ONTOLOGICAL PRINCIPLES OF THE PROBLEM OF CHOICE

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Abstract: *The complexity of the decision-making process for experts in various topics is in the problem of the initial processing of unstructured information. This problem of Big data requires the latest solutions that provide a structural reflection and integrative use of information descriptions under consideration by experts. It takes the process of information and analytical evaluation and decision-making beyond one subject area and raises questions about the ontological consideration of the problem itself. The ontological principles of such consideration are based on multiple hyperproperties of information for the realization of the categories of integration, systematicity and continuity and providing tracking and modification of their structures and their behaviour during activation. For this, there is the ontology of the problem of rational choice based on means of a ranking of alternatives on a set of indicators. It provides the creation of a system designed to solve the rank-ing problem based on the ontological model of the subject area. From the point of view of software engineering, a software system is considered as a set of descriptions presented as mathematical models, formalisms and modelling techniques. Hyperproperties of ontological representation of large amounts of information ensure the implementation of meta-tasks of its analysis, its structuring, synthesis, and most importantly support the process of rational choice of the most optimal solutions. Such an approach to ensuring the information and analytical activities of experts in the decision-making process creates the conditions for optimal processing of a large number of diverse information arrays. The component architecture of decision-making support services is based on ontological management.*

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Introduction

The decision-making process is based on the analysis of information and requires structured information. Moreover, this structure must meet the conditions of systematology, i.e. to include processing rules. This presentation of information is the most effective when based on the ontological approach. Ontology [Guarino, 1997; Sismondo, 2015; Buckingham Shum, 2000; Gladun, 1994; Noy, 2001; Palagin, 2008] is an attempt for a comprehensive and detailed formalization of some knowledge area using a conceptual scheme. Typically, such a scheme consists of a hierarchical data structure containing all relevant classes of objects, their relations and rules of this area. The conceptual scheme means a set of concepts (concepts) and information about the concepts (properties, relations of constraints, axioms and statements about the concepts needed to describe the processes of solving problems in the selected subject area).

Ontology is the vision of an expert in the subject area SA, expressed formally, which allows the use of this understanding of the structure and meaning of information elements in its automatic processing. It's the formalization of the representation of connections between concepts in the ontology that makes it possible to use them in a wide range of decision-making support systems (hereinafter – DMSS).

The links between concepts have particular importance in ontology. Due to them, the ontology is not just a structure of concepts, but also reflects the complex relations between them and comprehensively represents the subject area. There are three main types of connections between concepts:

taxonomic relations – express "is a species of" relations or the "general/specific" relations, i.e. relations that put the two entities in "general – partial" relations;

compositional connections – express "is a part of" relations;

topological connections – reflect how different components of the system are connected to each other through certain connections, or show the "paths" of physical interactions between components, as well as provide information about the spatial location of these components.

N. Noy and D. McGuinness in their article pay a lot of attention to the method of creating ontologies [Noy, 2001]. The authors form the basic rules of ontology development:

1. There is no right way to model the subject area – there are always viable alternatives.

2. The development of an ontology is definitely an iterative process. The iterative process means a repeated passage through the ontology in order to clarify it: i.e. a "draft" version should be made at the initial stage; then you need to check and refine the compiled ontology, and by adding details, it's possible to partially or even completely change the initial ontology.

3. The elements of the ontology should be close to the objects (physical or logical) and relations in a particular subject area SA.

Therefore, it's possible to consider the problems with the construction and use of ontologies [Guarino, 1997; Palagin, 2008]:

1. The conceptual unit structure – the choice problem and level of detail of units (concepts), the status of service units, the boundary between concepts and lexical variants, the recognition of units in texts.

2. Formal model – formalized, using a certain language of knowledge, a description of the conceptual system, which specifies:

- used classification of concepts;
- a set of acceptable paradigmatic relations between concepts;
- axioms and inference rules.

3. Computational functionality, determined by the following:

- unification of terminology;
- representation and logical processing of taxonomic relations;
- representation and logical processing of volume compatibility/incompatibility relations;
- representation and logical processing of subject and associative relations;
- presentation and logical processing of quantitative data support;
- regulation of object description procedures;
- axiomatization of processes descriptions, causal links and procedures.

Taking into account that the ontology is an information and computing resource, which is based on expert knowledge and qualitatively represents the subject area and can be used by any intelligent information technology, in particular, as a source of information. Then objects (alternatives) and their properties can be separated from it for the direct solution of the multicriteria ranking problem. Thus, the use of ontology as a basis for presenting alternatives is a very important issue in the creation of decision making support systems.

1. Ontological Representation of the Choice Problem Based on Ranking Alternatives

Each ontology contains information descriptions, based on the object-oriented formalization procedure, as well as descriptions of interpretive functions, which are a functional manifestation of the objects (concepts) properties of the ontology, and which control the process of information resource delivery on all stages of decision-making. Therefore, it’s quite useful to present an information model in the DMSS environment as a certain ontology [Palagin, 2008; Stryzhak, 2019; Gorborukov, 2013].

As you know, the ontological methodology is based on an object-oriented approach, in which the subject area is represented as a set of objects, and interaction between them can be represented by semantic linking of expressions, statements and judgments [Noy, 2001; Stryzhak, 2019; Booch, 2007; Weisfeld, 2013].

An object is a certain essence (real or abstract) with its inherent state, behaviour and individuality.

The state of an object is a list of all its possible properties i.e. the structure and values of each property.

Object’s behaviour (or its functionality) characterizes how an object interacts with other objects or is influenced by other objects, showing its individuality. The behavior of the object is realized as functions called methods. The structure of the object is available only through its methods, which together form the interface of the object.

The individuality of an object is characterized by such properties that distinguish it from all other objects.

To form a proper DMSS operating environment, two types of hierarchical dependencies between objects are of particular interest:

- connections – denote equal relations between objects; the object cooperates with other objects through the connections with them;
- aggregation – describes the relations of whole and part, leading to the corresponding taxonomy (hierarchy of objects).

Computer ontology of a certain subject area (SA) will be considered as a certain non-empty set of objects that meet the following requirements:

1. Objects are organized in a hierarchical structure of a finite set of concepts that describe a given subject area;

2. The structure can be represented by a set of bigraphs, where the vertices are concepts, and the arcs are semantic relations between them;

3. Concepts and attitudes are interpreted according to the general functions of interpretation, taken from electronic sources of knowledge of a given SA;

4. Definition of concepts and relations is based on the axioms and limitations of their scope;

5. Functions of interpretation and axioms are described as a formal theory.

1.1. Taxonomy of the Choice Problem

In general, the ontology of the subject area is formally represented by an ordered trio [Guarino, 1997; Palagin, 2008; Stryzhak, 2019; Gorborukov, 2013; Gruber, 1993]:

$$O = \langle X, R, F \rangle \tag{1}$$

where X is a set of concepts (concepts, terms) of the subject area (SA), which is the basis for the subject component of the operating environment;

R is the set of relations and properties between them (we will assume that properties are the interpretation of relations, ie there is a transformation that matches each relation to certain property);

F – interpretation functions (definitions) X and/or R, which are a functional part of the operating environment.

Consideration of boundary cases of sets of expression (1): $R=\varnothing; R\neq\varnothing; F=\varnothing; F\neq\varnothing$ in all four combinations of R and F meanings and gives different variants of ontological constructions, starting from a simple dictionary and taxonomy to the formal structure of the conceptual knowledge base. According to the procedure of constructing an ontology-based on its certain functional completeness and degree of formality, there are following types of ontologies: primary, final and set of intermediate ontologies.

Thematic ontology ($R\neq\varnothing$; $F\neq\varnothing$) – is such an ontology where sets of concepts and conceptual relations are as complete as possible, and axioms, definitions and restrictions on the subject of this SA are added to the functions of interpretation. The description of all components is represented by some formal language, which can be interpreted by some procedure (algorithm). The scheme of the formal model of thematic ontology is described as follows:

$$O = \langle X, R, F, \tilde{A}, D, R_S \rangle \tag{2}$$

where X is the set of concepts, $X = \{x_1, x_2, ..., x_i, ..., x_n\}, i = \overline{1,n}, n = \mathrm{Card}X$, the finite set of concepts (concepts) given in the DMSS operating environment;

$R=\{R_1,R_2,...,R_k,...,R_m\}, R_k \subseteq X\times X, k=\overline{1,m}, m=\text{Card}R$ is a finite set of semantically significant properties (relations) between the SA concepts. They determine the type of interaction between concepts.

F is a finite set of interpretation functions given on concepts and/or relations.

$\tilde{A}$ is a finite set of axioms used to write always true statements (definitions and restrictions) in terms of the SA subject;

D is a set of additional definitions of concepts (concepts) in terms of the SA topic;

R_S is a set of constraints that determines the scope of conceptual structures of a particular SA topic.

R^+ is a set of properties that characterize the SA concepts $(R^+ \subseteq R_S)$.

One of the system components of the ontological system is a taxonomy [Shatalkin, 2012], which reflects a certain hierarchy of concepts interaction. In this case, the hierarchy itself is set using binary relations that determine the nature of the interaction between the concepts of the ontology.

Taxonomy is a nonempty subset $\breve{T}$ of the set of concepts X of the ontology O over which the ordering relations $R_t, R_t \subset R \mid R_t \neq \varnothing$. If a multiple binary weak order relation is set for a set of concepts X – $\tilde{p}$, then formally the taxonomy can be determined as follows:

$$T=\{t\in X \mid \exists y\left(y\in X\right): x\tilde{p}y \vee y\tilde{p}x\}, \tag{3}$$

where $\tilde{p}(\tilde{p}\in R_t \subseteq R)$ is a binary relation that determines the partial order for the set T.

To form a model of the ranking problem based on ontology, it’s necessary to determine a certain $\tilde{H}_{MR}$ transformation.

$$\tilde{H}_{MR}:\left\langle X,R,F,\tilde{A},D,R_S\right\rangle \to M_R, \tag{4}$$

where M_R is the ranking problem model.

The ranking problem is described by a set of alternatives A, and each one has certain indicators (criteria). The solution of such a problem is the establishment of a linear order over the set A. This order, in particular, allows determining the alternative(s) with the best (in total) values of the criteria, which in general are of different importance. Formally, the mathematical model of the ranking problem consists of the following objects:

$$M_R = \left\langle A, K, \tilde{F}, G \right\rangle, \tag{5}$$

where A is the set of alternatives;

K is the set of criteria;

$\tilde{F} : A \times K \to Q$ is a function that determines the value of alternatives by a certain criterion, Q is a set of values of criteria;

G is a ranking rule that allows you to set a linear order for many alternatives.

Depending on the degree of formation of the ranking problem model, the following cases can be distinguished:

$M_R^{1A}(A \neq \varnothing, K = \varnothing, \tilde{F} = 0, G = 0)$ – only alternatives are defined;

$M_R^{1K}(A = \varnothing, K \neq \varnothing, \tilde{F} = 0, G = 0)$ – only criteria are defined;

$M_R^{2}(A \neq \varnothing, K \neq \varnothing, \tilde{F} = 0, G = 0)$ – sets of alternatives and criteria are defined;

$M_R^{3}(A \neq \varnothing, K = \varnothing, \tilde{F} \neq \varnothing, G = \varnothing)$ – values for criteria are set for alternatives. In this model, it’s assumed that the ranking problem is formed;

$M_R^{4}(A \neq \varnothing, K = \varnothing, \tilde{F} \neq \varnothing, G \neq \varnothing)$ – the ranking rule is set for the formed ranking model.

In fact, the mechanism of modelling the ranking problem and its solution based on ontology (4) is a multi-stage process, each one requires separate procedures. The general scheme of such a transformation can be represented by the following process:

$$O \to M_R^{1A} \rightleftarrows M_R^{2} \to M_R^{3} \to M_R^{4} \to O^{*} \tag{6}$$

where O^{*} is an extension of the ontology O as a result of ranking, which defines new relations of advantages and properties for the concepts of ontology (alternatives) $A \subseteq X$. It should be also noted that the new properties can be established as a result of post-analysis based on the solution of the inverse ranking problem.

Thus, the first stage of forming an ontological model of the ranking problem is to separate elements that can be considered as alternatives from the set of concepts X. Alternatives can be considered homogeneous objects with common properties. Such choice can be based on the choice function

$$F_{ch}(X) = Y, Y \subseteq X \tag{7}$$

1.2. Interpreting the Functions of the Ontology of the Choice Problem

The classical rational choice functions include those with the following characteristic properties [Gorborukov, 2013; Aizerman, 1990; Malyshevsky, 1998; Saaty, 2001]:

Condition of imitation (H):

$$X' \subseteq X \Rightarrow F_{ch}(X') \supseteq F_{ch}(X) \cap X' \tag{8}$$

Condition of constancy (K), strict imitation:

$$(X' \subseteq X) \wedge (X' \cap F_{ch}(X) \neq \varnothing) \Rightarrow F_{ch}(X') = F_{ch}(X) \cap X' \tag{9}$$

Condition of consent (C):

$$(X' \cup X'') = X \Rightarrow F_{ch}(X) \supseteq F_{ch}(X') \cap F_{ch}(X'') \tag{10}$$

Condition of independence or rejection (R):

$$F_{ch}(X) \subseteq X' \subseteq X \Rightarrow F_{ch}(X') = F_{ch}(X) \tag{11}$$

To construct a class of choice mechanisms that generate a characteristic region of rejection (B) in the function space, you need such type of structures as a hyperrelation $Y\gamma Z$, which connects pairs of sets [Malyshevsky, 1998; Barendregt, 1985].

Then the choice rule on such a structure is determined by the expression:

$$F_{ch}(X) = Y, Y \subseteq X \Leftrightarrow Y\gamma Z, Z \subseteq X \tag{12}$$

Note that if the hyperrelation has the property of hypertransitivity, then the choice function also belongs to the characteristic domain of imitation (I).

Hypertransitivity condition:

$$Y\gamma Z, x \in X, X\gamma z \Rightarrow ((Y \cup X) / \{x\})\gamma z \tag{13}$$

Let us define the mechanisms of alternative variants choice for ontology (2) with the taxonomic structure $\breve{T}$, over which there is the binary relation of weak ordering $\tilde{p}$. The partial order $\tilde{p}$ can be extended to the transitive circuit relation p^{+} .

$$x_i^j \tilde{p} x_l^k \xrightarrow{\alpha} x_i^j p^{+} x_l^k \tag{14}$$

Then you can determine the choice function based on the hyperrelation γ_1^+ and γ_1^- and between the $Y \subseteq \breve{T}$ and $Z \subseteq \breve{T}$ sets such as:

$$\begin{gathered}\gamma_1^- Z, Y = \left\{ y \in \breve{T} \mid \exists z \left(z \in Z \right) : z p^+ y \right\} \\ Y \gamma_1^+ Z, Y = \left\{ \nexists x \left(x \in X, x \neq y \right) : y \tilde{p} x \right\}, Y^* \gamma_1^- Z \\ F_{ch}(\breve{T}) = Y, Y = \left\{ y \in Y_1 / Y_2 \right\}, Y_1 \gamma_1^+ Z_1, Y_2 \gamma_1^- Z_2, Z_1 \subseteq \breve{T}, Z_2 \subseteq \breve{T}\end{gathered} \tag{15}$$

The described choice function determines the finite elements of the taxonomy, which are descendants of classes (categories), included in the set Z_1 and aren’t descendants of classes defined by the set Z_2 respectively. Elements that belong to common classes (superclasses) can be considered as alternatives because this determines their homogeneity. Thus described function for the directed graph representing the taxonomic structure defines hanging points with common ancestors Z_1. If the sets Z_1 and Z_2 are not defined, they should be specified as follows: $Z_1 = \left\{ \nexists x \left(x \neq z \right) : x \tilde{p} z \right\}, Z_2 = \varnothing$.

Let R^+ be the set of all properties (attributes) that characterize ontology objects. Hyperrelation γ_2 can be used to specify a set of properties for a specific object $P_x = \left\{ P \gamma_2 x \right\}$. Similarly, as for the choice function in the previous case, you can set the corresponding hyperrelation γ_3^- and γ_3^+ keeping the choice rule (15): $Y \gamma_3^+ Z, Y = \left\{ y \in \breve{T} \mid \forall y : (Z \subseteq P_y) \right\}$.

The hyperrelation γ_3^+ can be modified to $\tilde{\gamma}_3^+$ with the threshold value q.

$$\begin{gathered}Y \tilde{\gamma}_1^+ Z, Y = \left\{ y \in \breve{T} \mid \left(\mid Z \cap P_y \mid > q \right) \right\} \\ Y \tilde{\gamma}_1^- Z, Y = \left\{ y \in \breve{T} \mid \exists z (z \in Z) : z \in P_y \right\}\end{gathered} \tag{16}$$

The choice mechanism generated by this hyperrelation suggests that the objects under consideration don’t necessarily have all the desired parameters. It’s easy to see that the choice mechanisms (15) and (16) are equal, since belonging to a certain class can be considered a property of an object. You can also build a corresponding taxonomy for elements with certain properties, where there will be objects at the lower level of the hierarchy and a hierarchical structure of attributes at the upper ones.

The described choice mechanisms can be adjusted using certain parameters. If such parameters are not specified, then all elements that have the type “object” for (16) and elements of the lower level of the hierarchy for (15) will be chosen as alternatives. Note that the lack of parameters is not a problem for successfully structural taxonomies.

Note that the main feature of alternatives is their homogeneity. Consider the hyper dominant mechanism for choice [Aizerman, 1990; Malyshevsky, 1998] optimal sets based on the hyperrelation γ_4 given by the scalar function $\Phi(Z)$ (hyperscale): $Y\gamma_4 Z \Leftrightarrow \{\Phi(Y) \geq \Phi(Z)\}$.

Then the choice function will look like this:

$$F_{ch}(\breve{T}) = Y, \Phi(Y) = \max_{Z \in \breve{T}} \Phi(Z) \tag{17}$$

$$\Phi(Z) = GH_1\left(|Z|, \left|\bigcap_{z\in Z} P_z\right| - \left|\bigcup_{z\in Z} P_z\right| + \breve{q}, \left|\bigcap_{z\in Z} P_z\right| - \hat{q}\right), Z \subseteq \breve{T} \tag{18}$$

where function $GH_1(x,y,z) = \begin{cases} x, (y \geq 0) \& (z \geq 0) \\ 0, \text{otherwise} \end{cases}$,

$\hat{q}, \breve{q}$ are integer threshold values, P_z is set of element's properties z $(z \in Z)$. Note that the function GH_1 can be determined in another way, which would take into account not only the common properties but also their number. This choice function focuses on finding the maximum number of homogeneous objects. If there is more than one group of such objects in the ontology, then due to the consistent application of this mechanism, they can also be identified.

The function $\Phi(Z)$ can also be determined by direct pairwise comparisons.

$$\Phi(Z) = \left(\sum_{x\in Z} \sum_{y \in Z, x \neq y} GH_2(x,y,Z) \bullet \Pr(x,y,Z)\right), Z \subseteq \breve{T} \tag{19}$$

where $\Pr(x,y,Z)$ is a function based on predicates, that determines whether a set of objects has minimum requirements.

$GH_2(x,y,Z)$ is a function that determines the degree of similarity of objects x and $y (x \in Z, y \in Z)$ and which can take into account the number of elements in the set, by default it can be set into $GH_2(x,y,Z) = 1$.

$$\Pr(x,y,Z) = \begin{cases} 1, \Pr_1(x,y) \wedge \Pr_2(x,Z) \wedge \Pr_2(y,Z) \\ -\infty, \neg\Pr_1(x,y) \vee \neg\Pr_2(x,Z) \vee \neg\Pr_2(y,Z) \end{cases}$$

$$\Pr_1(x,y) = \begin{cases} \text{true}, \left(\left|P_x \cap P_y\right| \geq \breve{q}\right) \wedge \left(\left|P_x \cup P_y\right| - \left|P_x \cap P_y\right| \leq \hat{q}\right) \\ \text{false, otherwise} \end{cases}$$

$$\Pr_2(r,Z) = \begin{cases} \text{true}, |P_r| - \left|\bigcup_{z\in Z} P_z\right| \geq \tilde{q} \\ \text{false, otherwise} \end{cases}$$

where P_x , P_y , P_r are sets of properties of elements x , y , and r $(x \in Z, y \in Z, r \in Z)$, parameters $\breve{q}$, $\hat{q}$, $\tilde{q}$ are integer threshold values.

The described functions (15) – (16) have all the characteristic properties described in (8) – (11), provided that the properties of the objects are unchanged. The choice function described in (17) – (20) has only the property of rejection (R). But keep in mind that this choice mechanism is focused on finding the optimal set unlike the usual situation, where objects can be considered independent. That is, discarding certain elements from a set of objects significantly changes its qualitative characteristics, so the sets themselves must be integral. If this requirement is imposed, then the described function can be considered rational.

The next step after defining a set of alternatives is to establish a set of attributes that can be considered as criteria K. This can be done as follows:

$$K = \bigcup_{z \in A} P_z, F_{ch}\left(\breve{T}\right) = A, A \subseteq \breve{T} \tag{20}$$

Thus, a set K is formed, which can be used to determine the set of alternatives. That is, it can be used to determine the sets Z_1 and Z_2, which are used to choose alternatives. Thus, these stages of formation of alternatives and criteria are interrelated $M_R^{1A} \rightleftarrows M_R^2$, because the properties of objects allow you to set many alternatives, which in turn are characterized by common features.

1.3. Ontological Representation of Alternatives

The next step in forming a model of the ranking problem is to establish criteria for alternatives. For this, you should separate the values of attributes from the relation ("object", "attribute", "value") from the ontological model. Subsequent paragraphs, however, are indented.

$$(x, y, \tilde{F}(x, y)) \in R_\nu \tag{21}$$

where x is an alternative from the set A; y is a criterion from the set K; $\tilde{F}$ is a function, which defines the criterion value for the given alternative; R_ν are the trinary relations given for the set X.

After defining a set of criteria and alternatives, and their criteria values, we will assume that the model of the ranking problem is actually set. Further refinement and resolution depend on the direct involvement of the statement.

Based on our formal definition of ontology (1) and the category of choice, which is represented by expressions 7–19, we present the ontology of the *OTPCh* choice problem [Gorborukov, 2013; Velichko, 2017] as the following expression:

$$OTPCh = \left\langle \breve{T}, (\tilde{p} \vee R^+), F_{ch}(\breve{T}) \right\rangle \tag{22}$$

where $\breve{T}$ is a taxonomy, $\tilde{p}$ is a partial order relation, R^+ are the properties of concept objects, F_{ch} is a choice function.

So, the object components that determine the ontology of the problem of choice, allow us to interpret them as different information resources, the concepts of which are associated with certain binary relations of partial order and may have unary properties that characterize them in a certain qualitative form.

Now we directly specify the mathematical model of the ranking problem (5), which can be formally described as follows. Each alternative $x \in A\{A_1, A_2, ..., A_n\}$ (A is a set of alternatives) is defined by the values of some set of indicators $f_1(x), f_2(x), ..., f_m(x)$. Each $f_j(x), j \in J = \{1...m\}$ function specifies the value of the j-th criterion [Gorborukov, 2013; Aizerman, 1990; Malyshevsky, 1998; Saaty, 2001; Stoyer, 1992], which belongs either to a predetermined set or is calculated according to certain mathematical rules [Malyshevsky, 1998; Saaty, 2001; Kini, 1981]. In the first case, the options are possible: the set of values is given by a point or linguistic scale [Gray, 2015] or as a numerical interval $\left[f_j^{\min}, f_j^{\max}\right]$, which is formed from all possible values of the function (from minimum to maximum). An example of the second case is the synthesis of local priorities in the method of analysis of hierarchies [Gorborukov, 2013; Malyshevsky, 1998; Saaty, 2001]. Therefore, we can assume that the value of the j-th criterion is always countable, denote it as Q_j:

$$Q_j = \left\{f_j^{(1)}, f_j^{(2)}, f_j^{(3)}, ..., f_j^{(n_j-1)}, f_j^{(n_j)}\right\}$$

where $f_j^{(1)} = f_j^{\min}, f_j^{(n_j)} = f_j^{\max}, f_j^{(1)} < f_j^{(2)} < ... > f_j^{(n_j)}, n_j = |Q_j|$ is the number of elements of the set Q_j.

So, the problem of ranking alternatives by a set of indicators is to establish a certain order

$$A_{i_1} \succ A_{i_2} \succ ... \succ A_{i_n}, i_1, i_2, ..., i_n \in \{1...n\} \tag{23}$$

based on the calculation of the values of some general indicator $G(x)$ for each element of the A set:

$$\begin{gathered} G(x) = G(f(x), W) = G((f_1(x), ..., f_m(x), (\omega_1, ..., \omega_m)), x \in A = \{A_1, ..., A_n\} \\ W = (\omega_1, ..., \omega_m), \sum_{j=1}^{m} \omega_j = 1, \omega_j > 0 \end{gathered} \tag{24}$$

where $G(x)$ values are calculated according to a certain rule (algorithm), determined by the mathematical method used in each case, and

$$G(A_{i_1}) \geq G(A_{i_2}) \geq \ldots \geq G(A_{i_n}) \quad (25)$$

and W is the normalized weight vector of criteria [Aizerman, 1990; Malyshevsky, 1998; Saaty, 2001; Stoyer, 1992].

The alternative A_{i_1}, is the best alternative, which is the first one in the order (23), alternative A_{i_n} is the worst respectively. is the worst respectively A_{i_k} in the order (23) is on the k -th place, and k will be called its rating.

It should be noted that no matter how the problem is solved (23) – (25), the reliability of the obtained result significantly depends on the method of converting the $Q_j, j \in J$ values into a unified scale for all criteria. Thus, the formula is often used [Opricovic, 2004; Nogin, 2005]

$$f'_j(x) = q_{\min} + (q_{\max} - q_{\min})\left(\frac{f_j(x) - \inf Q_j}{\sup Q_j - \inf Q_j}\right), x \in X = \{A_1, \ldots, A_n\} \quad (26)$$

however, it may not accurately reflect the “essentiality” of the advantage of one alternative over another after transforming the initial $f_j(x)$ values to the $[q_{\min}; q_{\max}]$ scale. As a result, when using the obtained $f'_j(x)$ values in complex factor (24), the solution of problem (23) – (25) may be incorrect. Therefore, there is a need to develop other approaches to the normalization of criteria, which take into account the actual ratio of the strength of alternatives for each criterion properly.

2. Ontological Decision-making Support System Based on Solving the Rational Choice Problem

The solution of rational choice problems based on means of a ranking of alternatives on a set of indicators is the system for the solution of a ranking problem based on the ontological model of the subject area.

From the point of view of software engineering, a software system is considered as a set of descriptions presented as mathematical models, formalisms and modelling techniques.

The structure of mathematical models of IAS DMS (Information Analytical System of Decision Making Support) includes the following models:

1. information model;

2. component function model.

2.1. Information Model of the Ontological System for Solving Problems of Ranking Alternatives

The information model is used to represent and describe data flows, data structures and software modules in the ontological system.

The general information model of the ontological system for solving the ranking problems S_A looks like (27). It’s represented by some finite set of integrated S_{A_i} software modules.

$$S_A = \sum_{i=1}^{n} S_{A_i} \quad (27)$$

The set of modules can be considered as a separate independent system created by TODOS technology [Stryzhak, 2019; Velichko, 2017; Mintser, 2018; Stryzhak, 2018]. This set looks like (28).

$$S_A = \{S_{MO}, S_{MT}, S_{OA}, S_{AL}, S_{CO}\} \quad (28)$$

S_{MO} is a module responsible for information representation of the ontological graph and basic operations on it [Gladun, 1994; Palagin, 2008; Velichko, 2017]. The set of functions of this module looks like (29).

$$F_{MO} = \{F_1^{S_{MO}}, F_2^{S_{MO}}, F_3^{S_{MO}}, F_4^{S_{MO}}, F_5^{S_{MO}}, F_6^{S_{MO}}, F_7^{S_{MO}}, F_8^{S_{MO}}, F_9^{S_{MO}}\} \quad (29)$$

Its functions include:

$F_1^{S_{MO}}$ – upload an ontological model from supported data formats (XML and JSON);

$F_2^{S_{MO}}$ – finding the graph node by its unique identifier;

$F_3^{S_{MO}}$ – finding the node (and its unique identifier) by name;

$F_4^{S_{MO}}$ – node attachment to the graph;

$F_5^{S_{MO}}$ – node removal from the graph;

$F_6^{S_{MO}}$ – finding the child elements of a particular graph node;

$F_7^{S_{MO}}$ – finding the parent elements of a particular graph node;

$F_8^{S_{MO}}$ – reading and editing the properties of the ontological graph point;

$F_9^{S_{MO}}$ – downloading of the ontological model into one of the supported formats (JSON);

S_{MT} is a module responsible for the model of information representation of the problem of ranking alternatives. This module consists of S_{MTA} and S_{MTC} (30).

$$S_{MT} = \{S_{MTA}, S_{MTC}\} \quad (30)$$

S_{MTA} is a module responsible for the model of information representation of alternatives in the ranking problem. The set of functions of this module looks like (31).

$$F_{MTA} = \left\{F_1^{S_{MTA}}, F_2^{S_{MTA}}, F_3^{S_{MTA}}, F_4^{S_{MTA}}, F_5^{S_{MTA}}, F_6^{S_{MTA}}\right\} \quad (31)$$

Its functions include:

$F_1^{S_{MTA}}$ –attachment of an alternative to this container;

$F_2^{S_{MTA}}$ – removal of alternative (s);

$F_3^{S_{MTA}}$ – finding an alternative by its unique identifier;

$F_4^{S_{MTA}}$ – finding an alternative (and its unique identifier) by name;

$F_5^{S_{MTA}}$ – uploading and downloading objects from/into supported data formats;

$F_6^{S_{MTA}}$ – reading, setting and editing the properties of objects (alternatives) that can be used as criteria or data for their calculations.

S_{MTC} is a module responsible for the model of information representation of criteria in the ranking problem. The set of functions of this module looks like (32).

$$F_{MTC} = \left\{F_1^{S_{MTC}}, F_2^{S_{MTC}}, F_3^{S_{MTC}}, F_4^{S_{MTC}}, F_5^{S_{MTC}}\right\} \quad (32)$$

Its functions include:

$F_1^{S_{MTC}}$ – creation of criterion objects using the input list;

$F_2^{S_{MTC}}$ – attachment of a criterion to this container;

$F_3^{S_{MTC}}$ – criterion (a) removal;

$F_4^{S_{MTC}}$ – criteria editing (setting scales, optimization direction, etc.);

$F_5^{S_{MTC}}$ – finding the criterion by its unique identifier.

S_{OA} is a module responsible for the analysis of the ontological graph based on its information model, represented by the S_{MO} module. This module consists of S_{TA} and S_{PI}).

$$S_{OA} = \{S_{TA}, S_{PI}\} \quad (33)$$

S_{TA} is a module responsible for taxonomic ontograph analysis and the choice of a subset of points that can be used as alternatives. The set of functions of this module looks like (34).

$$F_{TA} = \left\{F_1^{S_{TA}}, F_2^{S_{TA}}, F_3^{S_{TA}}, F_4^{S_{TA}}\right\} \tag{34}$$

Its functions include:

$F_1^{S_{TA}}$ – identification of alternatives based on taxonomy. Homogeneous objects at the lower level of the hierarchy are considered alternatives by default;

$F_2^{S_{TA}}$ – marking of valid points based on the initial set of "white" nodes, i.e. only their child nodes should be included in the consideration;

$F_3^{S_{TA}}$ – marking of valid points based on the initial set of "black" nodes, i.e. all their child nodes should be excluded from consideration;

$F_4^{S_{TA}}$ – filtering of the points based on the node(s) of a higher layer of the hierarchy.

S_{PI} is a module responsible for finding the properties of objects that can be used for criterion score. The set of functions of this module looks like (35).

$$F_{PI} = \left\{F_1^{S_{PI}}, F_2^{S_{PI}}\right\} \tag{35}$$

Its main functions include:

$F_1^{S_{PI}}$ – separation of "criterion" properties and their characteristics from the ontograph points;

$F_2^{S_{PI}}$ – creation (or updating) criteria in a ranking problem model.

S_{AL} is a set of modules responsible for the actual solving of a ranking problem.

The set of functions of this S_{AL} looks like (36):

$$F_{AL} = \left\{F_1^{S_{AL}}, F_2^{S_{AL}}, F_3^{S_{AL}}, F_4^{S_{AL}}, F_5^{S_{AL}}\right\} \tag{36}$$

Its main functions include:

$F_1^{S_{AL}}$ – calculation of criteria weighting;

$F_2^{S_{AL}}$ – calculation of criterion values, if there is a way to interpret criteria;

$F_3^{S_{AL}}$ – normalization of criterion values of alternatives on a common scale;

$F_4^{S_{AL}}$ – calculation functions (depending on the specific method) to calculate the “rating” value of the alternatives, used to rank them;

$F_5^{S_{AL}}$ – ranking and setting a certain “rating” value for alternatives based on the aggregation of the result of calculations used to solve mathematical methods of ranking alternatives;

S_{CL} is a set of modules (controllers) responsible for preparing data for display in the user’s web interface and for processing his requests. If necessary, controllers use other auxiliary service modules to perform their tasks. The set of functions of these modules looks like (37):

$$F_{CL} = \left\{F_1^{S_{CL}}, F_2^{S_{CL}}, F_3^{S_{CL}}, F_4^{S_{CL}}, F_5^{S_{CL}}, F_6^{S_{CL}}, F_7^{S_{CL}}, F_8^{S_{CL}}\right\} \tag{37}$$

The main functions of these modules include:

$F_1^{S_{CL}}$ – definition of a set of alternatives.

$F_2^{S_{CL}}$ – fixing a set of criteria and specifying their method of interpretation.

$F_3^{S_{CL}}$ – specification of the range where the criteria can receive values (maximum and minimum score).

$F_4^{S_{CL}}$ – determining the direction of criteria optimization.

$F_5^{S_{CL}}$ – setting the criteria importance using provided methods.

$F_6^{S_{CL}}$ – definition of the system of advantages.

$F_7^{S_{CL}}$ – filter and search for objects.

$F_8^{S_{CL}}$ – data preparation and templating for presentation in the web interface.

2.2. Component Function Model of the Ontological System for Solving Ranking Alternatives Problems

The component function model is used to represent the interactions, relations and dependencies of software modules, as well as to describe in detail the components of the system. In general, this model for the software package for solving the ranking alternatives problems can be represented by the structure.

$$S_R = \{M_D, M_S, M_p\} \tag{38}$$

Elements included in this model:

M_D is a model that determines the behaviour of the system;

M_S is a model that defines the structure of the system;

M_p is a model that defines the structure of program entities.

The model of system behaviour looks like (39).

$$M_D = \{d_{use}, d_{act}, d_{seq}\} \tag{39}$$

It includes:

d_{use} – UML-diagram of options for using the system of solving ranking alternatives problems;

d_{act} – UML-diagram of activity of the system of solving ranking alternatives problems;

d_{seq} – UML-diagram of the interaction of the system of solving ranking alternatives problems;

The main user of the system is DMP (decision making person), who is provided with the means to form and solve the ranking alternatives problems.

The formation of the ranking problem includes:

- identifying a set of alternatives;
- definition of criteria;
- defining a set of criteria;
- establishing the importance of criteria;
- definition of interpretive functions for criteria;
- setting criterion values for alternatives;
- formation of a system of advantages.

The process of identifying a set of alternatives involves an analysis of the taxonomic structure of the ontological graph and finding homogeneous objects with common properties. DMP can control this process, in particular through filtering.

The definition of a set of criteria based on ontology is to establish a set of properties of objects that can be used as criteria. DMP can also choose and define a mechanism for interpreting properties as criteria.

Using the provided methods, DMP can establish the importance of criteria and choose the direction of their optimization. The user can also create a system of advantages. This process involves choosing ways to establish the importance of the criteria.

Solving the ranking alternatives problem involves choosing a way to solve the problem and view the result directly. The ranking result can be presented using diagrams and attribute filtering mechanisms.

The SA expert performs the function of forming thematic ontologies [Palagin, 2008; Stryzhak, 2019; Opricovic, 2004; Mintser, 2018], which can later be published by the administrator in the public library. Forming can be done manually (with an ontology editor) or automatically based on the text structuring process, which is also done by a SA expert.

The expert assists DMP in forming the ranking problem and has all the appropriate DMP tools, but DMP determines the participation and role of the expert in solving the ranking problem. The scheme of using the tools of the ontological environment by experts is presented in Figure 1.

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The definition of a set of criteria based on ontology is to establish a set of properties of objects that can be used as criteria. DMP can also choose and define a mechanism for interpreting properties as criteria.

Using the provided methods, DMP can establish the importance of criteria and choose the direction of their optimization. The user can also create a system of ad-vantages. This process involves choosing ways to establish the importance of the criteria.

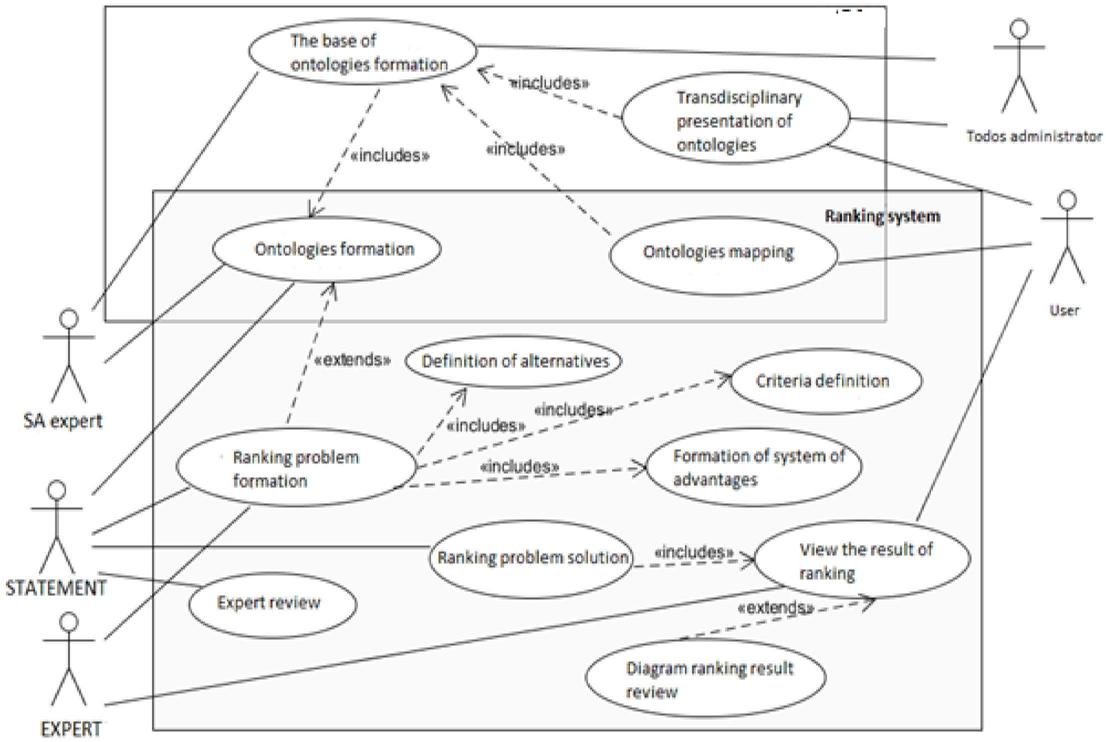

Figure 1. Ways to use the system

Solving the ranking alternatives problem involves choosing a way to solve the problem and view the result directly. The ranking result can be presented using diagrams and attribute filtering mechanisms.

The SA expert performs the function of forming thematic ontologies, which can later be published by the administrator in the public library. Forming can be done manually (with an ontology editor) or automatically based on the text structuring process, which is also done by a SA expert.

The expert assists DMP in forming the ranking problem and has all the appropriate DMP tools, but DMP determines the participation and role of the expert in solving the ranking problem.

The common user can view the ranking result if he has the appropriate access.

The UML-diagram of the activity of the software system for solving the ranking alternatives problems is shown in Figure 2. The diagram describes the algorithm for working with the system of solving ranking alternatives problems.

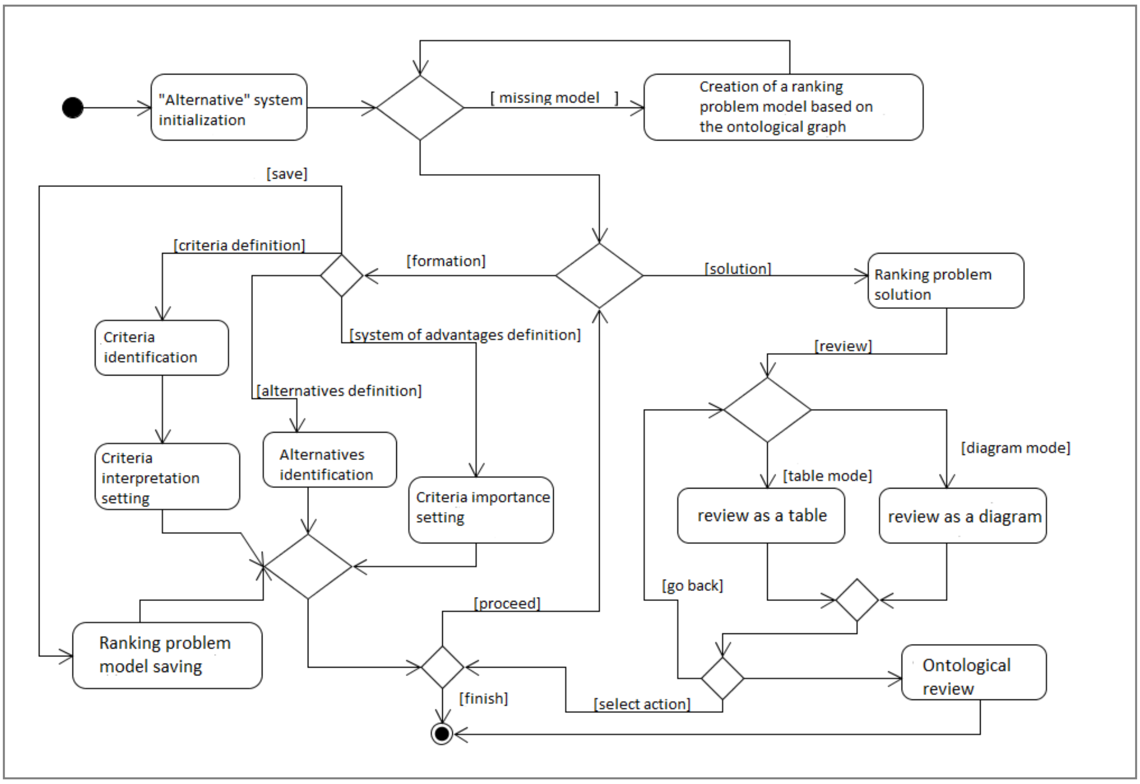

Figure 2. System activity diagram

After switching on the system, it is initialized and the problem of ranking alternatives is formed based on the corresponding ontological graph.

If the user does not save the corresponding model of the ranking problem for this graph, then the set of alternatives is based on the taxonomic analysis. The list of possible criteria is based on the properties of the objects that have been identified as alternatives.

Conclusions

A significant number of applied problems belonging to different spheres of human activity are formalized by models and methods of decision theory and are solved with the help of decision support systems. The most common problems here are related to the choice of a solution from a predetermined set based on multiple-criteria decision analysis and ranking of alternatives. Solving these problems occur in several stages, the key of which is the stage of structuring problems. In order to prevent possible inaccurate results in solving a multicriteria problem, it is convenient to represent the subject area as ontology.

Based on an object-oriented approach, the ontological methodology allows representing the subject area to which specific applications belong, in the form of a hierarchical data structure - a set of objects with certain attributes and relationships between them. Therefore, the effective use of decision support systems is not possible without the development of formal mechanisms for obtaining specific data from the ontological model of the subject area. This allows automating the process of building an information environment of the system, which ultimately increases the success of its use as a practical tool for solving problems of choice and ranking of alternatives.

In the article, the multi-stage process of forming a model of the problem of ranking alternatives by a set of indicators based on the ontological representation of the subject area is described. The method of constructing the information environment of the multiple criteria ranking problem, which carries out the transformation of the ontological model of the subject area based on the interpretive selection functions constructed using hyperrelation over elements of the taxonomic structure of the ontology and properties of its objects, is presented. An informational and functional-component model of the system for solving the problem of ranking alternatives is described.

Transdisciplinary analysis identifies a variety of criteria that characterize different sources.

Transdisciplinary analysis realizes the competitor's normalization of criteria, which ensures their use in processing various topics.

Transdisciplinary analysis provides an integrated use of a variety of criteria when ranking a large number of facts.

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