ECONOMIC ANALYSIS OF INFLUENCE OF IMPLEMENTATION OF INTERNATIONAL ENVIRONMENTAL

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Abstract. In the article the modified balanced ecologic-economical model is offered as "input-output" with the Paris agreement set limits on greenhouse gas emissions. The mathematical definition of the device changes the volume of gross output of main and auxiliary production in case of change of branch structure.

Keywords: sustainable development, the Paris Agreement, an ecological and economic system, Leontief "input-output" model, Leontief-Ford "input-output" model, simulation modeling.


Introduction

Energy sector plays the strategic role in the context of Ukrainian branch structure in the conditions of the socio-economic crisis. It fills the budget, provides additional export receipts, other economic and social benefits. However, the further exploitation of existing technologies for production, transportation and energy conservation has a negative impact on the state of the environment, which ultimately causes loss of quality of the energy resources itself and complicates their further production.

The existing economic mechanism cannot effectively use accumulated production potential and achievement in limiting the impact on an environment that holds back the intensification of production and exacerbates disproportion and the imbalance of the economic system. This economic mechanism was created in the period of largely extensive economic development and was focused on achieving quantitative indicators. Moreover, this analysis was carried out at increments to the base level on the basis of gross output products that provided for a recurring product account. Manifestation imbalances in the form of scarcity, lack of resources and products are usually caused by the existing mechanism, insufficient economic responsibility for the use of resources, overestimated demand for them, the weakening of incentives to improve environmental standards, etc.
Considering the achieved level of development of the production, created structure of the production the paramount importance in solving the problem of balancing is a definition of the right priorities in the development of industries and regions in order to achieve high end results and acceleration of environmentalizing of the economy. The establishment and support of the priorities and proportions require to determine the necessary resources for their implementation, which complicates the achievement of a balance. In such circumstances it is important not to allow considerable lag in all other branches and postponing the solving of non-priority but important issues. Priority development of individual industries and the intensification of industrial production anticipate certain structural changes in the economic system and require appropriate proportions in the growth rates of various structural components of the economy. The process of investing has an important influence on structural shifts, aimed at quick responding to changes in volumes and the structure of needs and influence the volume and structure of production in the certain direction. Investing actively influences the improvement of the structure of the economy and increases the balance of its development by introducing the main production funds and production facilities.

One way of attracting foreign investment, in particular, into energy branch is country’s participation in international environmental and economic agreements, aimed at reducing greenhouse gas emissions. The Framework Convention on Climate Change (UNFCCC) [1], whose main objective is establishment of the order of emission reductions in the atmosphere of greenhouse gases (namely, carbon dioxide), was created in order of the realization of the tasks set by United Nations. The accumulation of carbon dioxide is recognized as the cause of one of the main environmental problems of the present - global warming. The Paris Agreement [2], which defines rules for reducing greenhouse gas emissions globally, was signed as a result of the long-term negotiations of the participants of the climate change conference in France on December 12, 2015. The reported goal requires the implementation of appropriate structural changes in various branches of the economic system, the interaction of its constituents and the allocation of environmental branch as an independent unit.

It should also be noted that the current energy market of Ukraine still possesses the significant volumes of energy saving reserves that are able to meet additional requirements for increasing the efficiency of work and product quality. In addition, the share of energy is demanded in order to improve the sanitary condition of enterprises and related cities where emissions of harmful substances exceed the allowable rates dozens of times. It is necessary to study all of enterprise’s components, as direct (in a specific branch) and those that take place in adjacent and intersectoral industries chain in order of correct
estimation of energy costs. It is possible to get reliable data for further analysis and comparisons only by using comprehensive study of the impact of the introduction of those or other ecological and economic norms within the national economy.

It is possible to ensure a balanced economy in an intensification production only on resource-saving basis. Intensification and resource-orientation of the investment process are important factors for increasing the efficiency and balance of production. Accelerated and economical release of the final product of the investment process gives the opportunity to get bigger results with less resources, creates opportunities for accelerated environmental and economic problem solutions, meets the needs of the population in better way. The final result of the investment process introduces new products into operation capacity or their reconstruction, as well as ensuring conditions for their effective functioning. The technical equipment that meets new emission standards must meet these requirements too. Only these conditions will make the economic objects work with high ecological and economic efficiency and ensure balanced production development. Thus, the final result of the investment process directly affects resource-saving capabilities, as well as efficiency and balance development of the whole economic system.

Execution of established obligations for non-compliance of quotas on emissions of greenhouse gases creates certain restrictions on the main economic indicators of production activity. First of all, it concerns gross output, volume of investments, final product, their optimal distribution in the system of national wealth because of the connection between the volumes of gross output and CO2 emissions. Adhering to the general principle of the division of economic research into micro, meso- and macro levels, we will consider the functioning of production in the context of existing industries due to the complexity and multifactorial nature of the tasks of reducing greenhouse gas emissions in the national economy.

Objectives

The implementation of the provisions of the PA requires the application of an interdisciplinary approach because of the ecologically-economic content of the PA. An effective research tool in this case is a balance ecological and economic model of "input-output", which has a special role in solving the principal problems of long-term planning, taking into account the nature of use, namely the justification of the cost of environmental protection, considering the socio-economic effect and distribution of them in the territorial-branch section.
A number of issues related to the country’s participation in the Paris Agreement determines the assessment of the potential future level of the environmental services market, the identification of potential partners, the development of an economic strategy that would determine the priorities for each economic mechanism, the proportion of their application in order to attract the maximum amount of environmental investments.

A special role in solving the fundamental problems of nature use such as justification of the cost of environmental protection, taking into account the socio-economic effect and their distribution in the territorial-branch section – is played by the balance ecological-economic models of the type “input-output”, as well as regional and branch models.

Historically, the first and some kind of the simplest mathematical model of inter-branch industrial relations, acceptable for practical purposes calculations, was the balance model “input-output”.

The complexity and multifactorial nature of the problems of reducing greenhouse gas emissions in the national economy requires the consideration of production in the context of existing industries (types of economic activity), the inclusion the costs of implementing measures under the Paris Agreement in their composition and, first of all, the allocation of environmentally dirty groups among them. In this regard, it is proposed to take into account the costs of implementing emission limitations of greenhouse gases in the structure of the main production sectors in the form [4]:

\[
\begin{align*}
  x_1 &= A_{11}x_1 + A_{12}x_2 + C y_2 + y_1, \\
  x_2 &= A_{21}x_1 + A_{22}x_2 - y_2,
\end{align*}
\]

(1)

where \( x_1 = (x_1^1, x_2^1, \ldots, x_n^1)^T \) – vector column of production volumes;

\( x_2 = (x_1^2, x_2^2, \ldots, x_m^2)^T \) – vector-column of volumes of destroyed pollutants;

\( y_1 = (y_1^1, y_2^1, \ldots, y_n^1)^T \) – vector column of volumes of the final product; \( y_2 = (y_1^2, y_2^2, \ldots, y_m^2)^T \) – vector column of volumes of permanent pollution;

\( A_{11} = (a_{ij}^{11})^n \) – square matrix of coefficients of direct product costs per production of one unit \( j \);

\( A_{12} = (a_{ij}^{12})_{i, g=1}^{m,n} \) – a rectangular matrix of product costs per \( i \) unit of destruction of pollutants \( g \);

\( A_{21} = (a_{ij}^{21})_{k, j=1}^{m,n} \) – rectangular matrix of pollutant emissions \( k \) per unit of manufactured products \( j \);
\[ A_{22} = \left( a_{kg}^{22} \right)^n \] – rectangular matrix of pollutant emissions \( k \) per unit of destruction of pollutants \( g \).

\[ Cy_2 \] – costs related to greenhouse gas emissions (ie, costs for maintenance of greenhouse gas emissions, in particular, fee for emission permits);

\[ C = \left( c_{ig}^{12} \right)^{n,m} \] – rectangular product cost matrix \( i \) per unit of pollutant emissions \( g \);

Research results

Model (1) can be described in a vector-matrix model form:

\[
\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} E_1 & C \\ 0 & -E_2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix},
\]

where \( E_1 \) and \( E_2 \) – the corresponding unit diagonal matrices.

The first equation of the proposed model reflects the economic balance - the distribution of sectoral gross output to the production consumption of the main and auxiliary industries, the final consumption of the main production and the costs associated with the fulfillment of obligations under the Paris Agreement. The second equation reflects the physical balance of greenhouse gases, as the sum of emissions caused by the activities of the main and auxiliary industries, and their unchanged volumes.

The economic content of the variables model (1) requires consideration of their inalienable values. The latter is closely related to the question of the performance of balance models [5], which allows us to speak about the real functioning of a production system capable of providing intermediate consumption, a positive volume of the final product, and compliance with established limits on greenhouse gas emissions.

We have to express \( x_2 \) from the second equation and substitute into the first equation in order to study the issue of ensuring the integrality of solutions:

\[
x_1 = (E_1 - A_1)^{-1} \left( y_1 + Cy_2 - A_{12} (E_2 - A_{22})^{-1} y_2 \right),
\]

where \( A_i = A_{i1} + A_{i2} (E_2 - A_{22})^{-1} A_{2i} \) – square matrix of the \( n \)-th order.
We also express \( x_1 \) from the first equation and substitute into the second one:

\[
x_2 = (E_2 - A_2)^{-1} \left( A_{21}(E_1 - A_{11})^{-1} y_1 + A_{21}(E_1 - A_{11})^{-1} Cy_2 - y_2 \right),
\]

where \( A_2 = A_{22} + A_{21}(E_1 - A_{11})^{-1} A_{12} \) – square matrix of the \( m \)-th order.

Thus, the formal solution of system (3) can be written as:

\[
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} = \begin{pmatrix}
(E_1 - A_1)^{-1} & (E_1 - A_1)^{-1} A_2(E_2 - A_{22})^{-1} C \\
(E_2 - A_2)^{-1} A_2(E_1 - A_{11})^{-1} & (E_2 - A_2)^{-1} (E_2 - A_2)^{-1} C
\end{pmatrix} \begin{pmatrix}
y_1 \\
y_2
\end{pmatrix}.
\]

According to the classical method of studying balance models, we generalize the notion of productivity in the case of a block matrix with inseparable elements:

\[
A = \begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{pmatrix} \geq 0.
\] (2)

We consider an integral block matrix to be productive if matrices \( A_{11}, A_{12}, A_1 \) and \( A_2 \) are productive. Productivity of matrices \( A_1 \) and \( A_2 \) means the profitability of the main and auxiliary industries in the full cycle of production and in the full cycle of greenhouse gas destruction. If matrices \( A_{11}, A_{12}, A_1 \) and \( A_2 \) – productive, than matrices

\[
(E_1 - A_{11})^{-1} \geq 0, \ (E_2 - A_{22})^{-1} \geq 0, \ (E_1 - A_1)^{-1} \geq 0, \ (E_2 - A_2)^{-1} \geq 0
\]

exist and have integral elements.

Productivity of the block matrix (2) does not guarantee the integrity of the system solutions (2). Let’s analyze the resulting expressions for \( x_1 \) and \( x_2 \). System (2) gives us:

\[
x_1 = (E_1 - A_{11})^{-1} (A_{12} x_2 + C y_2 + y_1).
\]

Hence it follows that when \( x_2 \geq 0, \ y_1 \geq 0, \ y_2 \geq 0 \) the condition is fulfilled \( x_1 \geq 0 \).

Thus, the necessary and sufficient condition for the integrality of the solutions of the model (1) with the productivity of the block matrix (2) and with \( y_1 \geq 0, \ y_2 \geq 0 \) will be the condition \( x_2 \geq 0 \), namely

\[
(E_2 - A_2)^{-1} \left( A_{21}(E_1 - A_{11})^{-1} y_1 + A_{21}(E_1 - A_{11})^{-1} Cy_2 - y_2 \right) \geq 0.
\]
From the last inequality we obtain a sufficient condition for the existence of inseparable solutions:

\[ A_{21} (E_1 - A_{11})^{-1} (y_1 + Cy_2) \geq y_2 , \]

which can be replaced by an even more rigorous sufficient condition:

The latest inequality means that a sufficient condition for the functioning of the main and auxiliary industries is the non-excess of unleaded greenhouse gas emissions over total greenhouse gas emissions resulting from the production of the final product and the costs of servicing obligations under the Paris Agreement.

Let’s consider the problem of determining how vectors of gross output and volumes of greenhouse gas utilization will change, if we change the coefficients of technological matrices, in particular with the strengthening of environmental standards and the need to increase the costs of fulfilling obligations under the Paris Agreement. For example, assume that elements of one or more technological matrices \( A_{11}, A_{12}, A_{21}, A_{22}, C \) were changed.

Let’s determine how such changes affect the values of vectors \( x_1 \) and \( x_2 \). We use the procedure proposed in the mathematical literature on matrix analysis [6] in order to do this.

Model (1) can be represented in the form:

\[ Au = G . \quad (3) \]

where \( A = \begin{pmatrix} E_1 - A_{11} & -A_{12} \\ -A_{21} & E_2 - A_{22} \end{pmatrix} \), \( u = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \) - \((n + m)\)-dimension vector, \( G = \begin{pmatrix} E_1 & C \\ 0 & -E_2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \).

\( E_1, E_2 \) – block unit matrices of the corresponding dimension, \( 0 \) – block null matrix.

We will also consider the perturbed system (in the elements of the matrices \( A_{11}, A_{12}, A_{21}, A_{22} \) and vector column \( G \)) in relation to the system of linear algebraic equations (3) of the form:

\[ \bar{A}u = \bar{G} . \quad (4) \]

where \( \bar{A}, \bar{G} \) – corresponding perturbed matrix and vector-column. Suppose that the system (3) has a reference solubility and an inverse matrix. Then the following theorem takes place (details in [7]).
Theorem 1. The following relationships exist between the development coefficients of vector-normals of constraints on the rows of the basis matrix, the elements of the inverse matrices, the basic solutions, and the nonconstraints of the constraints in two adjacent basic solutions:

\[ \alpha_{rk} = \frac{\alpha_{rk}}{\alpha_{ik}}, \quad \alpha_{ri} = \frac{\alpha_{ri}}{\alpha_{ik}}, \quad r = 1, n + m, \quad i = 1, n + m, \quad i \neq k. \]  

(5)

\[ \bar{e}_{rk} = \frac{e_{rk}}{\alpha_{ik}}, \quad \bar{e}_{ri} = \frac{e_{ri}}{\alpha_{ik}}, \quad r = 1, n + m, \quad i = 1, n + m, \quad i \neq k. \]  

(6)

\[ \bar{u}_{0j} = u_{0j} - \frac{e_{jk}}{\alpha_{ik}} \Delta_j, \quad j = 1, n + m. \]  

(7)

\[ \bar{\Delta}_k = -\frac{\Delta_i}{\alpha_{ik}}, \quad \bar{\Delta}_r = \Delta_r - \frac{\alpha_{rk}}{\alpha_{ik}} \Delta_i, \quad r = 1, n + m, \quad r \neq k. \]  

(8)

In this case, the condition of the base matrix support when entering the normal vector \( a_i \) restriction \( a_i u \leq g_i \) on \( k \)-th position of basic matrix \( A \) is the implementation of inequality \( \alpha_{ik} \neq 0 \).

An algorithmic scheme for the study of systems (4) (with changes in the model) can be constructed on the basis of the given ratios. The algorithm will be based on the ideology of the simplex method, with some special features of the organization of the iterative process. In particular, the transition from system (3) to system (4) will be carried out by consistent replacing of the corresponding perturbed strings \( i, i+1, i+2, \ldots, i+i_0 \). This means that the vectors of the normalities of the hyperplanes that form the rows of the base matrix and its corresponding inverse matrix will be replaced by the corresponding "perturbed" normal vectors. On the basis of simplex relations (5) - (8), the following basic solvations and inverse matrices will be counted. With the preservation of the property of the support, on the iterations of the substitution, solvability of the system (4) will be found after the \( i_0 \) iterations. As a result, we obtain a new basic solution and an inverse matrix (without first solving the modified problem (4)).

Formulas (5) - (8) can be used as the basis for the algorithm for determining the new solution in the case of perturbation of elements of the basic matrix, which allows to determine changes in the volumes of gross output when the technological matrices of the ecological-economic model change (2).
Step 1. Find a solution \( u_0 \) of the output system (3) and its inverse matrix \( A^{-1} \).

Step 2. If perturb matrix \( A \) in element \( a_{ij} \), then \( \overline{a}_{ij} = a_{ij} + a'_{ij} \).

Step 3. Determine the coefficient \( \overline{\alpha}_{kk} = 1 + a'_{ij} \cdot e_{jk} \neq 0 \), where \( e_{jk} \) - corresponding element of matrix \( A^{-1} \).

Step 4. Find new vector-column \( \overline{e}_k = \frac{e_k}{\alpha_{kk}} \) of matrix that inversed to \( \overline{A} \).

Step 5. Determine the irregularity of the perturbed string in the element \( a'_{ij} \cdot \Delta_j = \overline{\Delta}_k = a'_{ij} \cdot u_{kj} \), where \( u_{kj} - j \) - and component \( u_0 \).

Step 6. Find a new solution based on the relationship \( \overline{u}_0 = u_0 - \overline{e}_k \cdot \Delta_j \).

Let’s illustrate the proposed algorithm for determining the volumes of gross sector output in the case of technological interbranch changes on conditional data. Suppose, the coefficients of the technological matrices of the ecological-economic model (3) have the following meanings:

\[
A_{11} = \begin{pmatrix} 0.2 & 0.1 \\ 0.3 & 0.2 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} 0.1 & 0.2 \\ 0.2 & 0.2 \end{pmatrix}, \quad A_{21} = \begin{pmatrix} 0.1 & 0.3 \\ 0.2 & 0.3 \end{pmatrix}, \quad A_{22} = \begin{pmatrix} 0.2 & 0.3 \\ 0.3 & 0.1 \end{pmatrix},
\]

the matrix for maintenance of greenhouse gas emissions and sectoral final output vectors and greenhouse gas emission limitations respectively:

\[
C = \begin{pmatrix} 0.3 & 0.2 \\ 0.1 & 0.5 \end{pmatrix}, \quad y_1 = \begin{pmatrix} 12 \\ 23 \end{pmatrix}, \quad y_2 = \begin{pmatrix} 5 \\ 8 \end{pmatrix}.
\]

We will check the fulfillment of the performance condition for the ecological-economic system in the case of selected numerical data.

We turn to step-by-step implementation of algorithm 1-6.

1. Find the solution of the source system and the inverse of block technological matrix:

\[
u_0 = \begin{pmatrix} 38.17 \\ 60.43 \\ 32.67 \\ 30.62 \end{pmatrix}, \quad A^{-1} = \begin{pmatrix} 1.79 & 0.73 & 0.6 & 0.76 \\ 1.08 & 2.0 & 0.74 & 0.93 \\ 1.04 & 1.32 & 1.99 & 1.19 \\ 1.1 & 1.27 & 1.04 & 1.99 \end{pmatrix}.
\]
2. Assume that the element $a_{21}^{11} = 0.3$ experiences the perturbation in the model (3), increases by 0.1. The latter means an increase in the costs of the 2nd branch per unit of issue of the 1st branch. Thus, $\bar{a}_{21} = 0.3 + 0.1 = 0.4$.

3. Find $\alpha_r = \bar{a}_{4k} = 1 + 0.1 \cdot a_{12}^{-1} = 1 + 0.1 \cdot 0.74 = 1.074$.

4. Define vector-column: $\vec{r}_2 = \begin{pmatrix} 0.73 \\ 2.0 \\ 1.32 \\ 1.27 \end{pmatrix} \quad / \quad 1.074 = \begin{pmatrix} 0.68 \\ 1.86 \\ 1.23 \\ 1.18 \end{pmatrix}$.

5. Count the irregularity of the perturbed string: $\Delta_r = \Delta_2 = 0.1 \cdot 38.17 = 3.817$.

6. A new solution is obtained in the form: $\bar{r}_0 = \begin{pmatrix} 38.17 \\ 60.43 \\ 32.67 \\ 30.62 \end{pmatrix} - 3.817 \cdot \begin{pmatrix} 0.68 \\ 1.86 \\ 1.23 \\ 1.18 \end{pmatrix} = \begin{pmatrix} 35.57 \\ 53.33 \\ 27.97 \\ 26.12 \end{pmatrix}$.

The analysis of the solution obtained allows us to draw the following conclusions. The increase in the cost of production of the 2nd branch per unit of production of the 1st sector within the framework of the balance ecological and economic system (2) leads to a decrease in the gross output of the 1st and 2nd branches of material production by 2.6 and 7.1 conventional units, as well as the volume of utilization of GHGs of the 1st and 2nd types by 4.7 and 4.5 conventional units, respectively.

We propose the application of (5) - (8) to construct a method for estimating the transformation of the production structure in the model (2) under the conditions of the change of the line of the technological matrix.

The result of the concretization of the given technology is the algorithm for determining the new solution in the case of perturbation of the rows of the base matrix for a specific line of constraints (3), which allows to determine changes in the volumes of gross output when the technological matrices of the ecological-economic model change (2).
Algorithm

**Step 1.** Find a solution \( u_0 \) of output system (3) and its inverse matrix \( A^{-1} \).

**Step 2.** Perturb matrix \( A \) in the elements of the 1st line in the form \( \bar{a}_i = a_i + a'_i, \ g_i = g_i + g'_i \), \( i = 1 \).

**Step 3.** Determine the coefficient \( \bar{\alpha}_{kk} = a_i e_k + a'_i e_k = 1 + a'_i e_k \), where \( e_k \) – matrix column \( A^{-1} \).

**Step 4.** Determine
\[
\bar{\Delta}_k = (a_k + a'_k) \cdot u_0 - (g_k + g'_k) = (a_k \cdot u_0 - g_k) + (a'_k \cdot u_0 - g'_k) = \Delta_k + \Delta'_k = \Delta'
\]

**Step 5.** Find \( \lambda = -\bar{\Delta}_k / \bar{\alpha}_{kk} \).

**Step 6.** Find new vector-column \( \bar{e}_k = \lambda \times e_k \).

**Step 7.** Form a new solution based on the relationship \( \bar{u}_0 = u_0 + \bar{e}_k \).

**Remark.** It's easy to make sure that the new solution (perturbed problem) is based on the old solution and the impact of the vector of the column of the inverse matrix and parameter. The "selection" of the column of the inverse matrix and the parameter (direction and stretch of the corresponding vector) can form certain result dominant, which means to make changes predictably in a certain direction.

We illustrate the proposed algorithm for determining the volumes of gross sector output in the case of technological interbranch changes on conditional data. Suppose the coefficients of the technological matrices of the ecological-economic model (3) have the following meanings:

\[
A_{11} = \begin{pmatrix} 0.2 & 0.1 \\ 0.3 & 0.2 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} 0.1 & 0.2 \\ 0.1 & 0.2 \end{pmatrix}, \quad A_{21} = \begin{pmatrix} 0.1 & 0.3 \\ 0.2 & 0.3 \end{pmatrix}, \quad A_{22} = \begin{pmatrix} 0.2 & 0.3 \\ 0.3 & 0.1 \end{pmatrix},
\]

the matrix of greenhouse gas emissions maintenance costs and sectoral final output vectors and greenhouse gas emission limitations respectively:

\[
C = \begin{pmatrix} 0.3 & 0.2 \\ 0.1 & 0.5 \end{pmatrix}, \quad y_1 = \begin{pmatrix} 12 \\ 23 \end{pmatrix}, \quad y_2 = \begin{pmatrix} 5 \\ 8 \end{pmatrix}.
\]
We turn to step-by-step implementation of algorithm 1-6.

1. Find the solution of the original system, and the inverse block technological matrix:

\[
\begin{bmatrix}
38.17 \\
60.43 \\
32.67 \\
30.62
\end{bmatrix}
\]
\[A^{-1} = \begin{bmatrix}
1.79 & 0.73 & 0.6 & 0.76 \\
1.08 & 2.0 & 0.74 & 0.93 \\
1.04 & 1.32 & 1.99 & 1.19 \\
1.1 & 1.27 & 1.04 & 1.99
\end{bmatrix}.
\]

2. Assume that the first line becomes perturbed in the model (3), that is, the perturbed matrix takes the form of:

\[
\bar{A} = \begin{bmatrix}
0.1 & 0.05 & 0.3 & 0.3 \\
0.3 & 0.2 & 0.1 & 0.2 \\
0.1 & 0.3 & 0.2 & 0.3 \\
0.2 & 0.3 & 0.3 & 0.1
\end{bmatrix}.
\]

The applied content of changes in the ecological and economic indicators is as follows: the reduction of the factor 1 of the industry per unit of production of the 1st industry by 0.1 units, the coefficient of expenditure of the 1st industry per unit of production of the 2nd branch by 0.05 units, respectively; there will be an increase in the expenditure ratio of the 1st branch for the destruction of the 1st type of pollution by 0.2 units, the coefficient of expenditure of the 1st branch for the destruction of the 2nd type of pollution by 0.1 units, respectively.

3. Determine the scalar product of the perturbed string and the solution of the output system:

\[
\bar{a}_k \cdot u_0 = \begin{bmatrix}
0.1 & 0.05 & 0.3 & 0.3
\end{bmatrix} \begin{bmatrix}
38.17 \\
60.43 \\
32.67 \\
30.62
\end{bmatrix} = 25.83
\]

4. Determine the product of the original process matrix and the output solution and calculate the non-interference of the perturbed string \(\Delta\):

\[
a_k \cdot u_0 = \begin{bmatrix}
0.2 & 0.1 & 0.1 & 0.2
\end{bmatrix} \begin{bmatrix}
38.17 \\
60.43 \\
32.67 \\
30.62
\end{bmatrix} = 23.068
\]

\[
\bar{\Delta}_k = \bar{a}_k \cdot u_0 - a_k \cdot u_0 = 25.83 - 23.068 = 2.7575
\]
5. Determine the coefficients $\overline{x}_k$, $\lambda$:

$$
\overline{x}_k = \overline{a}_k \cdot e_k = \begin{bmatrix} 0.1 \\ 0.05 \\ 0.3 \\ 0.3 \end{bmatrix}
= \begin{bmatrix} 1.79 \\ 1.08 \\ 1.04 \\ 1.1 \end{bmatrix} = 0.875, \quad \lambda = \frac{\overline{A}_k}{\overline{x}_k} = -\frac{2.7575}{0.875} = -3.15
$$

6. Determine the coefficient $\overline{e}_k$:

$$
\overline{e}_k = \lambda \cdot e_k = -3.15 \cdot \begin{bmatrix} 1.79 \\ 1.08 \\ 1.04 \\ 1.1 \end{bmatrix} = \begin{bmatrix} -5.64 \\ -3.4 \\ -3.28 \\ -3.47 \end{bmatrix}
$$

7. Determine the solution of the perturbed system $\overline{u}_0$:

According to the formula $\overline{u}_0 = u_0 + \overline{e}_k$, $\overline{u}_0 = u_0 + \overline{e}_k = \begin{bmatrix} 38.17 \\ 60.43 \\ 32.67 \\ 30.62 \end{bmatrix} + \begin{bmatrix} -5.64 \\ -3.4 \\ -3.28 \\ -3.47 \end{bmatrix} = \begin{bmatrix} 32.53 \\ 57.03 \\ 29.39 \\ 27.15 \end{bmatrix}$.

Research on the impact of changes of $k$-th column of the matrix of constraints $A$ in the form of $\overline{A}_k = A_k + \overline{A}_k$ on solution $u_0$, where $A_k = (a_{i1}, a_{i2}, ..., a_{im})^T$, $\overline{A}_k = (a'_{i1}, a'_{i2}, ..., a'_{im})^T$, which means $A$ replaced by $\overline{A}$ in this form.

**Algorithm**

1. We have a known vector $u_0 = (u_{i0}, u_{i2}, ..., u_{im})^T$, $A_k, A_k^{-1}$ - direct and inverse basic matrix (3).
2. Let’s replace the $k$-th column of the matrix of constraints $A_k$ with the column $\overline{A}_k$. Find vector $L_k = (L_{k1}, L_{k2}, ..., L_{km}) = A_k^{-1} \times \overline{A}_k$.
3. Form new solution $\overline{u}_{ik} = \frac{u_{i0} + u_{ik}}{1 + (A_k^{-1})_i \times L_{ik}}$, $i = k$.

$$
\overline{u}_{ik} = \frac{u_{i0} + \frac{u_{i0} \times \overline{L}_{ik}}{L_{ik}} - \frac{u_{i0} \times (A_k^{-1})_i \times A_k}{(A_k^{-1})_i \times A_k}}{1 + (A_k^{-1})_i \times A_k}, \quad i \neq k.
$$
We illustrate the proposed algorithm for determining the volumes of gross sector output in the case of technological interbranch changes on conditional data. Suppose the coefficients of the technological matrices of the ecological-economic model (2) have the following meanings:

\[
A_1 = \begin{pmatrix} 0.2 & 0.1 \\ 0.3 & 0.2 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0.1 & 0.2 \\ 0.1 & 0.2 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 0.1 & 0.3 \\ 0.2 & 0.3 \end{pmatrix}, \quad A_4 = \begin{pmatrix} 0.2 & 0.3 \\ 0.3 & 0.1 \end{pmatrix},
\]

the matrix for maintenance of greenhouse gas emissions and sectoral final output vectors and greenhouse gas emission limitations respectively:

\[
C = \begin{pmatrix} 0.3 & 0.2 \\ 0.1 & 0.5 \end{pmatrix}, \quad y_1 = \begin{pmatrix} 12 \\ 23 \end{pmatrix}, \quad y_2 = \begin{pmatrix} 5 \\ 8 \end{pmatrix}.
\]

1. We find the solution of the output system (4) and the inverse block technological matrix:

\[
u_0 = \begin{pmatrix} 38.17 \\ 60.43 \\ 32.67 \\ 30.62 \end{pmatrix}, \quad A = \begin{pmatrix} 0.2 & 0.1 & 0.1 & 0.2 \\ 0.3 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.2 & 0.3 \\ 0.2 & 0.3 & 0.3 & 0.1 \end{pmatrix},
\]

\[
A^{-1} = A_8^{-1} = \begin{pmatrix} 1.25 & 3.125 & -3.125 & 0.625 \\ -11.25 & 6.875 & 3.125 & -0.625 \\ 8.75 & -8.125 & -1.875 & 4.375 \\ 5.0 & -2.5 & 2.5 & -2.5 \end{pmatrix}.
\]

2. Assume that the third column \(k = 3\) in model (3) becomes perturbed: we carry out the replacement of the \(k\)-th column of the matrix of constraints \(A_3 = \begin{pmatrix} 0.1 \\ 0.1 \\ 0.2 \\ 0.3 \end{pmatrix}\) by the column \(A_5 = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.1 \\ 1.0 \end{pmatrix}\). Find vector \(\bar{L}_k = (L_{k1}, L_{k2}, L_{k3}, L_{km}) = A_5^{-1} \times \bar{A}_8 :\)

\[
\bar{A}_5 = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.1 \\ 0.1 \end{pmatrix}, \quad \bar{L}_k = \begin{pmatrix} 1.25 & 3.125 & -3.125 & 0.625 \\ -11.25 & 6.875 & 3.125 & -0.625 \\ 8.75 & -8.125 & -1.875 & 4.375 \\ 5.0 & -2.5 & 2.5 & -2.5 \end{pmatrix} \times \begin{pmatrix} 0.2 \\ 0.2 \\ 0.1 \\ 0.1 \end{pmatrix} = \begin{pmatrix} 0.625 & -0.625 & 0.375 & 0.5 \end{pmatrix}.
\]
3. Determine the solutions:

\[ \overline{u}_1 = 38.17 - \frac{32.67}{0.375} \cdot 0.625 = -16.28 \]
\[ \overline{u}_2 = 60.43 - \frac{32.67}{0.375} \cdot (-0.625) = 114.88 \]
\[ \overline{u}_3 = \frac{32.67}{0.375} = 87.12 \]
\[ \overline{u}_4 = 30.62 - \frac{32.67}{0.375} \cdot 0.5 = -12.94 \]

The obtained solutions coincide with the solutions obtained directly by direct calculations. They indicate a significant change in the functioning of auxiliary production, in particular, negative indicators require changes in the structure of technological matrices in both the main and the auxiliary spectrum of industries.

**Conclusion**

Thus, the transition to a policy of energy efficiency through the use of "green investments" needs to take into account the environmental factor in the modern system for the further development of civilization makes the relevance of the consideration of the industrial activity of society within the framework of a single socio-ecological and economic system. At the same time, an important requirement for its existence is the need to balance the interests of each of the subsystems. An effective tool for this is the balance method and the corresponding models based on it. The change in the coefficients of the technological matrixes reflects the change in the inter-branch relationships, their structure, which could be caused by relevant regulatory decisions regarding the reduction of greenhouse gas emissions, energy efficiency improvement, the implementation of the results of scientific and technological development, etc. The proposed algorithm allows an estimation of the solution in the case of such a structural adjustment. As a further development of the proposed theory, we can point out the path to the study of the problems of aggregation of the balance scheme "input-output", the definition of a certain direction for permissible changes in order to achieve the target benchmark by volume of industry issues.

**References**


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