MATHEMATICAL MODEL OF MULTICRITERIAL DISTRIBUTION OF TRANSPORT FLOWS IN THE MULTIMODAL TRANSPORT NETWORK

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Abstract: *The article proposes a mathematical model of multicriteria distribution of transport flows in a multimodal transport network. The model is based on the presentation of the problem in a multicriteria form according to the efficiency-cost model using a nonlinear scheme of compromises. The problem of choosing the optimal route is formalized and solved as a problem of multicriteria choice of the graph bypass option. The assessment of the reliability of the optimal route is carried out by an integrated assessment using a fundamental rating scale.*

Keywords: *Multimodal transportation, multicriterial optimization, indicator, criterion.*

Introduction. Problem statement

Intensive growth of demand for freight transportation by rail, road, sea (river) modes of transport necessitates the interaction of two or more modes of transport in one transport process. These modes of transport can be discharged by the shipper as an alternative, however, there are certain resource limitations in the use of each of the modes of transport, which make it impossible to deliver goods "door to door" only one mode of transport. In such cases, multimodal transportation is appropriate, which makes it possible to combine the advantages of interacting modes of transport.

The distribution of traffic flows will be considered on the example of the rail freight system, which is indicative, as hub railway stations are points of interaction of rail with other modes of transport, and passenger and freight railway rolling stock is shared, and require internal distribution of passengers and freight. This organization of traffic overloads railway infrastructure facilities and causes deviations in train schedules and subsequent successive reloading of cargo on other modes of transport. Therefore, for the effective organization and management of freight and passenger traffic in a multimodal transport network, the solution requires the applied problem of choosing the optimal route for the transportation of goods.

Therefore, there is a need to find new solutions for the efficient distribution of traffic flows in a multimodal transport network. One such solution is the use of mathematical approaches to solve the problem of allocation of limited resources, namely the separation of freight and passenger traffic in Ukraine with the allocation of separate tracks for exclusively freight trains. Progressive in this sense are the methods of multicriteria (vector) selection, evaluation and optimization [Voronin, 1997]. Methods for solving multicriteria problems are considered in the works of G.I. Ankudinov G.I., Antushev G.S., Voronin A.N., etc. [Ankudinov, 1986], [Antushev, 1986], [Vermishev, 1982]. In the field of view of modern domestic and foreign scientists are the issues of the mode of operation of modes of transport. In particular, it is worth noting the contribution of the American scientist Jean-Paul Rodrigo, who studies the modality of road and rail modes of transport [Rodrigue, 2013]; theoretical and practical problems related to the strategy of international integration of the transport and road complex of Ukraine in the processes of integration into the world and European transport systems are considered by scientists Yu. Ye. Pashchenko, O.I. Nikiforuk in [Pashchenko, 2008]; rational schemes of distribution of train flows are considered [Tkhoruk, 2019], [Kujala, 2018], [Kyrychenko, 2018], [Papakhov, 2016], [Galaburda, 1996]. The work of scientists Vorkuta A.I. is devoted to the study of the process of interaction of modes of transport during freight transportation, in particular the organization, transportation management, and operational work scientists [Vorkut, 1986], [Abramov, 2004], [Pravdin, 1997]. These approaches are used to increase the efficiency of the transport process on certain modes of transport, which does not provide a solution to the complex problem of the distribution of traffic flows in a multimodal transport network. The methods used in solving multicriteria and methods of parametric synthesis and design are discussed in detail in [Brahman, 1992], [Baranov, 2018], [Baranov, 2015], [Berdnik, 2016], [Ziatdinov, 2015]. Their analysis shows that the most effective are the methods of multicriteria optimization, which consist of reducing the problem to a single-criterion form by a certain criterion and finding its extremum [Keeney, 1981], [Voronin, 1997], [Brahman, 1992]. Since the traditional formalization of the optimization problem in a single-criteria form does not provide a solution to the complex problem of traffic flow distribution and selection of the optimal route of cargo delivery in a multimodal transport network, it is necessary to solve the problem using multicriteria approaches [Voronin, 2017], [Kharchenko, 2015], [Saaty, 1984]. Therefore, it is necessary to solve the problem of choosing the optimal route of transportation using multicriteria approaches.

Presentation of the main research material

Multimodal transportation of goods is carried out with the participation of two or more modes of transport involved in one transportation process and is an integral part of a single transport system of Ukraine. The points of interaction of modes of transport and segments of freight routes form a multimodal network, which by its nature is an undirected graph, the vertices of which are the points of interaction, and the edges - the paths. Roads and other infrastructure facilities (port, station, station, etc.) of multimodal transportation are a limited resource that can pass a certain number of units of rolling stock per unit time.

To characterize the subject area, we use the following terminology and take into account the following features.

Multimodal transportation of goods – transportation of goods by two or more modes of transport, which is organized under the responsibility of the operator of multimodal transportation on the basis of a single contract for multimodal transportation.

The stage of multimodal transportation is a segment of the route of multimodal transportation of cargo by one mode of transport.

The route of multimodal transportation is the way of transportation of cargo determined by the contract of multimodal transportation.

Multimodal terminal is a production and transshipment complex of an economic entity of any form of ownership, which provides the organization of multimodal transportation of goods.

To solve the applied problem of choosing the optimal route of cargo delivery in a multimodal transport network, a multicriteria mathematical model of traffic flow distribution has been developed. The model belongs to the class of optimization and involves determining the optimal delivery route and confirmation of the authenticity of the determined optimal route. The developed model is based on the presentation of the problem in a multicriteria form and its solution by a nonlinear scheme compromises.

The essence of the formation of a multicriteria model is explained by the structural-logical scheme shown in Figure 1.1.

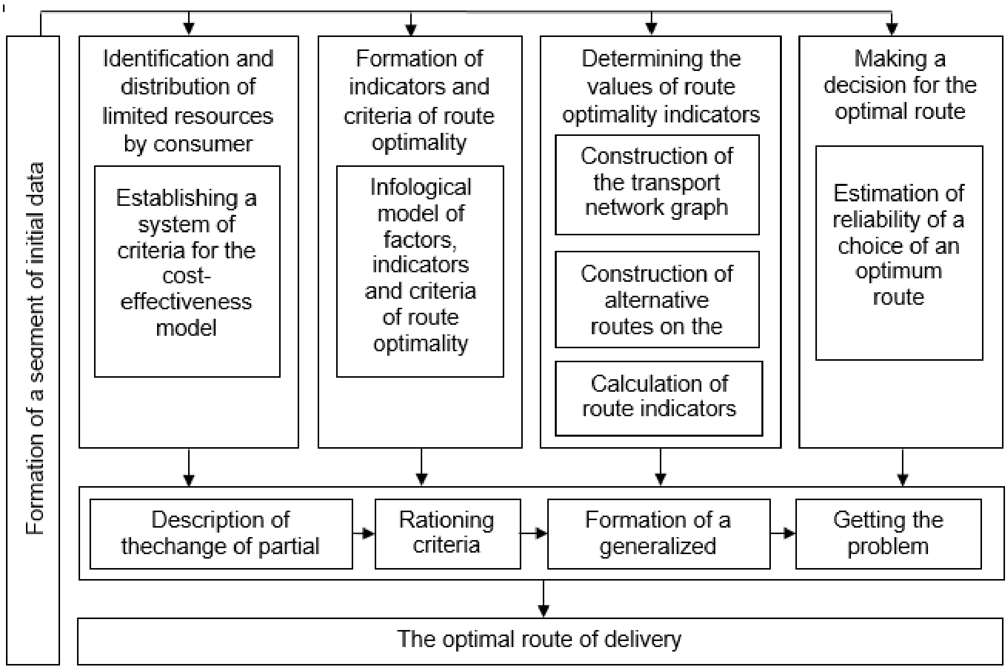

Figure 1.1. Structural and logical scheme of formation of the multicriteria mathematical model of traffic flow distribution

1. Identification and distribution of limited resources by consumer groups

Railway tracks, which carry freight and passenger rolling stock – a limited resource. Rational allocation of limited resources is needed by consumers of transportation services - passengers and shippers.

The optimal (rational) distribution of limited resources in complex the ergatic systems is performed to organize the efficient allocation of resources. Given the wide variety of tasks on the optimal allocation of resources, it should be noted some of its features:

- Diverse resources (stocks) are subject to distribution.
- The distribution of resources is characterized by the diversity of needs of consumers of stocks.
- The requirement of optimality (rationality) of resource allocation is the highest efficiency of performance of target tasks by consumers on

purpose after distribution of resources at their minimum expenses. There are no conditions for the distribution of all stocks (resources).

- The situation is possible when the target tasks are performed by the consumer for the intended purpose in the case of meeting at least one of his needs (in general, the supplier's resource is allocated).

Given the above, we can formalize the problem of optimal allocation of limited resources, as follows. Let the set of consumers be given $SP=\{SP_i\}$, $i=1\ldots I$ – number of consumers) each of which has limited set of disparate needs $SP_i \subset \{P_{otij}\}$, $j=1\ldots J$ – number of needs i-th consumer) certain limited resources (stocks). It is possible to set priorities in customer service and meet their needs. The set of suppliers is set $PS=\{PS_k\}$, $k=1\ldots K$ – number of suppliers) with a list of heterogeneous limited stocks $PS_k \subset \{Z_{apkm}\}$, $m=1\ldots M$ – the number of stocks of a certain type). The number of suppliers and their stocks of resources is much smaller than the number of consumers and the list of their needs. It is necessary to distribute limited stocks Z_{apkm} suppliers PS_k by consumers SP_i so as to meet their needs P_{otij} provided the best efficiency of performance of target tasks by consumers. The distribution of stocks should be implemented taking into account the priorities in customer service. Thus the condition of distribution of all available stocks (resources) is not put forward and the situation of effective performance of target tasks by the consumer at the satisfaction of at least one need is possible.

To solve the formulated problem, the first step is to form a table of consumer rankings by selecting from the set $SP=\{SP_i\}$ – $l=1\ldots L$ groups for each of which there are requirements to meet their needs and efficiency in the implementation of target tasks P_l. The ranking table is formed following the directive requirements for the efficiency of distribution, the experience of solving this class of problems, and external factors.

In each l-th group, the needs of each i-th consumer are ranked on priority, and efficiency of performance of target tasks at the satisfaction of its j-th needs P_{otij} and resource consumption k-th supplier Z_{apkm}. Thus, we will have a ranking of consumers and their needs according to the priority of service, which provides a conditional distribution of limited resources, based on the requirements for the effectiveness of this process, Table 1.1.

Table 1.1.Consumer ranking table

Group number / consumer number in the group	Priority of consumer needs	Hierarchy of consumer needs	The effectiveness of the target tasks by the consumer	Consumed resource of the supplier
Consumer groups 1				
1/1	Consumer 1			
	1	P_{ot11}	E_{11}	R_1
	2	P_{ot12}	E_{12}	R_2
	…	…	…	…
	J_1	P_{ot1J1}	E_{1J1}	R_K
1/2	Consumer 2			
	1	P_{ot21}	E_{21}	R_1
	2	P_{ot22}	E_{22}	R_2
	…	…	…	…
	J_2	P_{ot2J2}	E_{2J2}	R_K
…	…		…	
Consumer groups 2				
2/1	Consumer 1			
	…	…	…	…
	…	…	…	…
…				
Consumer groups L				
L/1	Consumer 1			
	…	…	…	…
	…	…	…	…

The second stage involves the formation of a system of partial criteria of optimality and a description of their change. To do this, set a variable parameter - the number of suppliers allocated to serve the needs of the consumer n_i, with restrictions on the total number of suppliers N:

$$n_i, \sum_{i=1}^{I} n_i \le N; \tag{1.1}$$

therefore, it will be allocated for the first consumer i_1, for the second – i_2, supplier, and the value n_i = 2, means selection for *i*-th consumer suppliers to meet P_{ot1} needs (with the first priority) and needs P_{ot2} with the second priority. A cost-effective model of partial criteria will be used to optimally allocate limited resources. The criterion of efficiency will be the quality of the consumer's performance of target tasks while meeting his needs $P(n) \Rightarrow max$. The criterion of cost is the generalized cost of the supplier's resource to meet the needs of the consumer for high-quality performance of their target tasks $S(n) \Rightarrow min$. Then, for several consumers within one group of importance, we will have a system of partial criteria

$$\begin{cases} P_1(n_1) \Rightarrow max, S_1(n_1) \Rightarrow min, P_2(n_2) \Rightarrow max, S_2(n_2) \Rightarrow min, ..., \\ P_i(n_i) \Rightarrow max, S_i(n_i) \Rightarrow min, ..., P_I(n_I) \Rightarrow max, S_I(n_I) \Rightarrow min. \end{cases} \tag{1.2}$$

The contradiction of partial criteria (1.2) is a sign of bringing the problem of the optimal distribution of limited resources to a multicriteria form. The description of the change of criteria (1.2) is determined by the specifics of the problem and in the first approximation is carried out in discrete form with subsequent approximation by polynomial models

$$P_i(n_i) = P_o + P_1 n_i + P_2 n_i^2 + ..., S_i(n_i) = S_o + S_1 n_i + S_2 n_i^2 + ..., \tag{1.3}$$

where the parameters of the models – P_0, P_1, P_2 ; S_0 S_1 S_2 consistent with discrete data, such as the norm.

The third stage of the methodology is a direct solution to the optimization problem by reducing the set of partial criteria to a generalized functional. To form a generalized criterion for the optimal distribution of limited resources, a nonlinear scheme of compromises will be used following the convolution of Professor A.M. Voronin, [Voronin, 1997], [Voronin, 2017]. Compared to other optimization schemes, the convolution has the following advantages: optimization problems are solved in the presence of constraints within which the unimodality of the generalized criterion function is guaranteed; relatively small computational complexity of the solution search algorithm. The convolution for the analog representation of partial criteria has the form

$$\chi^* = \underset{\chi \in G}{\operatorname{argmin}} \sum_{l=1}^{b} \gamma_{l0} \left(1-\varphi_{ol}(\chi)\right)^{-1} = F(\chi), \tag{1.4}$$

where l =1…b – the number of included in the convolution of partial optimality criteria; χ – optimized parameter; G – area for determining the functions of partial optimality criteria; $\varphi_{ol}(\chi)$ – normalized function l-th partial criterion of optimality; χ^* – the optimal value of the system parameter. The rationing of partial criteria is carried out to bring them to a dimensionless minimized form relative to the maximum (minimum) on a limited interval of consideration of the criterion. Thus, the rationing of analytical models (1.3), from the list (1.2) is carried out on a limited (when obtaining discrete values) interval of change of argument n_i, which allows to take into account the needs i-th consumers in the rationing of efficiency criteria $P(n_i)$, and limited stocks k-th supplier when rationing the cost criterion $S(n_i)$.

The formation of the generalized functionality according to criteria (1.2) will be carried out using the technology of nested convolutions (convolution convolutions) in stages for each subgroup of consumers. Then, the use of convolution (1.4) provides a system of generalized criteria for important groups.

$$\begin{cases} \delta_1(n_{11}, n_{12}, ..., n_{1i}, ..., n_{1l1}) \Rightarrow min, \delta_2(n_{21}, n_{22}, ..., n_{1i}, ..., n_{1l2}) \Rightarrow min, ..., \\ \delta_l(n_{l1}, n_{l2}, ..., n_{li}, ..., n_{ll1}) \Rightarrow min, \delta_L(n_{L1}, n_{L2}, ..., n_{Li}, ..., n_{LlL}) \Rightarrow min, \\ S_1\begin{pmatrix} n_{11}, n_{12}, ..., n_{1i}, ..., n_{1l1}, n_{21}, n_{22}, ..., n_{1i}, ..., n_{1l2}, ..., n_{l1}, n_{l2}, ..., n_{li}, \\ ..., n_{ll1}, n_{L1}, n_{L2}, ..., n_{Li}, ..., n_{LlL} \end{pmatrix} \Rightarrow min. \end{cases} \tag{1.5}$$

where

$$\delta_1(n_{l1}, n_{l2}, ..., n_{li}, ..., n_{ll1}) = \gamma_{ol1}\left(1-\varphi_{ol1}(n_{l1})\right)^{-1} + \gamma_{ol2}\left(1-\varphi_{ol2}(n_{l2})\right)^{-1} + ... +$$
$$+ \gamma_{oli}\left(1-\varphi_{oli}(n_{li})\right)^{-1} + ... + \gamma_{oll}\left(1-\varphi_{oll}(n_{ll})\right)^{-1},$$

criterion of efficiency of customer service l-th groups of importance,

$$S_1(n_{11}, n_{12}, ..., n_{1i}, ..., n_{1i1}, n_{21}, n_{22}, ..., n_{1i}, ..., n_{1l2}, ..., n_{l1}, n_{l2}, ..., n_{li}, ..., n_{ll1}, n_{L1}, n_{L2}, ..., n_{Li}, ..., n_{LlL}) =$$
$$= \gamma_{oS11}\left(1-S_{o11}(n_{11})\right)^{-1} + \gamma_{oS12}\left(1-S_{o12}(n_{12})\right)^{-1} + ... + \gamma_{oS1i}\left(1-S_{o1i}(n_{1i})\right)^{-1} + ... + \gamma_{oS1l1}\left(1-S_{o1l1}(n_{1l1})\right)^{-1} + ...$$
$$+ \gamma_{oS21}\left(1-S_{o21}(n_{21})\right)^{-1} + \gamma_{oS22}\left(1-S_{o22}(n_{22})\right)^{-1} + ... + \gamma_{oS2i}\left(1-S_{o2i}(n_{2i})\right)^{-1} + ... + \gamma_{oS2l1}\left(1-S_{o2l1}(n_{2l1})\right)^{-1} + ...$$
$$+ \gamma_{oSl1}\left(1-S_{ol1}(n_{l1})\right)^{-1} + \gamma_{oSl2}\left(1-S_{ol2}(n_{l2})\right)^{-1} + ... + \gamma_{oSli}\left(1-S_{oli}(n_{li})\right)^{-1} + ... + \gamma_{oSll1}\left(1-S_{oll1}(n_{ll1})\right)^{-1} + ...$$
$$+ \gamma_{oSL1}\left(1-S_{oL1}(n_{L1})\right)^{-1} + \gamma_{oSL2}\left(1-S_{oL2}(n_{L2})\right)^{-1} + ... + \gamma_{oSLi}\left(1-S_{oLi}(n_{Li})\right)^{-1} + ... + \gamma_{oSLl1}\left(1-S_{oLl1}(n_{Ll1})\right)^{-1} + ...$$

criterion of generalized value (costs) to meet the needs of consumers, γ_{0ij}, γ_{0sij} – ration weights that characterize the advantage (importance) of the relevant criterion in solving the problem of optimal allocation of resources. Double indexing of parameters n_{li} by index l – characterizes the number of selected groups, and the index i – number of consumers in the group.

From the generalized criteria (1.5) the integrated criteria of optimality of distribution of limited resources are formed (expression (1.6). Rationing (1.5) is carried out concerning the size which is defined at the worst variant of distribution of resources of suppliers between consumers:

$$\Omega\begin{pmatrix} n_{11},n_{12},...,n_{1i},...,n_{1l1},n_{21},n_{22},...,n_{2i},...,n_{2l2},...,n_{l1},n_{l2},..., \\ n_{li},...,n_{ll1},n_{L1},n_{L2},...,n_{Li},...,n_{LlL} \end{pmatrix} = \\ = G_{o1}\left(1-\delta_{o1}(n_{11},n_{12},...,n_{1i},...,n_{1l1})\right)^{-1} + \\ + G_{o2}\left(1-\delta_{o2}(n_{21},n_{22},...,n_{2i},...,n_{2l2})\right)^{-1} + \\ + G_{ol}\left(1-\delta_{ol}(n_{l1},n_{l2},...,n_{li},...,n_{ll1})\right)^{-1} \\ + G_{oL}\left(1-\delta_{oL}(n_{L1},n_{L2},...,n_{Li},...,n_{LlL})\right)^{-1} + \\ + F_o\left(1-S\begin{pmatrix} n_{11},n_{12},...,n_{1i},...,n_{1l1},n_{21},n_{22},...,n_{2i},. \\ ..,n_{2l2},n_{l1},n_{l2},...,n_{li},...,n_{ll1},n_{L1},n_{L2},...,n_{Li},...,n_{LlL} \end{pmatrix}\right)^{-1}. \tag{1.6}$$

In (1.6) G_{01}, G_{02}, G_{0l}, G_{0L}, F_0 – normalized weights, consistent with the requirements of the effectiveness of the target tasks by consumers after the distribution of resources of suppliers. Integrated criterion (1.6) is an optimization model of optimal allocation of limited resources. Determination of the optimal values of the varied parameters according to model (1.6) is carried out by solving a system of nonlinear equations:

$$\frac{\partial^{(l_1+l_1+...+l_l+l_L)}\Omega\begin{pmatrix} n_{11},n_{12},...,n_{1i},...,n_{1l1},n_{21},n_{22},...,n_{2i},...,n_{2l2},..., \\ n_{l1},n_{l2},...,n_{li},...,n_{ll1},n_{L1},n_{L2},...,n_{Li},...,n_{LlL} \end{pmatrix}}{\partial^{(l_1+l_1+...+l_l+l_L)} n_{il(1,2,...,l,...,L)}} = 0. \tag{1.7}$$

The result of the solution of the system (1.7) is the value of the varied parameters n^{*}_{lil}, which, if necessary, are rounded to whole numbers n^{opt}_{lil} with control of condition (1.1).

2. Formation of indicators and criteria of route optimality

The structure of System of Rail Freight Transportation (SRFT) can be represented by the following components - transport work, material and technical base, operational work, economic efficiency. These components of the structure of SRFT, following the main purpose of the system, are determined by a set of factors that affect its effectiveness as a whole and form a system of indicators and criteria of optimality.

The main categories of factors influencing the choice of the route of transportation of goods by rail rolling stock: organizational (technological), technical (operational) and economic.

The category of organizational (technological) factors includes those that characterize the participants in the transportation process, namely the number of modes of transport involved in the transportation process, available rolling stock, responsibility and professionalism of persons planning operations, routes, documentation and directly transportation.

The category of technical (operational) factors should include those determined by the technical component of the system, the capabilities of technical devices. The following groups of factors are included in this category: rolling stock and equipment, infrastructure, cargo, cargo handling point.

The category of economic factors that affect the efficiency of the functions performed by the system reflects the cost of operations, including the cost of transportation of passengers and goods, the cost of achieving high productivity, deductions for system maintenance and more.

To achieve a positive effect from the functioning of the system when considering both organizational (technological) and technical (operational) factors, there is a need to increase certain costs. This resource is limited, so it is natural to try to reduce costs in the system. Thus, the category of economic factors affects the efficiency of the system, and the reduction of economic costs often leads to a deterioration of its properties.

A limited resource is an infrastructure (a category of technical factors) that can pass a certain number of units of rolling stock per unit time (organizational factors), and exceeding this limit leads to depreciation of fixed assets, and instead of a positive economic effect leads to increased costs. Thus, there is an impact of technical factors on the economic and organizational components of the rail freight system.

According to the structure of SRFT and the factors influencing the choice of route, groups of indicators of railway transportations are:

Indicators of transport work (turnover of goods *Pl*, turnover of passengers *Al*, volume of cargo transportation q_g, volume of passenger traffic q_p);

Indicators of material and technical base (operating length of tracks between stations L_E, network density d_S, load capacity of a rolling stock unit P_n);

Indicators of exploitation work (average distance of cargo transportation l_c, average distance of passenger transportation l_{cp}, speed of cargo delivery v_d, traffic intensity *W*);

Indicators of economic efficiency and financial (the cost of transporting passengers C_p, the cost of transportation of goods C_v, productivity of work P_p, revenues *D*, costs *E*, profit *Y*, cargo transportation fee *N*).

Each factor influencing the choice of the optimal route of transportation is reflected by one or more indicators and criteria. Therefore, indicators and criteria can be unitary or combinatorial, ie those that are generalized for a particular decomposition link and are formed from partial indicators and criteria. For the case of combinatorial indicators and criteria, a hierarchy is built for each of them, from which a system of efficiency criteria is formed. The generalized hierarchy of partial and generalized criteria for organizational (technological), technical, operational, and economic categories can take the form of structural schemes (Figure 2.1).

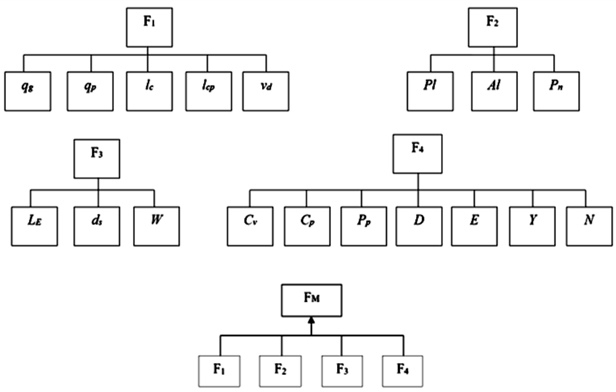

Figure 2.1 Hierarchy of partial and generalized criteria

Following, we obtain a system of criteria that will characterize the optimal choice of the route of transportation of goods by rail rolling stock F_M for indicators of the unitary type in the form:

$$F_M = \begin{cases} F_1 \to min; \\ F_2 \to min; \\ F_3 \to min; \\ F_4 \to min. \end{cases} \quad (2.1)$$

$$F_1 = \begin{cases} q_g \to max; \\ q_p \to min; \\ l_c \to max; \\ l_{cp} \to min; \\ v_d \to max. \end{cases} F_2 = \begin{cases} Pl \to max; \\ Al \to max; \\ Pn \to max. \end{cases} F_3 = \begin{cases} L_E \to max; \\ d_s \to max; \\ W \to max. \end{cases} F_4 = \begin{cases} C_v \to min; \\ C_p \to min; \\ P_p \to max; \\ D \to max; \\ E \to min; \\ Y \to max; \\ N \to max. \end{cases} \quad (2.2)$$

Combinatorial indicators of the optimal route will be characterized by the expansion of the system of criteria (2.1) following the model subsystems of partial criteria (Figure 2.1).

Thus, partial criteria of efficiency of the process of choosing the route of cargo transportation by rail rolling stock in the form of unitary (2.1) and combinatorial systems of partial criteria (2.2) are formed.

The main criteria for the effectiveness of the choice of the route of transportation of goods by rail rolling stock are the following:

- F_1 – duration of cargo delivery;
- F_2 – productivity of rolling stock and equipment;
- F_3 – capacity of the transport network (infrastructure facilities, track facilities);
- F_4 – economic efficiency.

The model shown in Figure 2.1 allows to form a system of contradictory partial criteria of route optimality, which indicates the receipt of a multicriteria optimization problem for determining the optimal route of transportation, equation (2.3):

$$\delta = \begin{cases} q_p \to max; q_p \to min; l_c \to max; l_{cp} \to min; \\ v_d - max; Pl \to max; Al \to min; \\ P_n \to max; L_E \to max; d_s \to max; \\ W \to max; C_p \to min; C_v \to min; P_p \to max; \\ D \to max; E \to min; Y \to max; N \to max. \end{cases} \quad (2.3)$$

The formed system of partial criteria is contradictory, which follows from such considerations. The simultaneous increase in freight and passenger traffic on one line will negatively affect the capacity and carrying capacity of the elements of the transport network. Thus, the set of contradictory partial criteria of route optimality indicates the reduction of the synthesis problem to a multicriteria form. Then the formation of the mathematical model of SRFT will consist in establishing the relationship between the partial criteria of system optimality by solving a multicriteria optimization problem.

The explanation of the physical essence of established indicators and criteria of optimality according to the notations and explanations of the order of calculation, describing the change of partial criteria, are given in Table 2.1. Data are formed for each point of interaction of modes of transport and are initial for the further calculations of alternative routes.

Table 2.1. Calculation of partial criteria of route optimality

Indicator	Equation	Explanation of the calculation
q_g	$q_g = \sum_1^n q_E$,	q_E – elementary cargo flow (volume of transported cargo per unit time).
q_p	$q_p = n$,	n – number of tickets sold.
l_c	$l_c = \frac{Pl}{q_g}$,	Pl – turnover of goods; q_g – the volume of cargo transportation.
l_{cp}	$l_{cp} = \frac{Al}{q_p}$,	Al – turnover of passengers; q_p – the volume of passenger traffic.
v_d	$v_d = \frac{l_g}{T_g}$,	l_g – distance shipment; T_g - total time of cargo stay on transport.
Pl	$Pl = p_1 l_1 + ... + p_i l_i$	$p_1 l_1, p_i l_i$ – turnover consignments of goods (p_1, p_i), transportation distance (l_1, l_i).
Al	$Al = A_1 l_1 + ... + A_i l_i$,	A_i – number of passengers sent, distance l_i .
P_n	$P_n = \frac{p_0 m_0}{1 + k_t}$,	p_0 – permissible axial load; m_0 - the number of wheel pairs; k_t - technical coefficient of packaging.
L_E	L_{Ei}	reference data, values of real distances between stations
d_s	$d_S = \frac{1000 \cdot L_E}{S}$,	L_E – operating length of tracks between stations; S – area of territory.
W	$W = \frac{\sum Pl + \sum Al}{L_E}$,	$\sum Pl$ – turnover of goods; $\sum Al$ – turnover of passengers; L_E – operating length of tracks between stations.
$C_{p(v)}$	$C_{p(v)} = \frac{C_i}{l} + c_{p(v)}$,	C_i – cost of initial and final operations; $c_{p(v)}$ – cost of passenger (goods) movement operations; l –

Indicator	Equation	Explanation of the calculation
		distance
P_p	$P_P = \frac{\sum Pl_c}{H_e}$,	$\sum Pl_C$ – the yearly volume of transport work; H_e – the average number of workers employed in operational work
D	$D = D_G + D_P$,	D_G – revenues from freight transportation; D_P – revenues from passenger traffic
E	$E = E_G + E_P$,	E_G – freight costs; E_P – costs of passenger transportation
Y	$Y = D - E$,	D – total income for manufactured and sold products, work, and services; E – operational expenses
N	N	reference data, according to Tariff rules №4 of railways of Ukraine

3 Determining the values of route optimality indicators. Construction of the transport network graph

To construct a graph of a multimodal transport network, the points of interaction of modes of transport are chosen by the vertices, namely, nodal railway stations with access roads (interaction of railway transport - road transport); sea (river) port with access railways and highways (interaction sea (river) transport - railway transport - road transport). Segments of the route of multimodal transportation of goods by one mode of transport - the edges of the graph. The characteristics of the edges of the graph are the real distances between the vertices.

Table 3.1. Generalized table of indicators of transportation routes

Indicator \ Route		M1			Σ_1	…	Mn			Σ_{14}
		Vertice i_1	…	Vertice j_1		…	Vertice i_{14}	…	Vertice j_{14}	
Generalized indicator	Partial indicator									
F1	q_g	q_g^{i1}	…	q_g^{j1}	Σq_g^{ij1}	…	q_g^{i14}	…	q_g^{j14}	Σq_g^{ij14}
	q_p	q_p^{i1}	…	q_p^{j1}	Σq_p^{ij1}	…	q_p^{i14}	…	q_p^{j14}	Σq_p^{ij14}
	l_c	l_c^{i1}	…	l_c^{j1}	Σl_c^{ij1}	…	l_c^{i14}	…	l_c^{j14}	Σl_c^{ij14}
	l_{cp}	l_{cp}^{i1}	…	l_{cp}^{j1}	Σl_{cp}^{ij1}	…	l_{cp}^{i14}	…	l_{cp}^{j14}	Σl_{cp}^{ij14}
	v_d	v_d^{i1}	…	v_d^{j1}	Σv_d^{ij1}	…	v_d^{i14}	…	v_d^{j14}	Σv_d^{ij14}
F2	Pl	Pl^{i1}	…	Pl^{j1}	ΣPl^{ij1}	…	Pl^{i14}	…	Pl^{j14}	ΣPl^{ij14}
	Al	Al^{i1}	…	Al^{j1}	ΣAl^{ij1}	…	Al^{i14}	…	Al^{j14}	ΣAl^{ij14}
	P_n	P_n^{i1}	…	P_n^{j1}	ΣP_n^{ij1}	…	P_n^{i14}	…	P_n^{j14}	ΣP_n^{ij14}

F_3	L_E	L_E^{i1}	…	L_E^{j1}	ΣL_E^{ij1}	…	L_E^{i14}	…	L_E^{j14}	ΣL_E^{ij14}
	d_s	d_s^{i1}	…	d_s^{j1}	Σd_s^{ij1}	…	d_s^{i14}	…	d_s^{j14}	Σd_s^{ij14}
	W	W^{i1}	…	W^{j1}	ΣW^{ij1}	…	W^{i14}	…	W^{j14}	ΣW^{ij14}
F_4	C_v	C_v^{i1}	…	C_v^{j1}	ΣC_v^{ij1}	…	C_v^{i14}	…	C_v^{j14}	ΣC_v^{ij14}
	C_p	C_p^{i1}	…	C_p^{j1}	ΣC_p^{ij1}	…	C_p^{i14}	…	C_p^{j14}	ΣC_p^{ij14}
	P_p	P_p^{i1}	…	P_p^{j1}	ΣP_p^{ij1}	…	P_p^{i14}	…	P_p^{j14}	ΣP_p^{ij14}
	D	D^{i1}	…	D^{j1}	ΣD^{ij1}	…	D^{i14}	…	D^{j14}	ΣD^{ij14}
	E	E^{i1}	…	E^{j1}	ΣE^{ij1}	…	E^{i14}	…	E^{j14}	ΣE^{ij14}
	Y	Y^{i1}	…	Y^{j1}	ΣY^{ij1}	…	Y^{i14}	…	Y^{j14}	ΣY^{ij14}
	N	N^{i1}	…	N^{j1}	ΣN^{ij1}	…	N^{i14}	…	N^{j14}	ΣN^{ij14}

The graph of the multimodal transport network, Figure 3.1 is undirected, as transportation can be carried out both in the forward and in the reverse directions.

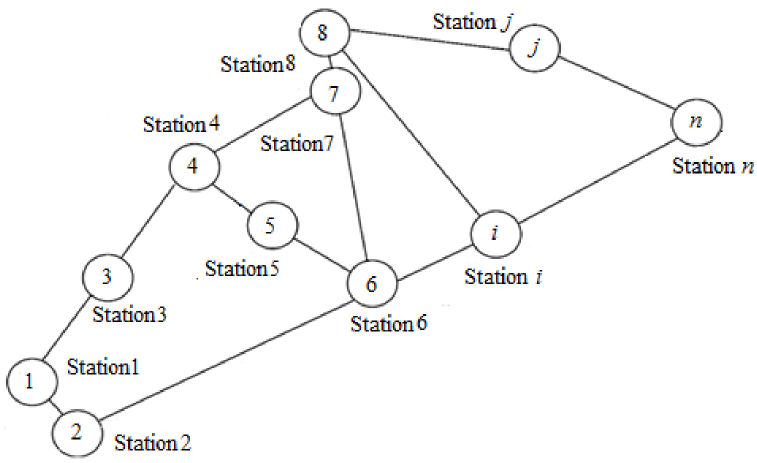

Figure 3.1. Graph of the multimodal transport network

Construction of alternative routes of cargo delivery on the graph of the multimodal transport network is carried out from the top of the shipment to the top of the destination. Partial indicators of stations of the laid routes are generalized in Table 3.1.

4. Elaboration of a decision on the optimality of the route

To make a decision about the optimality of the route according to contradictory criteria (3.1), their reduction to an integrated assessment according to a nonlinear scheme of compromises following the convolution of Professor A.M. Voronin [Voronin, 1997], [Voronin, 2017], using the technology of nested convolutions. The convolution uses a nonlinear scheme of trade-offs, which allows to obtaining a Pareto-optimal solution at low computational costs. The optimization problem is solved in the presence of constraints, which ensures the unimodality of the function of the generalized criterion and in any case guarantees a single solution. Convolution allows to using the minimax approach, ie to concentrate on extremization of the dominant partial criterion of optimality.

The rationing of partial optimality criteria is carried out by expression (1.4), rationing of weights coefficients according to equations (4.4) – (4.4):

$$Y(y_0) = \sum_{l=1}^{b} \gamma_{0l} (1 - y_{0l})^{-1} \to min \tag{4.1}$$

where $l=1…b$ – the number of included in the convolution of partial optimality criteria; γ_{0l} – normalized weighting coefficient; y_{0l} – normative partial criterion.

The requirement to use the convolution (4.1) is the normalization of the parameters that are part of them. Rationing provides a reduction to a single dimensionless scale of estimates of partial criteria of optimality with different physical essence and different absolute values. This allows you to equally influence the result of solving the optimization problem of each of the partial criteria.

Weights are normalized by the equation:

$$\gamma_{0l} = \frac{\gamma_l}{\sum_{l=1}^{b} \gamma_l}, \tag{4.2}$$

where γ_l – the current (unregulated) value of the weighting coefficient.

Since the convolution (4.1) is minimized, ie will include criteria requiring their minimization, then the partial discrete optimality criteria are normalized relative to the sum of current values separately for those that are minimized:

$$\varphi_{0l}^{min} = \frac{\varphi_l^{min}}{\sum_{i=1}^{N} \varphi_{li}^{min}},, \tag{4.3}$$

and for those who maximize:

$$\varphi_{0l}^{max} = \left(\varphi_l^{max} \cdot \sum_{i=1}^{N} \frac{1}{\varphi_{li}^{max}} \right)^{-1}. \tag{4.4}$$

The formation of generalized optimality criteria (1.1) for each route is carried out by equations (4.5) – (4.8).

$$F_{1_i} = (1 - q_{g_i 0})^{-1} + (1 - q_{p_i 0})^{-1} + (1 - l_{c_i 0})^{-1} + (1 - l_{cp_i 0})^{-1} + (1 - v_{d_i 0})^{-1} \to min; \tag{4.5}$$

$$F_{2_i} = (1 - Pl_{i0})^{-1} + (1 - Al_{i0})^{-1} + (1 - P_{n_i0})^{-1} \rightarrow min; \quad (4.6)$$

$$F_{3_i} = (1 - L_{E_i0})^{-1} + (1 - d_{s_i0})^{-1} + (1 - W_{i0})^{-1} \rightarrow min; \quad (4.7)$$

$$F_{4_i} = (1 - C_{v_i0})^{-1} + (1 - C_{p_i0})^{-1} + (1 - P_{p_i0})^{-1} + (1 - D_{i0})^{-1} + (1 - E_{i0})^{-1} + (1 - Y_{i0})^{-1} + (1 - N_{i0})^{-1} \rightarrow min; \quad (4.8)$$

In equations (4.5) – (4.8) the index $i=1\ldots N$ characterizes the corresponding partial and generalized criteria for different variants of construction of a route, the index normalized criteria are marked by an index zero.

The maximum and minimum values of the minimizing and maximizing criteria are grouped in the form of an abstract (worst) M_A route. The rationing of partial criteria is carried out relative to the maximum (minimum) values by equations (4.9), (4.10), which makes it possible to obtain generalized criteria for the optimal route.

$$\varphi_{0l}^{min} = \frac{\varphi_l^{min}}{max\varphi_l^{min} + \Delta}; \quad (4.9)$$

$$\varphi_{0l}^{max} = \frac{min\varphi_l^{max} - \Delta}{\varphi_l^{max}}. \quad (4.10)$$

where $max\varphi_l^{min}$, $min\varphi_l^{max}$ – maximum and minimum values of minimizing and maximizing criteria in the interval of their consideration; Δ – stock ratio, $\Delta = 0,03$; $i=1\ldots N$ – the number of discrete values describing the change of partial optimality criteria.

The rationing of the generalized criteria is carried out under the conditions of their minimization according to the equation (4.11):

$$F_{0i} = F_i / F_{max}, \quad (4.11)$$

where F_{max} – the worst assessment of a particular option for building a system.

The parameter F_{max} is determined separately for each generalized criterion (4.5) – (4.8) and is formed from the worst (for normalized - maximum) values that describe the change of partial criteria when considering all the formed options for its construction.

The integrated optimality criterion is determined by normalized generalized criteria, which is a multicriteria optimization model (4.12).

$$F_{Mi} = (1 - F_{10})^{-1} + (1 - F_{20})^{-1} + (1 - F_{30})^{-1} + (1 - F_{40})^{-1} \rightarrow min. \quad (4.12)$$

The choice of the optimal route is to analyze the values that describe the change of the integrated criterion of optimality. In this case, the optimal route will be assumed (*i=1…N*), for which the requirement of minimization of criterion (4.12) will be fulfilled.

$$N_{opt} = i,\ if\ \ F_{M\,i} \rightarrow min. \tag{4.13}$$

To assess the reliability of the choice of the optimal route of transportation in a multimodal transport network and make a final decision on the integrated assessment (4.12) it should be normalized according to the equation:

$$F_{M0} = 1 - \frac{F_{Mi}}{maxF_M}\ ; \tag{4.14}$$

$$\begin{aligned} maxF_M = (1-[maxF_{10}-\Delta])^{-1} + (1-[maxF_{20}-\Delta])^{-1} + (1-[maxF_{30}-\Delta])^{-1} \\ + (1-[maxF_{40}-\Delta])^{-1}. \end{aligned} \tag{4.15}$$

Integrated evaluation is normalized to the worst-case scenario of evaluating the effectiveness of the system as a whole. Such rationing allows to make the decision on the optimality of a route and to establish a comparative estimation of alternative routes. After normalization of the integrated estimate, we will have its change within in the range from zero to one with the highest score - close to one, and the lowest - close to zero. The integrated assessment of system efficiency takes into account its tasks, functions, and factors, as well as indicators of optimality criteria. It is according to the integrated assessment that the final decision on the optimality of the delivery route is made. The decision is made subjectively on the basis of an integrated assessment in the form of a linguistic category "good-bad". Therefore, it is natural to use a fundamental scale of assessment, which allows you to move from a numerical measure of efficiency to a linguistic category. This transition scale may have the form given in Table 4.1.

Table 4.1. Fundamental rating scale

Integrated assessment of the effectiveness, F_{M0}	Linguistic category of efficiency
1,0 – 0,7	High
0,7 – 0,5	Good
0,5 – 0,4	Satisfactory
0,4 – 0,2	Low
0,2 and less	Unsatisfactory

As a result of calculations, the correspondence of alternative delivery routes to certain categories is determined, Table 4.2.

Table 4.2. Distribution of routes by linguistic categories

Integrated assessment of the effectiveness, F_{M0}	Linguistic category of efficiency	Route
1,0 – 0,7	High	M_i
0,7 – 0,5	Good	…
0,5 – 0,4	Satisfactory	…
0,4 – 0,2	Low	…
0,2 and less	Unsatisfactory	M_j

Provided that the evaluation of the effectiveness of the i-th route M_i is belongs to the linguistic category "High", it is considered that the optimal route is established reliably.

Thus, the mathematical model of multicriteria distribution of transport flows in a multimodal transport network includes the following steps:

1. Definition and distribution of limited resources by consumer groups Table 1.1, equation (1.1) - (1.7).
2. Formation of indicators and criteria of route optimality, Figure 2.1, Table 2.1, equation (2.1) - (2.3).
3. Calculation of the values characterizing change of the established indicators and criteria of optimality of a route:

   3.1 Construction of the graph of the transport network, Figure 3.1;

   3.2 Calculation of partial criteria for the optimality of alternative routes Table. 3.1.
4. Development of a decision on the optimal route:

   4.1 Formation of an integrated criterion of route optimality according to the mathematical model (4.12);

   4.2 Selection of the optimal route from the set of alternatives under condition (4.13);

   4.3 Estimation of the reliability of the defined optimal route on the integrated estimation of optimality of a route in the form (4.14), (4.15) and definition of a linguistic category of efficiency on a fundamental scale, Table 4.1, Table 4.2.

Conclusion

To solve the applied problem of choosing the optimal route of cargo delivery, a mathematical model of the distribution of traffic flows in a multimodal transport network was proposed. The model based on multicriteria optimization obtained using nested convolutions according to a nonlinear scheme of compromises. This ensures that a significant number of criteria are taken into account in the solution, the diversity of consumer needs and suppliers' stocks, and limited resources. The calculation of partial criteria of route optimality is carried out on the graph of the transport network. Elaboration of the decision on the optimality of a route of transportation is carried out on the integrated criterion of optimality on the performance of requirement of minimization of criterion. The reliability of the determined optimal route of transportation is carried out according to the integrated criterion in the form of the linguistic category "good-bad".

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