EXPERT MODELS OF VECTOR OPTIMIZATION

Albert Voronin, Yuriy Ziatdinov

Abstract. *An approach is proposed to solve the vector optimization problem of complex technical and economic systems in cases where there is insufficient (or lacking) information on the experimental and statistical data necessary for building regression models. To solve the problem under consideration, a multi-criteria optimization approach is undertaken using a non-linear compromise scheme. A model example is provided.*

Keywords: *vector optimization, regression models, expert estimation method, approximation polynomials, least squares method, nonlinear compromise scheme.*

ACM Classification Keywords: *H.1 Models and Principles – H.1.1 – Systems and Information Theory; H.4.2 – Types of Systems.*

Problem Content

When optimizing complex technical and economic systems we are often faced with the fact that for the construction of the necessary mathematical models is not enough experimental and statistical data. The situation is exacerbated in the case where the optimization is performed over several conflicting performance criteria.

In conditions of the acute shortage of experimental data, we propose to obtain necessary information ("quasi-experimental" data) from the experts – professionals who have sufficient experience in the design and operation of complex systems of this class.

A study is carried out on the example of vector optimization of space facilities objects by the generalized criterion of "reliability-value", but the results could easily be used in other subject areas. By the term "reliability" we mean the probability of finding the defining parameters of all elements of the object within the permissible limits under the terms of operability.

Please note that in this case we are talking about designing a fundamentally new technology, for which technical and economic characteristics significantly different from those with which we have dealt earlier or developers are working today. One of the specific aspects is extreme limitation (and sometimes complete lack of it) of experimental and statistical data by which we could identify the mathematical models of reliability and cost.

In these hard formalizable conditions we have to resort to non-traditional approaches, one of which is considered in this section. Naturally, in this case, we can talk only about rough calculations, approximate determination of the main trends in the choice of factors affecting the reliability and value of developed space facilities.

Statement of the Problem

To solve the optimization problem, you must have the following starting data:

1. Mathematical models:

$$y_1' = f_1(x);$$

$$y_2 = f_2(x),$$

where y_1' – the reliability of the space facility object (criterion to be maximized); y_2 – the cost of measures that affect the reliability (criterion to be minimized); f_1 and f_2 – some criteria functions; $x = \{x_i\}_{i=1}^{n}$ – *n*-dimensional vector of independent variables (optimization arguments).

2. Restrictions on independent variables $x \in X$, where

$$X = \{x | x_{i\min} \le x_i \le x_{i\max}, i \in [1,n]\}.$$

3. Restrictions on criteria $y \in M$, where $y = \{y_k\}_{k=1}^{s}$ – *s*-dimensional vector of the minimized nonnegative criteria (here *s*=2). We explain that as a single method of extremalization criteria in our task is selected minimization. To make the reliability criterion minimized too, we define

$$y_1 = 1 - y_1^{'}$$

(if one hundred percent reliability is expressed in the unit). Then

$$M = \{y | 0 \le y_k \le A_k, k \in [1;2]\}.$$

Limitations $x_{i\min}, x_{i\max}$ on the arguments $x \in X$ and A_k on criteria $y \in M$ are set for physical reasons.

If all this takes place, there are all prerequisites for the optimization of space objects on the criteria of reliability and cost, i.e., to determine a compromise-optimal parameters values $x^* = \{x^*_i\}_{i=1}^{n}$.

Method of Solution

In view of the apparent inconsistency of the criteria, it is necessary to resort to specific methods of multicriteria (vector) optimization theory. If you are using a method of scalar convolution, then a mathematical model for solving the problem of vector optimization is represented as

$$x^* = \arg\min_{x \in X} Y[(x)], \tag{12.1}$$

where $Y(y)$ – a scalar function, which has the meaning of the scalar convolution of partial criteria vector, which depends on the kind of compromise scheme. It should be ensured that its minimization leads to a Pareto optimal solution: $x^* \in X^K$. In (Voronin, A.N. at al 1999), a scalar convolution on nonlinear compromise scheme is proposed

$$Y[y(x)] = \sum_{k=1}^{s} A_k [A_k - y_k(x)]^{-1}, \tag{12.2}$$

where *s* – the dimension of the criteria vector. Convolution (12.2) makes it possible to obtain in formalized procedure the Pareto-optimal solutions, appropriate to given situations. For *s*=2 model (12.1) has the form

$$x^* = \arg\min_{x \in X} [\frac{A_1}{A_1 - y_1(x)} + \frac{A_2}{A_2 - y_2(x)}]. \tag{12.3}$$

The qualitative composition of the vector *x* is rather variegated and, consequently, the dimension *n* of the vector is generally high. Taking full account of the parameters *x* would lead to unnecessary complication of criteria functions f_1 and f_2, and to excessive difficulties in solving the optimization problem. Therefore, natural is selection only the most informative parameters *x* – coordinates of space in which optimization will be carried out of the criteria y_1

and y_2 , while the other parameters are assumed to be fixed and predetermined.

Selection will be done with the involvement of experts. They are introduced to the conditions of the problem, i.e., is called a developed particular type of space object (rocket or spacecraft), are described the terms of its design, manufacturing, testing and operation. Experts are asked to write down those activities that, in their view, may affect the reliability and cost of the space object at different stages of the product lifecycle.

This, for example, is the redundancy multipleness of control systems x_1, the values of stock on structural strength coefficient x_2 and power energy sources coefficient x_3; the relative volume of incoming inspection of materials and components x_4 , the selection of production technologies and the relative volume of the control of their stability x_5, the amount of control-sample testing x_6; the volume of experimental testing of components and systems in all modes x_7, the amount of material incentives for staff x_8 and so forth. As a result of a special procedure (Voronin, A.N. at al 1999), is determined an adequate qualitative structure and dimension *n* of the vector of independent variables of criterion functions $f_1(x)$ and $f_2(x)$.

Kind of criterion functions depends on what information a researcher has to build the model. The range is wide – from a full knowledge of the mechanisms of phenomena (deterministic model) to the total uncertainty ("black box"). Between these two poles of the information is a probability level of uncertainty. Deterministic mathematical model of any characteristic of the space facility is extremely difficult to develop because of the complexity of the physical processes taking place and of the object reactions to complex of internal and external factors.

Let us consider, for example, the reliability criterion function $f_1(x)$ and approximate it on the set of arguments $x \in X$ by an approximating function $F_1(x,a)$ known to an accuracy of a vector of constants (coefficients) $a = \{a_j\}_{j=1}^{m}$.

When choosing the type of a function $F_1(x,a)$ you need to keep in mind the following. It is established (Voronin, A.N. at al 1999) that the best results are obtained if the regression model is based on some information known about the mechanisms of the studied phenomena. Then the model is called substantional. If such information is not available, then we have to work in the class of formal regression models, and to pay for the lack of information, a large amount of computation.

For a formal model are presented two contradictory requirements. On the one hand, the approximating function should be simple enough to computing processes are not proved to be too cumbersome. On the other hand, the approximating relationship must have sufficient accuracy and prognostic properties. In most practical cases, both these requirements are met in the class of the second order polynomial regression:

$$F_1(x,a) = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i,j=1,i<j}^{n} a_{ij} x_i x_j, \qquad (12.4)$$

where a_0, a_1, a_{ij} – coefficients. Function (12.4) is well adapted to the topography of the objective function $f_1(x)$, it is capable of expressing such features as the ravine, and so on. In practice, are used different truncations of regression polynomial (12.4), mainly linear approximations.

Determining of coefficients *a* may be performed as by the interpolation methods and by the method of least squares (MLS). Interpolation formulas provide an exact match to approximating and objective functions in the reference points (interpolation nodes), the amount of which *N*, as well as the number of unknown constants *a*, is equal to *m*. The coefficients *a* are determined by solving a determine system of equations for reliability criterion

$$F_1(x^{(u)}, a) = f_1(x^{(u)}), u \in [1, N = m], \qquad (12.5)$$

where $x^{(u)}$ – the interpolation nodes.

It is assumed that the values of the objective function in the approximation nodes $f_1(x^{(u)}), u \in [1, N]$ are known. The determination of these values with the help of experts is a key point in this study and is discussed below.

MLS includes *N* control points (approximation nodes), the number *N* may be less, greater than or equal to (as a special case) constants number *m*. The unknown coefficients approximating functions are determined from the condition

$$E(a) = \sum_{u=1}^{N} [F_1(x^{(u)}, a) - f_1(x^{(u)})]^2 = \min_{a}. \qquad (12.6)$$

Using the necessary condition for a minimum of functions, we get called in the MLS theory a system of *normal* equations for reliability criterion

$$\frac{\partial E(a)}{\partial a_j} = 0, j \in [1, m], \tag{12.7}$$

whose solution determines the coefficients of the approximating function. Note that regardless of the number of selected reference points *N* the system of normal equations (12.7) is always determined.

For the cost criterion $f_2(x)$ are valid all the foregoing considerations, but instead of $a = \{a_j\}_{j=1}^{m}$ in expression of the approximating function $F_2(x,b)$ in the general case appears another vector of unknown constants $b = \{b_h\}_{h=1}^{p}$.

The specificity of the problem is that to get the values of the objective functions in reference points is very difficult. Indeed, even for the one single point, which corresponds to the established to date set of measures to ensure the reliability of the space object of this class, there is no a sufficient statistic for a confident assessment of reliability. It particularly relates to newly developed objects not having a long operating period. And it is quite illusory a possibility of objective evaluation of reliability for the other points of the region of existence of optimization arguments $x \in X$.

As always in cases when the task is difficult to formalize, one has to resort to the methods of expert estimates. Skilled specialist (expert), having sufficient experience in the design, manufacture and operation of the objects of this class can produce a **thought experiment** and imagine what will be the object reliability levels for various combinations of factors $x \in X$. Thus, the method is based on individual opinion (postulate), expressed by a qualified expert on the assessed value on the basis of his professional experience. The main drawback of the postulation is subjectivity and arbitrariness.

The procedure of expert assessments processing method allows reducing this disadvantage. The method consists in the fact that for quantitative assessment of some characteristic are used the postulates of not one but *several* persons competent in this matter. It is assumed that the "true" value of unknown to us quantitative characteristic is within the range of expert opinions and a "generalized" collective opinion is more reliable. In (Voronin, A.N. at al 1999) is proposed a procedure for processing the data of expert estimations, during which are obtained more accurate aggregated assessments, and (as a by-product) the coefficients of confidence to the opinion of individual experts.

Applying this method to the treatment of expert assessments of reliability and cost in each of the *N* nodal points in domain $x \in X$, we obtain two ratings vectors (quasi-experimental-data):

$$\{f_1(x^{(u)})\}_{u=1}^{N};$$
$$\{f_2(x^{(u)})\}_{u=1}^{N},$$

which serve as the basis for determining the vectors of the constants *a* and *b*, by condition (12.5), if applicable is interpolation method, or by the condition (12.6) - (12.7), if applicable is MLS. So are determined the mathematical regression models

$$y_1' = f_1(x) \approx F_1(x);$$
$$y_2 = f_2(x) \approx F_2(x),$$

which are involved in the optimization procedure (12.3). Since the information about the objective functions at the nodal points of the approximation area is not obtained experimentally but by expert estimation, the models $F_1(x)$ and $F_2(x)$ are called *expert regression models*.

Let us separately discuss the problem of choosing the method of approximating criterion functions in the specified circumstances. At various truncations of a regression polynomial (12.4), the number of unknown constants, as a rule, exceeds the number of approximation nodes *N,* in which the expert can give his quite confident assessment of the value of a criterion function. Therefore, using the method of interpolation, we obtain underdetermine system of equations in which the number of equations is less than the number of the unknown constants.

To get out of this situation, it is necessary to apply the method of least squares, which in mathematics is seen as a way to solve underdetermine, overdetermine and determine (as a special case) equations systems. The solution can be both analytical (by (12.6) - (12.7)), and numerical, if the residual function $E(a)$ has a complex expression. In the latter case, a Levenberg-Marquardt algorithm is used.

Illustrative Example

The problem of optimizing the process of developing a perspective carrier rocket on the generalized criterion "reliability-cost" is considered. With the help of expert procedures (Voronin, A.N. at al 1999) are selected and fixed values of the factors that affect the reliability of the product, except for the following three (hereinafter all numeric data is arbitrary):

1. Multiplicity of reservation of missile control systems, x_1 is selected from a possible range from one to ten: $x_1 \in [1;10]$;

2. The safety factor in strength of design x_2 is in the range of one to five: $x_2 \in [1;5]$;

3. The coefficient of material incentives for developer company personnel x_3 may be selected from the range of one to six: $x_3 \in [1;6]$.

It is required, using inconsistent reliability $y_1^{'} = f_1(x)$ and cost $y_2 = f_2(x)$ criteria to evaluate the compromise-optimal values of these three factors: $x^* = \{x_i^*\}_{i=1}^{3}$.

A criteria reliability function is approximated by a linear approximation with a free term

$$y_1^{'} = f_1(x) \approx F_1(x) = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 ,$$

the number of unknown constants $m = 4$.

In determining the objective function values in the approximation nodes $f_1(x^{(u)}), u \in [1, N]$ we keep in mind that with sufficient confidence experts can estimate these values only in three (N = 3) points of the region $x \in X$:

$$x_1^{(1)} = x_{1\max} = 10; x_2^{(1)} = x_{2\max} = 5; x_3^{(1)} = x_{3\max} = 6;$$
$$x_1^{(2)} = x_{1\min} = 1; x_2^{(2)} = x_{2\min} = 1; x_3^{(2)} = x_{3\min} = 1;$$
$$x_1^{(3)} = x_{1\text{nom}} = 6; x_2^{(3)} = x_{2\text{nom}} = 3; x_3^{(3)} = x_{3\text{nom}} = 3.$$

The last point corresponds to the ideas of experts about the nominal values of reliability factors.

Let the experts gave the following their assessments of the product reliability levels in these approximation nodes (unit corresponds to 100 percent reliability):

$$f_1(x^{(1)}) = 0{,}8; f_1(x^{(2)}) = 0{,}6; f_1(x^{(3)}) = 0{,}7 .$$

Note that in this problem *N* <*m* and interpolation method can not be applied. MLS method involves minimizing on the unknown constants the function (6) of squared residuals, which has the form under specified conditions

$$E(a) = (a_0 + 10a_1 + 5a_2 + 6a_3 - 0{,}8)^2 + (a_0 + a_1 + a_2 + a_3 - 0{,}6)^2 + \\ + (a_0 + 6a_1 + 3a_2 + 3a_3 - 0{,}7)^2 .$$

Application of the necessary condition of minimum for function (12.7) leads to the following determine system of normal equations:

$$\begin{aligned} &3a_0 + 17a_1 + 9a_2 + 10a_3 = 2{,}1; \\ &17a_0 + 137a_1 + 69a_2 + 79a_3 = 12{,}8; \\ &9a_0 + 69a_1 + 35a_2 + 40a_3 = 6{,}7; \\ &10a_0 + 79a_1 + 40a_2 + 46a_3 = 7{,}5. \end{aligned}$$

It is a system of linear algebraic equations (SLAE), for solution of which the standard computer software is developed. Successive elimination of variables by Gauss method is the basis of on-line program (http://www.webmath.ru/web/prog13_1.php.). Cramer's Formula for solving linear systems are used in the program (http://www.webmath.ru/web/prog12_1.php). Applying Gauss method for solving our system of normal equations, we obtain the values of the unknown constants:

$$a_0 = 0{,}46; a_1 = -0{,}06; a_2 = 0{,}25; a_3 = -0{,}06.$$

The minimized criterion on the reliability characteristic is determined by the formula

$$y_1(x) = 1 - y_1^{'}(x) \approx 0,54 + 0,06x_1 - 0,25x_2 + 0,06x_3$$

(is meant that the unit corresponds to 100 percent reliability of the product).

Similar calculations are made for the cost criterion. Criterion function of cost we approximate by linear approximation without constant term:

$$y_2(x) \approx F_2(x^{(u)}, b) = b_1x_1 + b_2x_2 + b_3x_3,$$

where b_1, b_2, b_3 – unknown constants (p=3). Evaluation levels experts gave in the same approximation nodes (N = 3), as in the case of reliability criterion. Let these estimates are as follows:

$$f_2(x^{(1)}) = 1; f_2(x^{(2)}) = 0,3; f_2(x^{(3)}) = 0,6$$

(Normalized value of cost at the maximum factors is equal to unity). Since in this case p=N, to determine the constants you can use an interpolation method. Equation (12.5) with respect to the cost criterion is converted to the form

$$F_2(x^{(u)}, b) = f_2(x^{(u)}), u \in [1, N = p],$$

and with given numerical data

$$10b_1 + 5b_2 + 6b_3 = 1;$$
$$b_1 + b_2 + b_3 = 0{,}3;$$
$$6b_1 + 3b_2 + 3b_3 = 0{,}6.$$

Solving this SLAE by Cramer (http://www.webmath.ru/web/prog13_1.php), we obtain

$$b_1 = -0{,}1; b_2 = 0{,}3; b_3 = 0{,}1$$

and the expression for the cost criterion is of the form

$$y_2(x) \approx -0{,}1x_1 + 0{,}3x_2 + 0{,}1x_3$$

We got analytical expressions for criteria of reliability and cost, allowing you to apply the formula (12.3) for the determination of a compromise-optimal values of factors *x** under the obvious limitations $A_1 = A_2 = 1$:

$$x^* = \arg\min_{x \in X}\left(\frac{1}{1 - 0.54 - 0.06x_1 + 0.25x_2 - 0{,}06x_3} + \frac{1}{1 + 0{,}1x_1 - 0{,}3x_2 - 0{,}1x_3}\right).$$

Having carried out minimization of the scalar convolution of criteria on the optimization arguments by Nelder-Mead method (Voronin, A.N. at al 1999), we obtain

$$x_1^* = 9{,}99; x_2^* = 3{,}69; x_3^* = 1{,}01.$$

This result of illustrative example can be interpreted as follows. When optimizing the design of the future launcher, special attention should be paid to the reservation of the product control systems x_1. Safety factor for structural strength to choose roughly a little more than mid range $x_2 \in [1;5]$. Do not place much hope in the possibility of material incentives for personnel x_3.

Thus, this study makes it possible to identify the main trends in the development and optimization of new space technology. The proposed procedures allowed practically solve the problem of vector optimization of compulsory insurance process for developed space facilities objects (Voronin, 2017).

We must be aware that expert models are less informative than regression models identified with the help of real experiments. However, firstly, expert assessments still reflect, albeit with a possible distortion, the reality, and, secondly, the expert model is only an initial approximation and with the accumulation of statistics is amenable to improvement.

Acknowledgements

The paper is published with partial support by the project ITHEA XXI of the ITHEA ISS (www.ithea.org) and the ADUIS (www.aduis.com.ua).

Bibliography

[Voronin at al, 1999] Voronin A.N., Ziatdinov Ju.K., Kozlov A.I. Vector optimization of dynamic systems, Kiev, Tehnika.

[Voronin, 2017] Voronin A.N. Multi-Criteria Decision Making for the Management of Complex Systems, IGI Global.

Authors' Information

Voronin Albert Nikolaevich *– professor, Dr.Sci (Eng), professor of the department of National aviation university of Ukraine, Komarova Ave, 1, Kiev-58, 03058 Ukraine; e-mail: alnv@ukr.net*

Major Fields of Scientific Research: Multi-Criteria Decision Making; Man-Machine Complex Systems

Ziatdinov Yuriy Kashafovich – *professor, Dr.Sci (Eng), head of the department of National aviation university of Ukraine, Komarova Ave,1, Kiev-58, 03058 Ukraine; e-mail: oberst@nau.edu.ua*