A MULTI-CRITERIA APPROACH TO THE RESOURCE DISTRIBUTUON

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***Abstract**: The problem of distribution of the given global resource of the system under the constraints, imposed on individual resources is considered. It is shown, that the problem lies in constructing an adequate objective function for optimization of the resources distribution under their limitations. For solving the considered problem, the multicriteria optimization approach is undertaken with the nonlinear trade-off scheme. The proposed nonlinear compromise scheme has the property to adapt to the situation of multicriteria decision-making. The adaptation to the situation of a nonlinear scheme is carried out continuously, while the traditional selection of compromise schemes is done discretely that adds to subjective errors the errors, associated with the quantization compromise schemes. Model examples are given.*

***Keywords**: Multicriteria Allocation, Limited Resources, Criteria of Performance, Multicriteria Optimization, Nonlinear Compromises Scheme.*

***ITHEA Keywords**: H.1 Models and Principles – H.1.1 – Systems and Information Theory; H.4.2 – Types of Systems.*

Introduction

In various areas of management and economics the problem of such resource distribution of the controlled system between the individual elements (objects), which provides the most effective functioning of the system under the given conditions, is urgent [Voronin, 2017]. The problem of allocating of limited resources is a main problem of economics. It is believed that the proper distribution and redistribution of resources – this is just the economics. Similar problems arise in other subject areas. The art is to allocate properly limited resources, depending on the circumstances.

Often the problem is solved subjectively, on the basis of the experience and professional qualifications of a decision maker (DM). In simple cases, such approach may be justified. However, when there is a large number of objects and for important cases, the price of the error of management decisions sharply increases. The development of the formalized methods of decision making support, for competent resource distribution between objects, taking into account all the given circumstances, becomes urgent.

One of such circumstances is usually resources limitation. The most prevalent is the case of upper limitation of a total (global) system resource to be distributed among the individual objects. The problem of redistribution of resources, while decreasing the previously planned level of projects funding, is considered, in particular, in [Voronin, 2004].

In practical cases, constraints are imposed not only on the global resource, but also on the individual resources, given to individual objects. The constraints may be imposed both from below and from above. Such constraints either are known in advance, or are determined by technical and economic calculations or by peer review methods. One should distinguish the conditional limitations (when the violation of limits is not desirable) and limitations unconditional (when their violation is physically impossible).

Example 1

To run several flights to different cities the airport has a certain fuel resource to be distributed between the aircrafts. For every flight there is a lower limit below which the fuel providing is pointless, because the plane just will not fly to its destination. This is the essence of the lower limit for every individual resource. If the given flight obtains the fuel above the certain lower limit, it has, on the one hand, an opportunity to maneuver freely by echelons, bypass a thunderstorm, going away to an alternate airfield, etc. On the other hand, the partial resource can not be increased unlimitedly too, since there is an upper bound of the resource. This is understandable, since every aircraft has a certain capacity of tanks and physically it cannot take on board more fuel.

But usually the upper limit is introduced as conditional and assigned by the flight plan. Taking into account this set of constraints, it is required to allocate the global resource of fuel between flights to ensure the most effective operation of the airport as a whole.

Example 2

In the planning and designing organization the order for the development of several projects is received. To fulfill the order, the specific funding is provided, which is to be distributed among the individual projects. For every project the minimum level of funding, below which fulfillment of the project is impossible, is known. Usually there are protected items of the estimate – the salary of employees, rent, utility payments, cost of an absolutely necessary equipment, etc. It is clear that, with minimal funding the quality of the project would be appropriate. The funding increase makes the development of the project more effective. But it is possible to increase the funding amount to the certain limit, constraint by the total estimated cost of the project. Exceeding this limit is called the non purposeful spending funds and threatens sanctions. Taking into account the mentioned limitations from above and below, it is necessary to distribute the global amount of funding between projects so, that the work of the planning and designing organization as a whole would be the most effective.

It easy to see, that for all the individual resources the sum of the constraints from below is a lower bound for the global resource, and the sum of the constraints from above restricts the global resource from above.

The problem lies in constructing an adequate objective function to optimize resource distribution under the condition of their limitation. A simple uniform distribution in this case is not suitable, since it can put some objects on the verge of the impossibility of their functioning, while other objects obtain an unreasonably great resource.

In the present work for solving the considered problem, the approach of multicriteria optimization, using the nonlinear trade-off scheme [Voronin, 2017], is used.

Problem Formulation

Since the considered problem is urgent for different domains, we shall present the problem formulation in a general form (Figure 1).

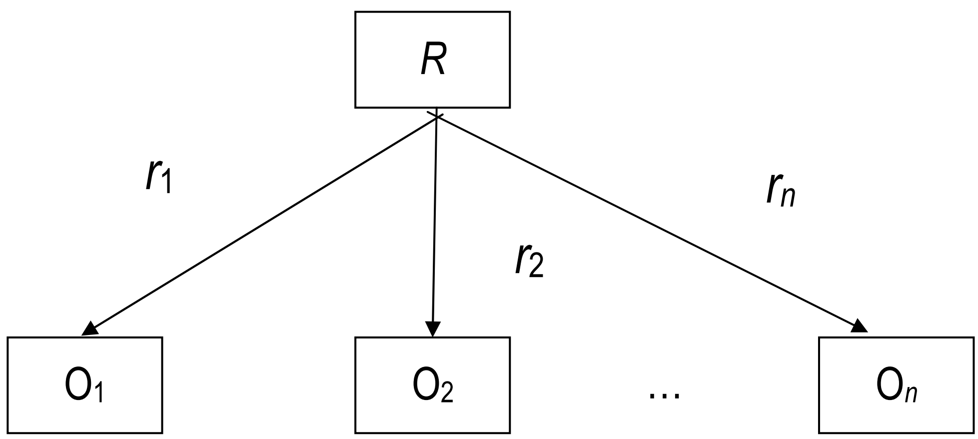

Figure 1. Resource distribution

The global resource *R* is given, which is to be allocated between *n*≥2 elements (objects) O_1 , O_2 … O_n of the system, every one of which is provided with the individual resource r_i , their set forming the vector $r = \{r_i\}_{i=1}^n$.

The formula for determining the domain of this vector has the form

$$r \in X_r^\circ = \{r | 0 \le r_i \le R, i \in [1, n]\}. \quad (1)$$

At the same time, the condition

$$\sum_{i=1}^{n} r_i = R \quad (2)$$

holds true.

For every object it is known (or determined by the peer review method), the maximum permissible value of the resource $B_{i\min}$, below which the given object can not function. Thus, the given system is constraint from below

$$r_i \ge B_{i\min}, \sum_{i=1}^{n} B_{i\min} \le R, i \in [1, n]. \quad (3)$$

At the same time, for every object the quantity $B_{i\max}$ is known, which can not or should not be exceeded by the object resource. The system of constraints from above has the form

$$r_i \le B_{i\max}, \sum_{i=1}^{n} B_{i\max} \ge R, i \in [1, n]. \quad (4)$$

From (3) and (4) it follows that

$$B_{i\max} \geq r_i \geq B_{i\min}, i \in [1,n], \tag{5}$$

$$\sum_{i=1}^{n} B_{i\max} \geq R \geq \sum_{i=1}^{n} B_{i\min}. \tag{6}$$

In view of (5). expression (1) is transformed to the form

$$r \in X_r = \{r | B_{i\max} \geq r_i \geq B_{i\min}, i \in [1,n]\}. \tag{7}$$

Let us consider the polar (degenerate) cases of inequality (6). If $R = \sum_{i=1}^{n} B_{i\min}$, then the considered problem is reduced to such distribution of the global resource, for which every object obtains its minimum allowable individual resource: $r_i{*} = B_{i\min}, i \in [1,n]$.

If the global resource can fully satisfy the needs of the objects, i.e., $R = \sum_{i=1}^{n} B_{i\max}$, then the problem is solved as $r_i{*} = B_{i\max}, i \in [1,n]$.

Thus, in the polar cases of inequality (6) the considered problem has trivial solutions. And only if the expression (6) becomes a *strict* inequality

$$\sum_{i=1}^{n} B_{i\max} > R > \sum_{i=1}^{n} B_{i\min} \tag{8}$$

the problem of optimizing distribution of limited resources gets the sense.

The problem is formulated: under the condition (8) to define such individual resources $r{*} \in X_r$, for which requirement (2) is fulfilled and some objective function $Y(r)$, which type should be selected and justified, takes the extreme value.

Method of Solution

In the problem of optimizing the distribution of limited resources the limit $r_i \le B_{i\max}, i \in [1,n]$ from above, is considered as a simple optimization constraint, the approaching to which does not threaten the system very much. Quite a different meaning has the limit $r_i \ge B_{i\min}, i \in [1,n]$ from below. The resource approaching this limitation threatens the very possibility of the appropriate object functioning. One can say, as in [Voronin, 2017] is said, that “the limitation from below is “criteria-forming” in the sense that the objective function must increase the difference between the individual resource and its limit from below”.

Therefore, the expression of the desired objective function should: 1) include constraints from below in the explicit form, 2) penalize the system for the partial resources approaching these constraints, 3) be differentiable by its arguments.

The simplest objective function, satisfying these requirements, is

$$Y(r) = \sum_{i=1}^{n} B_{i\min}(r_i - B_{i\min})^{-1}. \quad (9)$$

The analysis of formula (9) shows that this is nothing but an expression of the scalar convolution of the *maximized* individual criteria $r_i, i \in [1,n]$, by the nonlinear trade-off scheme (NTS) in the problem of multicriteria optimization [Voronin, 2014]. Indeed, in the considered problem the resources $r_i, i \in [1,n]$, have a dual nature. On the one hand, they can be considered as independent variables, the *arguments of optimization* of the objective function $Y(r)$. On the other hand, for everyone of the objects it is logical the desire to maximize its individual resource, to go away as far as possible from the dangerous limit $B_{i\min}$ to improve the efficiency of its operation. From this point of view, the resources $r_i \ge B_{i\min}, i \in [1,n]$ can be regarded as individual quality *criteria* of operation of the corresponding objects. These criteria are subject to maximization, they are limited from below, nonnegative and contradictory (the increase of one resource is possible only at the expense of reducing the other).

The NTS concept is based on the principle “away from the constraints”. It is assumed that the DM utility function estimates as preferable those solutions that give the greater remoteness of the criteria from hazardous constraints. The scalar convolution $Y(r)$ is a model of the utility function and includes in the explicit form the difference $r_i - B_{i\min}$, as a characteristic of tension of the decision making solution. This allows one to penalize the criteria for the approximation to their limits.

On the ground of the told above, the problem of vector optimization of limited resources, taking into account the isoperimetric constraint for arguments, becomes

$$r^* = \arg\min_{r \in X_r} Y(r) = \arg\min_{r \in X_r} \sum_{i=1}^{n} B_{i\,\min}(r_i - B_{i\,\min})^{-1}, \sum_{i=1}^{n} r_i = R. \qquad (10)$$

Problem (10) can be solved both analytically, using the Lagrange method of multipliers, and by numerical methods, if analytical solution is difficult.

The analytical solution involves the construction of the Lagrange function in the form

$$L(r,\lambda) = Y(r) + \lambda\left(\sum_{i=1}^{n} r_i - R\right),$$

where λ is an indefinite Lagrange multiplier, and solving the system of equations

$$\frac{\partial L(r,\lambda)}{\partial r_i} = 0, i \in [1,n];$$

$$\frac{\partial L(r,\lambda)}{\partial \lambda} = \sum_{i=1}^{n} r_i - R = 0.$$

For solving multicriteria problems by numerical methods, using the NTS concept and the constraints on the arguments and criteria, the algorithms are developed and the computer program is written [Voronin at al, 1999].

The Program of Vector Optimization

For solving a vide range of optimization problems the vector optimization program TURBO-OPTIM is developed and described in [Voronin at al, 1999]. The program is executed in language Borland C++3.1, using the Turbo Vision library, which provides the efficient use of computer resources, standardized and convenient environment for the user, the ease of modification and debugging.

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For working with the program, the following steps are to be fulfilled:

- select a set of individual criteria so that they all would take positive values and require minimization;
- determine the admissible limit value for every criterion;
- select a set of parameters (independent variables), on which the particular criteria depend;
- determine the range of variation of every parameter (minimum, initial (starting) and maximum values);
- set for the parameters the constraints in the form of the inequalities $g_j(r) \leq 0, j \in [1,k]$, where k is a number of constraints;
- determine the type of the dependence of the individual criteria upon the parameters.

The program allows solving the optimization problems for the following cases of connection of the individual criteria with the arguments of the optimization (parameters):

- the criteria are expressed through the parameters explicitly, analytical dependences are known;
- the criteria are some functionals, and for calculation of their values, the solution of the system differential equations is needed;
- the dependences of the criteria upon the parameters are not known, and for parameters determination the experiments are needed;
- the criteria values can be obtained by running the program written by a user;
- there is a table of the dependences of the individual criteria upon the parameters.

In every of the cited cases, the program gives a user the means of finding the minimum of the generalized criterion, built by the nonlinear trade-off scheme, by one of the methods of optimization: 1) the method of simplex planning in the Nelder-Mead modification and 2) the nonlocal method of nonlinear programming (dual optimization method, ([Voronin at al, 1999]).

Illustrative Examples

Example 1. To perform two flights (*n*=2) the airport has fuel, totaling *R*=12 tons (figures are conditional). The minimum requirement of the first flight is $r_1 \geq B_{1\min}$ =2 tons, the second is $r_2 \geq B_{2\min}$ =5 tons. They are limits from below for the individual resources. The oil tanks capacity of the first aircraft is $B_{1\max}$ =7 tons, the second is $B_{2\max}$ =10 tons. They are limits from above.

Condition (8) as a strict inequality (dimensions are omitted)

$$7 < R = 12 < B_{1\min} + B_{2\min} = B_{1\max} + B_{2\max} = 17$$

is observed. Hence, the problem of optimizing the distribution of limited resources can be posed and the solution will be nontrivial.

Problem 1. It is necessary to get the analytical solution of compromise-optimal distribution of fuel between the flights.

The Lagrangian function is built

$$L(r,\lambda) = B_{1\min}(r_1 - B_{1\min})^{-1} + B_{2\min}(r_2 - B_{2\min})^{-1} + \lambda(r_1 + r_2 - R).$$

The system of the equations is obtained

$$\frac{\partial L(r,\lambda)}{\partial r_1} = -B_{1\min}(r_1 - B_{1\min})^{-2} + \lambda = 0;$$
$$\frac{\partial L(r,\lambda)}{\partial r_2} = -B_{2\min}(r_2 - B_{2\min})^{-2} + \lambda = 0;$$
$$r_1 + r_2 - R = 0.$$

Substituting the numerical data

$$-2(r_1 - 2)^{-2} + \lambda = 0;$$
$$-5(r_2 - 5)^{-2} + \lambda = 0;$$
$$r_1 + r_2 - 12 = 0,$$

and solving this system by the Gauss method (successive elimination of variables), we obtain

$$r_1{*} = 3{,}94\,\text{tons}, r_2{*} = 8{,}06\,\text{tons}.$$

The posed problem is solved under assumption that the relative importance of both flights for DM is the same. Otherwise, the weighting coefficients α_1 and α_2, reflecting the individual DM preferences, are introduced in the objective function. These coefficients should be normalized and defined on the simplex:

$$\alpha_1, \alpha_2 \in X_\alpha = \{\alpha_i \left| \alpha_i \geq 0, \sum_{i=1}^{n=2} \alpha_i = 1, i \in [1;2]\right.\}.$$

Example 2. In the design office the order for the design and manufacture of scaled-down prototypes of aircrafts of the three species (*n*=3): 1) passenger, 2) transport, 3) sport and training, is received. To fulfill the order, the financing of the total volume R=10 million UAH (hereinafter figures are conditional) is provided.

The complete budget for every project (limits from above) is calculated:

$$r_1 \leq B_{1\max} = 7\,\text{m UAH}; r_2 \leq B_{2\max} = 5\,\text{m UAH}; r_3 \leq B_{3\max} = 4\,\text{m UAH}.$$

By means of economic calculations the minimum amounts of funding the individual projects, below which the design is not possible (limits from below), are determined:

$$r_1 \geq B_{1\min} = 2\,\text{m UAH}; r_2 \geq B_{2\min} = 1\,\text{m UAH}; r_3 \geq B_{3\min} = 0{,}5\,\text{m UAH}.$$

Condition (8) is a strict inequality (dimensions are omitted):

$$\sum_{i=1}^{n} B_{i\max} = 16 > R = 10 > \sum_{i=1}^{n} B_{i\min} = 3{,}5$$

so the above technique can be applied to non-trivial optimization of distribution of limited resources.

Problem 2. By using the vector optimization TURBO-OPTIM program, find the compromise-optimal values of the individual fundings r_1*, r_2* and r_3* for the design and manufacture of the scaled-down prototypes of the passenger liner, transport aircraft and sport, respectively.

On the basis of the stages of work with the program, one sets: the “analysis” mode, method of “simplex-planning” optimization (default) and then enter the numerical data (the dimensions are omitted):

$$r_{1\min} = B_{1\min} = 2; r_{1start} = 3; r_{1\max} = B_{1\max} = 7;$$
$$r_{2\min} = B_{2\min} = 1; r_{2start} = 3; r_{2\max} = B_{2\max} = 5;$$
$$r_{3\min} = B_{3\min} = 0.5; r_{3start} = 3; r_{3\max} = B_{3\max} = 4;$$
$$r_1 + r_2 + r_3 - 10 = 0;$$
$$y_1 = 1/r_1; y_2 = 1/r_2; y_3 = 1/r_3;$$
$$y_{1\max} = A_1 = \frac{1}{B_{1\min}} = 0.5; y_{2\max} = A_2 = \frac{1}{B_{2\min}} = 1; y_{3\max} = A_3 = \frac{1}{B_{3\min}} = 2.$$

After this, the command “execute” is given, and the program determines the desired values of the individual fundings of the projects:

$$r_1* = 4{,}945\,\text{m UAH}; r_2* = 3{,}083\,\text{m UAH}; r_3* = 1{,}972\,\text{m UAH}.$$

The obtained result corresponds to the unified version of the convolution by the nonlinear trade-off scheme, which is applied for the public use.

Individual Preferences and Unification

As noted above, the choice of a compromise scheme is the prerogative of a human, a reflection of his subjective utility function for solving specific multicriteria problem. Nevertheless, it was possible to identify some patterns and on this objective basis to construct a scalar convolution of criteria, the form of which is determined by the substantive views of essence of the studied phenomenon (NTS). The phenomenon of the individual preferences of the decision maker is formally represented by the presence of vector α in the structure of substantional model (11):

$$Y(a, y) = \sum_{i=1}^{n} a_i A_i [A_i - y_i]^{-1}. \quad (11)$$

There are various estimates of the role of subjective factors in solving multiobjective problems. Subjectivity is acceptable, and even desirable, if this problem is solved in the interests of a particular person. Indeed, if a human operator in ergatic system has the possibility to adjust the parameters of the workplace under his own individual characteristics, it improves the quality of the controlled process; suit made to measure in the customer's company, usually better than store-bought ready-made clothes, etc.

Therefore, the mechanism of individual preferences is rather intensively used in the practice of multicriteria decision problems solution.

However, subjectivity in their decision is permissible and desirable only as long as the result is intended for a specific decision-makers or narrow groups of people with similar preferences. If it is intended for general use, it has to be completely objective, standardized. In these cases, the mechanism of the individual preferences of the methods of solution of multicriteria tasks should be excluded in order to avoid arbitrariness and ambiguity of the results of decisions. According to Gilbert, the overall task of science is to "deliver us from chance, bias personal sentiments and habits and protect against subjectivism". The formalization of qualitative concepts eliminates the inevitable in intuitive considering the ambiguity of interpretations, and, that is much more important, allows you to explore the phenomena by mathematical methods, that usually already at the first steps brings new important results.

When the result of the multicriteria problem decision is intended for general use, it is unified and individual preferences are leveled on statistics; becomes applicable principle of insufficient reason by Bernoulli-Laplace: if a priori probabilities of possible hypotheses are not known, they should be set equal, i.e., all hypotheses should be regarded as equally probable. As applied to multiobjective problem, this means that all the weighting coefficients α_i, $i \in [1,n]$ in the expression for the scalar convolution of criteria to be minimized must be equal unless there is no prior information about different values of criteria. Since in the formulation of the problem we considered the criteria equal in importance, with the unification we should take in (11) all the weights equal: $\alpha_i = 1/n$, $\forall i \in [1,n]$. Then

$$Y(\alpha, y) = \frac{1}{n} \sum_{i=1}^{n} [A_i - y_i]^{-1}.$$

Taking into account that the multiplication by $1/n$ is a monotonic transformation, which, in Germeier theorem, does not alter the comparison of the results, we go to the unified expression for the scalar convolution of criteria

$$Y(y) = \sum_{i=1}^{n} [A_i - y_i]^{-1}. \qquad (12)$$

This formula is recommended in all cases where the multicriteria problem is solved not in favor of any one particular decision-maker, but for general use.

In [Voronin & Ziatdinov, 2013] it is proposed to start a multicriteria decision *in all cases* using formula (12). The result and the corresponding values of the partial criteria are imposed to decision maker to assess. If the decision maker believes that the obtained solution does not satisfy him and the correction according to his personal preferences is required, then, the procedure for determining the weighting coefficients α_i, $i \in [1,n]$ is organized. To optimize the formula (11) is used. Turning to our analogy, we note that the store bought ready to wear a suit usually needs only a minor adjustment.

For taking into account the DM individual preferences, the above program contains the appropriate option.

In the case when the global resource is allocated between the objects not directly, but through intermediaries, the system becomes hierarchical [Prilutskiy,1996]. The given approach can be applied also in this case.

Conclusion

The problem is solved of such the control system resource allocation between the individual elements (objects), which provide the most efficient operation of the system in the given circumstances. A nonlinear scheme of compromise approach is appropriate in the case [Voronin A.N 2009].

Acknowledgements

The paper is published with partial support by the project ITHEA XXI of the ITHEA ISS (www.ithea.org) and the ADUIS (www.aduis.com.ua).

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