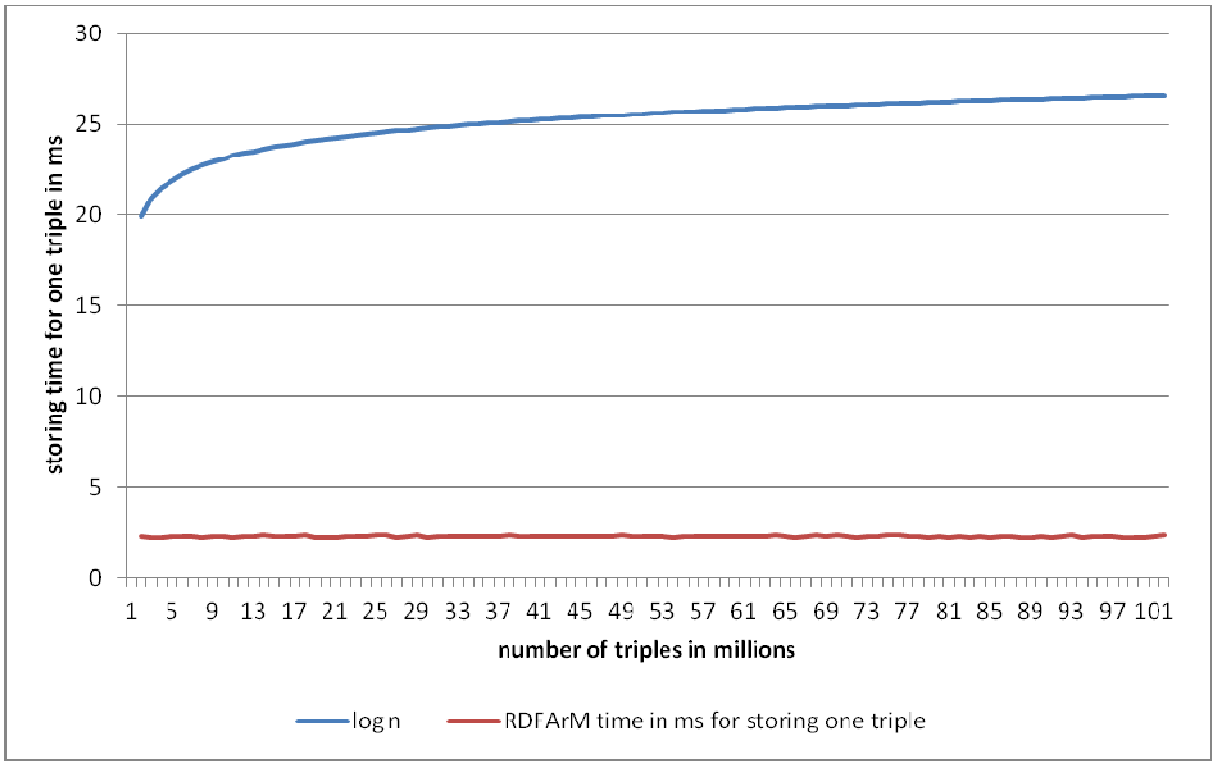


Figure 2. Comparison of log n and average time in ms for NLA storing one instance

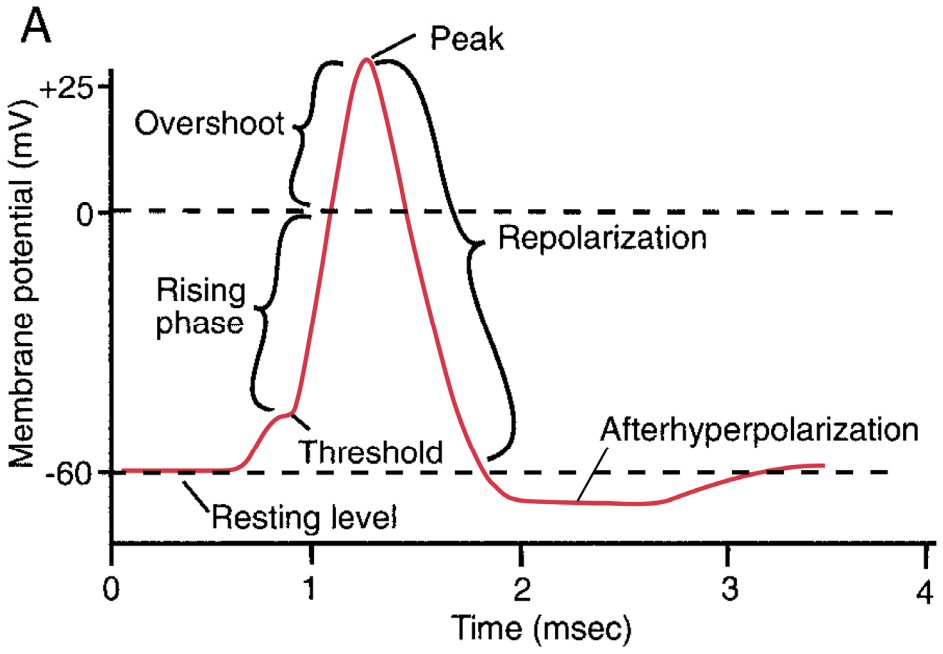


Figure 3. The phases of a neuron action potential:

Depolarization to threshold, the rising phase, overshoot, peak, repolarization, afterhyperpolarization, and return to the resting membrane potential. [Forehand, 2009]

## Practical Aspects of Implementation and Using of NL-Addressing

NL-addressing is approach for building a kind of so called "post-relational databases".

The NL-addressing software realization is a part of an instrumental system for automated construction of ontologies "ICON" ("Instrumental Complex for Ontology designatioN") which is under development in the Institute of Cybernetics "V.M. Glushkov" of NAS of Ukraine [Palagin et al, 2011].

The work with ontologies is supported by a tool called OntoArM. Using it, the ontological elements can be organized in ontological graph spaces with variable ranges. There is no limit for the ranges of the spaces. Every ontological element may be accessed by a corresponding multidimensional space address (coordinates) given via word or phrase. Therefore, we have two main constructs of the physical organizations of OntoArM – ontological spaces and ontological elements. A separated ontology may be represented by OntoArM in one single archive. In addition, the NL-addressing permits accessing the equal names in different ontologies without any additional indexing or using of pointers, identification and etc. Only the NL-words or phrases are enough to access all information in all existing ontologies (respectively, graphs) [Ivanova et al, 2013b].

## Conclusion

Storing information via natural language addressing as a step toward modeling human brain memory was discussed in this paper.

We remembered the features of Natural Language Addressing (NLA) and discussed storing ontological information by NLA, Some practical experiments and implementations were outlined.

The starting point of our work is the understanding of intelligence as a synergetic combination of: (1) *(primary) activity for external interaction; (2) information reflection and information memory*; (3) *information self-reflection*; (4) *information expectation;* and (5) ***resolving the information expectation***.

In this paper we discuss the second characteristic connecting the problem of developing the "smart" memory structures. Our hypothesis is that we have built a model of storing information which is similar but not the same as human brain memory.

We have implemented our new approach for storing ontologies using only the names but not pointers and this way to simplify and to speed up the corresponded computer programs. The approach is called "Natural Language Addressing" (NLA). This approach is a possibility to access information using natural language words or phrases as direct addresses of the information in the computer memory. For this purpose the internal encoding of the letters is used to generate corresponded address co-ordinates. The tool OntoArM, based on the idea of NLA approach, and experiments based on it were outlined.

What we gain and loss using NL-Addressing?

The loss is additional memory for storing internal NL-address structures. But the same if no great losses we will have if we will build balanced search threes or other kind in external indexing. It is difficult to compare with other systems because such information practically is not published.

The benefit is in two main achievements: (1) High speed for storing and accessing the information; (2) The possibility to access information immediately after storing without recompilation the database and rebuilding indexes.

NL-addressing looks like the information processing in the human brain. NLA has similar to the neurons' individual time for reaction and similar possibility to be in work condition permanently without timeouts for recompilation of the database after storing new information. The amount of stored information by NLA is practically not limited. All these characteristics of NLA give a good starting point for modeling human brain memory.

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# CREATING A TRAINING SIMULATOR FOR THE ECONOMIC DISCIPLINES AND ITS FUNCTIONALITY

**Tea Munjishvili; Zurab Munjishvili**

***Abstract***: *One way to improve the quality of distance learning subjects in economics is the development and application in the process of training of trainers and simulators. The article defines the necessary requirements to simulators for the subjects of economic profile. The technology training and learning using the software package developed by the author - trainer "Cyber2T". Attention is concentrated on the stage of the debriefing. A method of analyzing and summarizing the results of the training is done using logical probabilistic model to predict the possible errors and assumptions form the recommendations to correct them."Cyber2T" designed to "Cyber2" basis -. The system of identifying and evaluating the development of knowledge "Cyber2T" written in VB.NET 2010, the database is organized to SQL Server 2008.*

***Keywords***: *"Cyber2T"; Simulators-trainer; knowledge systems; Simulator of economic subject.*

## Introduction

Various kinds of trainers and simulators have been employed for training the complex technical system exploitation and operation specialists. The article describes [1] the trainers used in the Russian Federation. In the recent years, various medical universities worldwide have been widely employing simulators [2] for training and professional advancement of the medical personnel. It may be said, that mainly physical, electronic and combined simulators are being developed.

There are multiple type and purpose simulators employed not only for training the technical system exploitation and operation specialists and medical personnel but those to be used in their further professional activities.

It should be said though that merely electronic trainers and simulators can be developed and employed where training of economists is concerned. Electronic trainers are broadly developed and exploited in training the operators, e.g. the railway ticket or the passenger carriage heating system operators, etc [3].

There are some interesting studies (theses) depicting development of the trainers designed for various fields of economics [4, 5].

Analysis of the available materials reveals disparity of opinions concerning the concept of electronic trainers and simulators. Some of the scholars state that the electronic trainer is a combination of a hypertext-based manual and software and data complex. The electronic manual is for familiarization and comprehension of an issue, while the software complex is conducive to remembering it. In an interactive mode, a student solves tasks and answers theoretical questions. IN answering a student uses textual, graphical or video prompts and gets scores, etc. The Training may be performed repeatedly, with its content, time etc. changed at one's own discretion. It is distant, individual learning.

The simulation modeling implies the software and data environment in which a specific, real-life, typical situation is being modeled. In the instruction, the simulators are used for the purpose of learning how to manage the situations concerning a real, physical item, make the relevant decisions and analyze its results.

Based on the available data, the students are taught how to model a specific situation, see and assess the anticipated results. It should be said though that similarly to the electronic diagnosing, however accurate a situational model may be, adequate modeling of a real - life situation is virtually impossible. Therefore, a decision-maker has the final say and has to bear responsibility.

The topics discussed in a specific subject include consideration and comprehension of typical finite situations, with simulation models to be developed for them.

Therefore, there are two related tasks in terms of training and professional advancement of economic experts: firstly, a uniform software package of an electronic trainer has to be developed and secondly, a software package for development of simulators has to be made up.

The first task consists of two related sub-tasks: compilation of a hypertext-based e-manual and creation of a software environment for distance solving of problems in an interactive mode. Theoretical and practical aspects of compilation of hypertext-based e-manuals have been widely discussed in the professional literature. Some simple editing software are available by means of which, a trained professor of economics can compile an e- manual. A number of them containing texts, graphics and videos are available nowadays. After familiarization with the material in them, one should get over to some hands-on work, such as solving tasks, discussion and analysis of theoretical issues etc. and that's what the trainer software is helpful in.

For economic subjects, we developed operational and strategic management (Prof. Badri Ramishvili) included in the curriculum since 2013, financial functions in Excel (Prof. T. Munjishvili), a trainer (a manual and a trainer software [6-10]. The training is conducted by "Cyber 1" software package. [11]. Consideration of its operational strengths and weaknesses led to creation of "Cyber 2T" trainer software allowing:

1. an instructor to formulate a typical training assignment by subjects, topics and sub-topics;

2. prior to the training, a student:
   - selection of a training language (Georgian, English, Russian, Ukrainian etc.);
   - formulation of an individual task on the grounds of a typical assignment;
   - definition of the training time ;
   - according to the assignment, probabilistic arrangement and selection of the tests or performance of a training in the order specified in the assignment;
   - changing the training trajectory according to the student's answers;
3. diagnosis of mistakes and showing the ways to prevent them;
4. introduction of the concept of complexity of the assignment topics and sub-topics;
5. textual, graphic, video or combined subject or test-specific aids;
6. working on the open or closed tests;
7. setting a task by way of textual or graphic or video or textual and graphic or textual and video information;
8. in the closed tests, availability of at most three correct answers out of maximum seven options;
9. giving answers only upon marking the necessary number of correct answers and giving the "answer" order;
10. availability of any number of answers in the open tests;
11. marking the $q_i \in Q$ score for the $\forall n_i, n_i \in N$ answer. The score may be an integer or a decimal positive number;
12. use of words, numbers, a sentence or a combination thereof, as well as an abbreviation in an open test;
13. comprehension of an unsynchronized statement or in case of addition of words;
14. writing words used in a statement or generally answers in any case or using wrong spelling;
15. identification of the actual answers to the tasks depicting subjects, topics, sub-topics with regard to the reference value (standard meaning) thereof;
16. journaling the training proceedings upon its completion;
17. in the training, showing various diagnostic notices; e.g. in case of use of an unknown word or omission of words, numbers etc.
18. continuation of a test suspended for a technical etc. reason from the breakpoint.

G is the in advance known finite set. Elements of the set are natural language sentences. Number of words from $g_{i,\lambda}$, of dictionary L, used in $g_{i,\lambda} \in G$, are depended on subjects put in exam and on the sentences which are used in open answers of this subject tests. Every element of $g_{i,\lambda} \in G$ are different at least with one word.

$$\forall g_{i,\lambda}(g_{i,\lambda} \in G) \wedge g_{i_0}(g_{i,\lambda} \in G) \Rightarrow g_{i_0,\lambda} \setminus (g_{i_0,\lambda} \cap g_{i,\gamma}) \neq \otimes \wedge g_{i,\lambda} \setminus (g_{i_0,\gamma} \cap g_{i,\lambda}) \neq \otimes$$

Every pair can be consisted of words same by content.

$$< g_{i_0,\lambda}, g_{i,\lambda} > g_{i_0,\lambda} \subset G, g_{i,\lambda} \subset G$$

$$\exists \forall g_{i,\lambda}(g_{i,\lambda} \in G) \equiv g_{i_0,\lambda}(g_{i_0,\gamma} \subset g_{i,\gamma}) \qquad [1]$$

Here sign $\equiv$ means that, two, $< g_{i_0,\lambda}, g_{i,\lambda} >$ sentences are equal by content.

In the system of knowledge revealing and rating "Cyber-2T", for any answers, phrases, of each tests is relevant to $g_{i,\lambda} \in G$ sentence. Desired result can be reached, if $g_{i,\lambda}$ will be discussed as a product, and words $a_i \in L$ included in it as a condition. In this case, "Semantic analysis" will be reduced to searching of productive system, dictionary organization and relevant productions of received facts. Actually, particular production is selected, because we discuss particular answer with known meaning. It is needed to find out relevance of received facts (words) to existed facts. So:

Is given: non-empty set of predicates, natural language words of open tests in given subjects $\{L = \{a_k\} \neq \otimes \ k = 1...n$ , non-empty set of actions $K = \{k_{j_0}(a_{k_{j0}})\}$ $j_0 = 1....m$, and non-empty set of informativeness $K = \{k_{j_0}(a_{k_{j0}})\}$ $j_0 = 1....m$ . Where, elements of the set $m \in N$ are scalar functions on G and using them is measured some parameters with defined scalar. Every criteria of the in formativeness has some weight $h_0$ in way, that

$$h_0 \in Z, \quad \sum_{\alpha}^{\beta} h_\alpha = 1. \qquad [2]$$

Conditions of using a trainer:

1. Regular operation of computers, computer network, server and software. It's a must;

2. Discussion of the syllabus topics and sub-topics by an instructor, setting up a test (task)-base according to the e-manual;

3. Creation of a comprehensive textual, graphic, video aid to the tests;

4. Periodic information updating in the test aid base, adding up new ones and modification of those available;

"Cyber 2T" is multilingual. Various subjects may be presented in several languages.

Here we'll focus on conditions 1-4. For a topic, an instructor makes up a typical assignment specifying the training language, number of the topic/sub-topic, name, test numbers, including those to be used in the training, test type (closed /open), total number of questions for the closed tests, including the number of right answers, kind of appraisal (percentage /non-percentage), maximal score for the right answers in the open or closed tests, number of the right answers in the open test. Before the test, at a student's order the following table shows up:

Here the student can change the training time, appraisal (percentage /non-percentage), number of the training tests, maximal score or the training language. For instance: if a student doesn't want021 topic tests to be used in the training, he will write 0 in the relevant space. He can select any number of topics from those offered.

With "Cyber 2T", a subject is taught in the order below:

**Lecture**

Focusing on the key issues, analysis of the material contained in the e-manual. The video footage of the lecture is published on the website.

↓

**Seminar**

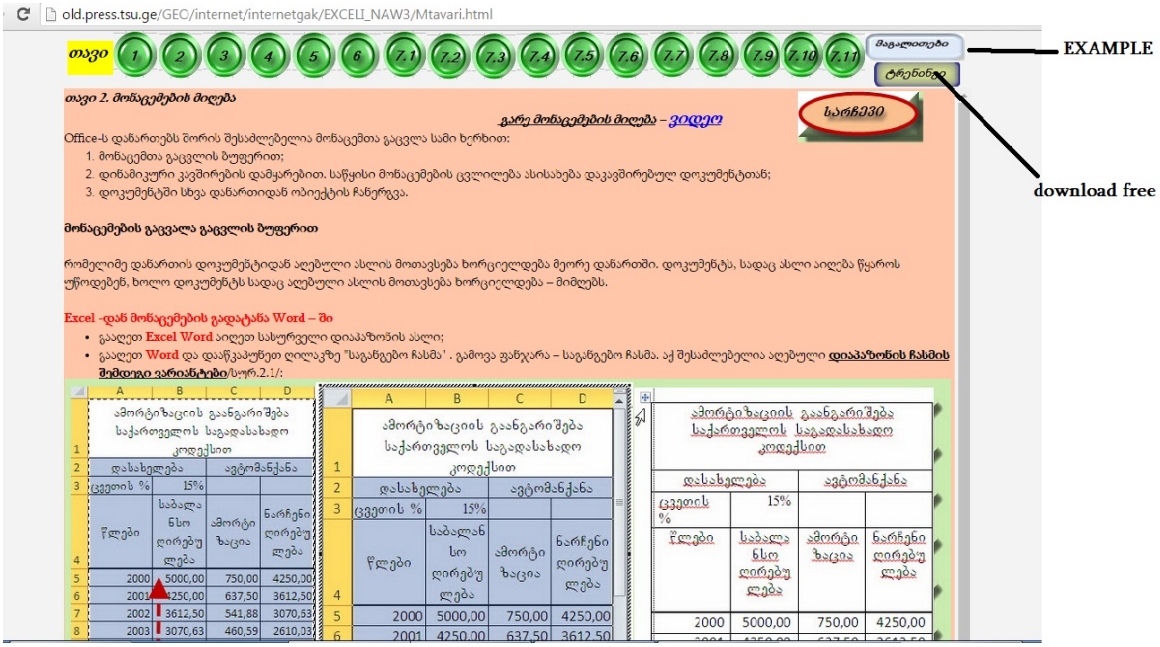
- Solving typical tasks, the lecturer points out at the peculiarities, the techniques etc. He focuses on the analysis of the results and presentation forms thereof;
- Students solve tasks in the "Cyber 2T" trainer environment.

↓

Debriefing

In the knowledge mastering scheme, we'll focus on debriefing, i.e. discussion of the training results, which takes place at actually every session without being set apart.

Figure1. Educational process




Pic.2. Example online course "EXCEL"

## Conclusion

In working with a trainer, the students' mistakes appear on the lecturer's computer screen as the training protocol, which the lecturer analyses and after the training discusses the results (debriefing). The debriefing is an informal procedure, in his publication [A.A. Svistunov] /2/ details its tasks and goals, focusing on the lecturer's (leader of the debriefing) role. He states that it is the leader's task to create a creative atmosphere, adding that emphasizing a student's mistake is unacceptable. Instead, the lecturer should describe the mistake as the one made by the group so that all the students could express their views about it. Such an approach is conducive brainstorming, search for better solutions to tasks etc.

The requirements to the debriefing leader depicted in /2/ are not new. They are detailed in the intellectual process regulation method - the study /12/ in which the psychoneurotic programming method is set out.

In preparing the subject trainer materials and making changes to them, the lecturer will take the students' views and proposals into account.

From 2016-2017 academic years Financial Analysis is going to be taught by 'Cyber 2T" trainer we have developed. The instruction at the National Commerce University in Kiev, the Tbilisi Iv., Javakhishvili State University and D. Robakidze University, Tbilisi will be in the Georgian , Ukrainian and English languages.

Given the trainer specifics, "Cyber 2T" is the modification /13/ of the "Cyber 2", the software package for presentation and assessment of knowledge that we developed. It was written on VB.NET 2010, the data base is on SQL Server 2008.

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