OPTIMIZATION PROBLEM: SYSTEMIC APPROACH

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Abstract. *A systemic approach to the problem of multi-criteria optimization allows us to combine the models of individual compromise schemes into a single holistic structure, adapting to the situation of multi-criteria decision making. The advantage of the concept of a non-linear compromise scheme is the possibility of making a multi-criteria decision formally, without direct human participation.*

Keywords: *system, optimization, multicriteria problem, utility function, scalar convolution, non-linear scheme of compromises.*

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Introduction

The essence of many practical problems in different subject areas is the choice of conditions that allow the object of research in a given situation to show its best properties (optimization problem). The conditions on which the properties of the object depend are expressed quantitatively by some variables $x_1, x_2, \dots , x_n$, given in the domain of definition X and called optimization arguments. External actions r do not depend on us, but it is known that they can take their values from a compact set R. Usually it is assumed that the calculations are carried out for a given and known external action vector $r^0 \in R$, which ultimately determines the decision-making situation.

In turn, each of the properties of the object in the domain M is quantitatively described by the variable y_k, $k\epsilon[1,s]$, the value of which characterizes the quality of the object O in relation to this property (Fig. 1):

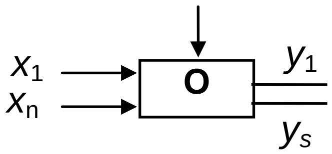

Fig. 1

In the general case, the parameters y_1, y_2, ... , y_s, called the quality criteria, form the vector $y = \{y_k\}_{k=1}^{s} \epsilon M$. Its components quantify the properties of the object for a given set of optimization arguments $x = \{x_i\}_{i=1}^{n} \in X$.

Systemic Approach

The term "systemic approach" means that a real object represented as a system is described as a set of interacting components that implements a specific goal. At the same time, a finite, but ordered set of elements and relations between them is "cut out" from the variety of components of a real object. We can say that the system is a model of a real object only in the aspect of the goal that it implements. The goal, requiring for its achievement certain functions, determines through them the composition and structure of the system.

The goal isolates, outlines the contours of the system in the object. In this system (object model) only what is necessary and sufficient to achieve the goal will come from the real object. If the same object can realize several goals, then with respect to each it acts as an independent system. The systemic approach assumes that not only the object, but also the research process itself acts as a complex system, the task of which, in particular, consists in combining in a single whole various models of the object.

Thus, with the systemic approach, the researcher receives only that information about a real object, which is necessary and sufficient to solve the task.

Optimization

If the object realizes only one goal, then the effectiveness of achieving the goal is quantitatively expressed by the single criterion of optimality y. The solution of the optimization problem involves reaching the extreme value of the criterion by choosing the set of optimization arguments.

The extremalization of the optimality criterion is often identified with the concept of goal realization, while in reality these are different concepts. We can say that the criterion and goal are correlated with each other as a model and an original with all the consequences that follow from this. This is understandable, if only because the original is usually put in line not one, but several models reflecting this or that aspect of the original. Some goals are difficult, and sometimes impossible to describe with the help of quantitative criteria. In any case, the criterion is just a surrogate of the goal. Criteria characterize the goal only indirectly, sometimes better, sometimes worse, but always approximately [1,2].

The decision of optimization problems assumes presence of some estimation of quality of work of a system from which it is possible to tell that one system works better, and another – is worse and how much. The fundamental problem of the quantitative assessment of objects and processes is that the notions of "better" and "worse" be put in line with the concepts "more" and "less." For certainty, it is believed that, for example, "better" means "less".

According to S. Stevens, if the description opens the way for measurement, then the discussions are completely replaced by calculations. In application to our problems, this means that if there are reasonable quantitative criteria for the quality of a complex system, then its study can be carried out through a formalized mathematical apparatus. Otherwise, subjective assessments, multivalued interpretations and arbitrary decisions are inevitable.

The function $y = f(x)$ relates the quality criterion to the optimization arguments. In estimation problems, the function $f(x)$ is called the *evaluation* function, and in optimization problems it is called the *objective* function. With some reservations, the optimization problem is formulated as finding such a combination of arguments from their domain, in which the objective function acquires an extreme value:

$$x^* = \arg \operatorname*{extr}_{\substack{x\in X \\ y\in M}} f(x)\Big|_{r^o \in R}$$

If "better" means "less", then in practice, for fixed $r^o \in R$ and guaranteed $y \in M$, expression

$$x^* = \arg \min_{x\in X} f(x)$$

is applied.

Multicriteria Problems

A complex object of research can not be characterized by any one (for example, the most "important" or "typical") attribute. When describing it, many inseparable properties must be taken into account simultaneously. In other words, to study complex objects, a modern systemic approach requires the involvement of the entire spectrum of their properties. A complex object and any fragment of it must be viewed not in isolation, but in numerous contradictory interactions and, importantly, in various possible situations.

Complex systems, being in different conditions (situations, modes), reveal different system properties, including those that are incompatible with none of the other situations separately. In their study, an approach is used that consists in the creation and simultaneous coexistence of not one but a set of theoretical models of the same phenomenon, some of which conceptually contradict each other. However, not one can be neglected, since each characterizes some property of the phenomenon under study and neither can be accepted as a single one, since it does not express the complete complex of its properties. It is interesting to compare what was said with the principle of complementarity introduced into science by Niels Bohr: "... To reproduce the integrity of the phenomenon it is necessary to apply mutually exclusive "additional" classes of concepts, each of which can be used in its own special condition, but only taken together, exhaust all the determinable information".

Multiple properties of a complex system in a given situation of its functioning are quantitatively estimated by corresponding partial criteria. In different situations, the rank of "the most important" acquire different properties and, accordingly, different partial criteria. Thus, mutually exclusive "additional" classes of concepts, in which the role individual theoretical models are presented, are characterized by conflicting partial criteria, each of which is applicable in its own special condition. And only a complete set of partial criteria (vector criterion) makes it possible to adequately assess the functioning of a complex system as a manifestation of the contradictory unity of all its properties. Therefore, it can be assumed that multicriteriality is the embodiment of the principle of complementarity in the methodology of research of the complex systems.

However, this possibility is only a necessary, but not sufficient, condition for vector estimation of the entire system as a whole. Indeed, let it be known the numerical values of all the partial criteria of the system. Does this mean that we, knowing these values, can evaluate the effectiveness of the system as a whole? No we can not.

It is appropriate to recall the old Indian parable about the blinds that got to know the elephant. One touched the trunk and decided that the elephant was like a snake. The second picked up an ear and said that the elephant reminds him of a sheet. The third felt a foot and said that the elephant is a pillar.

For a holistic assessment, it is necessary to rise to the next level, i.e. carry out the act of composing the criteria. Let us compare this with Kurt Gödel's incompleteness theorem: "... In any sufficiently complex first-order theory there is an assertion that can not be proved or disproved by the means of the theory itself. But the consistency of one particular theory can be established by means of another, more powerful, second-order formal theory. But then the question arises of the consistency of this second theory, and so on." Gödel's theorem seems to be the methodological basis for composing criteria, which is a sufficient condition for vector estimation of the system as a whole.

A scalar convolution of the criteria can serve as an instrument of the composition act. Scalar convolution is a mathematical method of compressing information and quantifying its integral properties by one number.

In general, the simultaneous description of the phenomenon (object) from several sides always gives a qualitatively new, more perfect idea of the described phenomenon (object) in comparison with any "one-sided" description. So, even two flat images that form a stereo pair make up a three-dimensional image of the object, not to mention the possibilities of holography. A multi-criteria approach that gives a "stereoscopic" look at the evaluation of the functioning of the system opens up new ways for improving complex management systems and decision making. So, for a holistic perception of a complex system in different conditions of its work, it is necessary to apply a multi-criteria approach.

In practical problems, a real object usually implements not one, but several goals and, accordingly, is characterized by several partial criteria of efficiency (quality). Let's pay attention to the fact that quality criteria, as a rule, are contradictory. The art of the researcher consists in the systemic linking models characterized by contradictory indicators. Thus, Jean Colbert (Minister of Louis XIV) in 1665 said: "The art of taxation is to get a maximum number of feathers while digging a goose, with a minimal hiss." At the systemic approach there is a task which consists in connection in a single whole of various models of object. The problem is solved by applying the act of criteria composition.

For the systemic linking in multicriteria problems, the scalar convolution of the particular criteria $Y=f[y(x)]$, where y is no longer a scalar, but an s-dimensional vector of the criteria $y = \{y_k\}_{k=1}^{s}$, is used as the objective (or evaluation) function instead of $y=f(x)$. Scalar convolution acts as an instrument of the act of composing criteria.

In the notion of optimality, in addition to the criteria, limitations $x \in X$ on the optimization arguments as well as $y \in M$ on the efficiency of the solution play an equally important role. Even small changes can significantly affect the solution. And very serious consequences can be obtained by removing certain restrictions and adding others with the same system of criteria. There is a great danger in the optimization of complex systems, as N. Wiener pointed out in his first publications on cybernetics. The fact is that, without setting all the necessary restrictions, we can, simultaneously with the extremization of the objective function, obtain unforeseen and undesirable accompanying effects.

To illustrate this N. Wiener liked to bring an English fairy tale about a monkey's foot. The owner of this talisman could fulfill any desire with its help. When he once wished to receive a large sum of money, it turned out that for this he paid the life of his beloved son. We will agree that it is often very difficult, and sometimes it is simply impossible to foresee in advance all the consequences of adopting multi-criteria decisions.

The idea of N. Wiener that in complex systems, we are fundamentally unable to determine in advance all the conditions and limitations that guarantee the absence of undesirable optimization effects, allowed him to make a gloomy assumption about the catastrophic consequences of cybernation of society.

Nevertheless, from the standpoint of system analysis, the attitude to optimization can be formulated as follows: it is a powerful means of increasing efficiency, but it should be used more cautiously as the complexity of the problem increases.

We formulate the formulation of the multicriteria optimization problem in a fairly general form.

Formulation of the Problem

A set of possible solutions $X \subset E^n$ consisting of vectors $x=\{x_i\}_{i=1}^{n}$ of n-dimensional Euclidean space is given. By the physical nature of the problem the vector holonomic (in static) or nonholonomic (in dynamics) connection $B(x) \leq 0$ is given. The decision is made at external influences, described by the vector r, given on the set of possible factors R.

The quality of the solution is estimated from the set of contradictory partial criteria that form the s-dimensional vector $y(x)=\{y_k(x)\}_{k=1}^{s} \subset F$, which is defined on the set X. The expression $y \subset F$ denotes the vector y belonging to the class F of admissible efficiency vectors. The partial criteria vector is bounded by the admissible domain: $y \in M$.

The situation that results from the adoption of a multi-criteria solution x under given external conditions r, is characterized by the Cartesian product $S=X \times R$.

The problem is to determine a solution $x^* \in X$, which, under given conditions, connections, and constraints, optimizes the efficiency vector $y(x)$.

This formulation is so general that, according to a famous comic expression, it can not be applied in any particular case. For the constructive solution of the task in various particular statements, it is necessary to carry out the structuring of certain concepts. To do this, we need to make additional special assumptions that help solve the following problems of vector optimization:

- determination of the range of Pareto optimal solutions;

- choice of the scheme of compromises;
- normalization of partial criteria;
- consideration of priority.

Difficulties in solving vector optimization problems are not computational, but conceptual in nature (this is not *how* to find the optimal solution, but *what* should be understood by it). Therefore, the development of a formal apparatus for solving multicriteria problems is one of the most difficult problems in the modern theory of decision-making and management. Its solution is important both in theoretical and applied terms.

Selection of the Scheme of Compromises

From the problems of vector optimization, we will pay special attention to the problem of choosing a scheme of compromises. One of the most important theses of the theory of decision-making under many criteria is that there is no best solution in some absolute sense. The decision made can be considered the best only for the person making the decision (decision maker, DM) in accordance with the goal set by him and taking into account the specific situation. The normative models for solving multicriteria problems are based on the hypothesis of the existence in the consciousness of the DM some utility function [3], measured both in nominal and in ordinal scales. The reflection of this utility function is the scheme of compromises and its model in a given situation – the scalar convolution of partial criteria $Y[y(x)]$, which allows constructively solving the problem of multicriteria optimization.

The determination of a multi-criteria solution is by its nature compromise and fundamentally based on the use of subjective information. Having received this information from the decision maker and choosing a scheme of compromises, one can move from a general vector expression to a scalar convolution of partial criteria, which is the basis for formation a constructive apparatus for solving multicriteria problems. If the scalar convolution method is used, the mathematical model of solving the vector optimization problem is represented in the form of the extremization of the function $Y[y(x)]$. This is a scalar function that has the meaning of a scalar convolution of the vector of partial criteria, the form of which depends on the chosen scheme of compromises.

The most commonly an **additive** (linear) scalar convolution is used

$$Y[y(x)] = \sum_{k=1}^{s} a_k y_k(x),$$

where a_K are the weight coefficients determined by the decision maker, starting from his utility function in the given situation. The Laplace principle in the theory of decision-making consists in the extremization of a linear scalar convolution. The drawback (specificity) of the application of linear scalar convolution is the possibility of "compensating" one criterion at the expense of others.

Multiplicative convolution

$$Y[y(x)] = \prod_{k=1}^{s} y_k(x)$$

is free of this shortcoming. The Pascal principle is the extremization of the multiplicative scalar convolution.

Historically, Blaise Pascal's principle was first described in the work of Pensees, published in 1670. It is believed that this work laid the foundation for the whole theory of decision-making. Here are introduced two key concepts of the theory: 1) partial criteria, each of which evaluates any one side of the effectiveness of the solution and 2) the principle of optimality, i.e. rule, allowing by the values of the criteria to calculate a single numerical measure of the effectiveness of the solution (act of criteria composition).

The Pascal principle is adequate in tasks with a cumulative effect, when the effect of certain efficiency factors is, as it were, increasing or decreasing the influence of other factors. When maximizing partial criteria, the zero value of any of them completely suppresses the contribution of all others to the overall effectiveness of the solution. In the aerospace industry, this approach can be partly justified when each criterion (for example, reliability and safety) is critical and no improvement in other criteria can compensate for its low value. If at least one of the partial criteria is zero, then the global criterion is also zero.

Shortcoming of application of multiplicative scalar convolution: a very expensive and very effective system can have the same estimation as a cheap and low effective. We will compare such "weapon systems" as an atomic bomb and a slingshot, which at a low cost has some damaging effect. Guided by the multiplicative convolution, it is possible to select a slingshot for the armament of the army.

Similar to the Laplace principle, one can generalize the Pascal principle by introducing weighting coefficients:

$$Y[y(x)] = \prod_{i=1}^{s} [y_i(x)]^{a_i} .$$

Convolution according to the **Charnes-Cooper** concept. The concept of Charnes-Cooper is based on the principle of "closer to the ideal (utopian) point." In the space of criteria under given conditions and constraints, an unknown a priori vector y^{id} is determined, for which the optimization problem is solved s times (by the number of partial criteria), each time with one (the next) criterion, as if the rest were not exist at all. The sequence of "single-criterion" solutions of the initial multicriteria problem gives the coordinates of an unattainable ideal vector $y^{id} = \{y_k^{id}\}_{k=1}^{s}$.

After that, the criterion function $Y(y)$ is introduced as a measure of approximation to the ideal vector in the space of optimized criteria in the form of some non-negative function of the vector y^{id}-y, for example, in the form of a square of the Euclidean norm of this vector:

$$Y(y) = \left\| \frac{y^{id} - y}{y^{id}} \right\| = \sum_{k=1}^{s} \left[\frac{y_k^{id} - y_k}{y_k^{id}} \right]^2 .$$

The disadvantage of this method is the cumbersome procedure for determining the coordinates of an ideal vector. In addition, the possibility of violation of restrictions is not ruled out.

The choice of the scheme of compromises is carried out by the person making the decision (DM) and has a conceptual character.

Formalization

Depending on the availability and type of information on the preferences of DM, the approaches to solving multicriteria tasks can be different. If there is no such information at all, then sometimes we are limited to finding any solution vector x^* that ensures only the fulfillment of the constraints $A = \{A_k\}_{k=1}^{s}$ condition:

$$y^* \in M = \{y | 0 \leq y_k(x^*) \leq A_k, k \in [1, s]\}, \; x^* \in X.$$

(Here we have the structuring of the concept of the domain of constraints M).

The disadvantages are obvious – the solution obtained is often rough and, as a rule, not Pareto-optimal. Consequently, the capabilities of the system in this case are not fully used.

The method is recommended to be used to optimize very complex systems, when it is far from easy to carry out even such a simple reconciliation of conflicting criteria ("just to get into limitations"). A variation of this approach is the widely accepted technique, when for optimization of the set y_k, $k \in [1,s]$, the decision maker chooses only one criterion (for example, the first one), and the remaining criteria are reclassified into the category of constraints. Thus, the original multicriteria problem is artificially replaced by a one-criterion problem with constraints:

$$x^* = \arg\min_{x \in X} y_1(x), 0 \leq y_k(x) \leq A_k, k \in [1, s].$$

A consequence of this approach is the solution in the form of a polar point of the Pareto region, i.e. frankly rude and subjective decision.

The scalar convolution approach with minimized criteria involves the use of the formula

$$x^* = \arg\min_{x \in X} Y[y(x)].$$

It is more reasonable in terms of formalization.

Analysis of the Scalar Convolution

The problem is that the form of the function $Y[y(x)]$ depends on the situation of the adoption of the multicriteria solution and is usually not known. Since the function $Y[y(x)]$ is difficult to obtain throughout the entire domain, we are often limited to an analysis of its behavior in the vicinity of that point in the arguments space that corresponds to the most typical situation. Since we are talking about *small* neighborhoods of the operating point, then, using the hypothesis of the smoothness of the criterion function, we replace it by a hyperplane tangent to the surface of equal values of $Y[y(x)]$ at the operating point. Then the approximating dependence $Y[\alpha, y(x)]$ takes the form of a linear scalar convolution

$$Y^o[a, y(x)] = \sum_{k=1}^{s} a_k^o y_k(x),$$

where $\alpha^o{}_k$ is the regression coefficient having the meaning of the partial derivative of the criterion function with respect to the k-th criterion, calculated at the base operating point. To calculate the coefficients α with the use of information from the DM, it is possible to solve the problem using the least squares method, but it is better to use the heuristic modeling technique described in [4].

Using the expression obtained, it must always be remembered that this is only a linear approximation of the scalar convolution of criterial functions, and in situations that differ from the base one, it can lead to significant distortions.

To obtain a criterion function over the entire domain, it is necessary to specify the form of the approximation dependence. As usual in the practice of approximation, success depends on how adequately the form of the given function reflects the physics of the phenomenon being studied. If you use information about the mechanisms of phenomena, then the model you specify is meaningful. In the absence of such information, the "black box" approach is used, and formal regression models of a general type (polynomial, power, etc.) are given for approximation. The quality of meaningful models is usually much better than formal ones.

Content Analysis of Utility Function

To improve the quality of the research, one should always involve a priori information about the physics of the phenomenon under investigation and, at every opportunity, move from formal models to meaningful ones. In our case, the subject of investigation is such a subtle substance as an imaginary utility function that arises in the mind of the decision-maker when solving a particular multicriteria problem. In addition, even if it does exist, then each DM has its own utility function. Nevertheless, it is possible to obtain information for specifying the type of the meaningful model of a criterial function if one reveals and analyzes some general laws observed in the process of making multicriteria decisions by various decision-makers in different situations.

Comparison of partial criteria of a different physical nature is possible only in a normalized (dimensionless) space. We normalize the efficiency vector y by the constraint vector A and obtain a vector of relative partial criteria (the normalized efficiency vector)

$$y_0(x) = \{y_k(x)/A_k\}_{k=1}^{s} = \{y_{0k}(x)\}_{k=1}^{s}.$$

This operation is monotonic, and, in accordance with the well-known theorem of Hermeyer, any monotonic transformation does not change the results of the comparison. Therefore, we replace the model of the solution of the vector optimization problem with the original criterion functions by the model

$$x^* = \arg\min_{x \in X} Y[y_0(x)],\ y_{0k}(x) \in [0;1], k \in [1,s],$$

in which the practically used schemes of compromises have a physical meaning. The form of the function $Y[y_0(x)]$ depends on the chosen scheme of compromises.

The scheme of compromises determines in what sense the multicriteria solution obtained is better than other Pareto-optimal solutions. At present, the choice of the scheme of compromises is not determined by theory, but is carried out heuristically, on the basis of individual preferences and professional experience of the developer, as well as information about the situation in which a multi-criteria decision is taken.

The main difficulty of the transition from the vector quality criterion to the scalar convolution is that the convolution should be a conglomeration of partial criteria, the significance (importance) of each of which in the overall assessment changes depending on the situation. In various situations, the rank of "the most important" can acquire different partial criteria. In other words, the scalar convolution of partial criteria must be an expression of a scheme of compromises that *depends on the situation*. When analyzing the possibilities of formalizing the choice of the scheme of compromises, let's put this thesis in the basis.

It is assumed that there are some invariants, rules that are usually common to all decision-makers, regardless of their individual inclinations, and which they equally adhere to in any given situation. The inevitable subjectivity of a decision maker has its limits [5]. In business decisions, a person must be rational in order to be able to convince others, explain the motives of his choice, the logic of his subjective model. Therefore, any preferences of decision-makers should be within the framework of a certain rational system. This makes possible formalization.

The concept of the situation, expressed by the deuce $S=<r,x>$ from the Cartesian product $R\times X$, is fundamental to the theory of vector optimization, since it, being objective, is the only support for attempts to formalize the choice of the compromise scheme. We introduce the concept of **tension of the situation** as a measure of the closeness of relative partial criteria to their limiting value (unit):

$$\rho_k(r,x)=1-y_{0k}(r,x),\ \rho_k\in[0;1],\ k\in[1,s].$$

This system is a structured characteristic of the concept of the situation $S=<r,x>$, $r \in R$, $x \in X$.

If a multicriteria solution is taken in a **stressful** situation, then it means that under given external conditions r, one or more partial criteria $y_{0k}(r,x), k \in [1,s]$, as a result of the solution x, may be in dangerous proximity to the limiting value ($\rho_k = 0$). And if one of them reaches the limit (or exceed it), then this event is not compensated by a possible low level of the remaining criteria – it is usually not allowed to violate any of the restrictions.

In this situation, it is necessary to prevent in every possible way the dangerous growth of the most unfavorable (i.e., closest to its limit) partial criterion, not taking very much into account the behavior of the others at this time. Therefore, in sufficiently stressful situations (for small values of ρ_k), the DM, if it admits the deterioration of the maximal (most important in the given conditions) partial criterion per a unit, then only compensating by a large number of units for improving the remaining criteria. And in a very tense situation (the first polar case: $\rho_k \approx 0$), the DM generally leaves only this one, the most unfavorable partial criterion, in view, without paying attention to the others.

Consequently, an adequate expression of the scheme of compromises in the case of a stressful situation is the minimax (Chebyshev) model

$$x^* = \arg\min_{x \in X} \max_{k \in [1,s]} y_{0k}(x). \qquad (1)$$

In less stressful situations, it is necessary to return to the simultaneous satisfaction of other criteria, taking into account the contradictory unity of all interests and goals of the system. In this case, the DM varies his estimate of the winnings according to one criteria and the losses on the other, depending on the situation. In intermediate cases, schemes of compromises are chosen, giving different degrees of partial equalization of partial criteria. With a decrease in the tension of the situation, preferences for individual criteria are aligned.

And, finally, in the second polar case ($\rho_k \approx 1$) the situation is so calm that the partial criteria are small and there is no threat of violation of the restrictions. DM here considers that the unit of deterioration of any of partial criteria is completely compensated by an equivalent unit of improvement of any of the others. This case corresponds to an economical scheme of compromises, which provides the minimum for the given conditions, the total losses by partial criteria. Such a scheme is expressed by the model of integral optimality

$$x^* = \arg\min_{x \in X} \sum_{k=1}^{s} y_{0k}(x). \tag{2}$$

Analysis shows that schemes of compromises are grouped at two poles, reflecting different principles of optimality: 1) egalitarian – the principle of uniformity and 2) utilitarian – the principle of economy.

The application of the principle of uniformity expresses the aspiration uniformly, i.e. equally reduce the level of all relative criteria in the functioning of the management system. A very important realization of the principle of uniformity is the Chebyshev model (1) – the polar scheme of this group. This scheme makes it necessary to minimize the worst (greatest) of the relative criteria, reducing it to the level of the others, i.e. leveling all the partial criteria. The disadvantages of egalitarian schemes of uniformity include their "economic inefficiency". Providing the closest to each other level of relative criteria is often achieved by significantly increasing their total level. In addition, sometimes even a small digression from the principle of uniformity can significantly improve one or more important criteria.

The principle of economy, which is based on the possibility of compensating for some deterioration in quality according to one criteria with a certain improvement for others, is devoid of these shortcomings. The polar scheme of this group is realized by the model of integral optimality (2). The utilitarian scheme provides the minimum total level of relative criteria. A common drawback of schemes of the principle of economy is the possibility of a sharp differentiation of the level of individual criteria.

The analysis reveals a pattern by which the decision maker varies from a model of integral optimality (2) in calm situations to a minimax model (1) in stressful situations. In intermediate cases, the decision maker chooses compromise schemes that give different degrees of satisfaction of individual criteria in accordance with his individual preferences, but in accordance with the given situation. If we take the conclusions from the analysis as a logical basis for formalizing the choice of the compromise scheme, then we can suggest various constructive concepts, one of which is the concept of a nonlinear scheme of compromises.

Nonlinear Scheme of Compromises

From the standpoint of the systemic approach, it is advisable to replace the problem of *choosing* the scheme of compromises with the equivalent problem of synthesizing a certain *single* scalar convolution of partial criteria, which in different situations would express different principles of optimality. Separate models of compromise schemes are combined into a single holistic model, the structure of which adapts to the situation of a multi-criteria decision making. The requirements for the synthesized function $Y[y_0(x)]$:

- it should be smooth and differentiable;
- in tense situations, it should express the principle of minimax;
- in calm conditions – the principle of integral optimality;
- in intermediate cases it should lead to Pareto-optimal solutions, giving various measures of partial satisfaction of the criteria.

In other words, such a universal convolution should be an expression of a scheme of compromises that *adapts* to the situation. We can say that adaptation and the ability to adapt are the main substantive essence of the study of multi-criteria systems. For this it is necessary that the expression for the scalar convolution explicitly include the characteristics of the tension of the situation. We can consider several functions that satisfy the above requirements. The simplest of these is a scalar convolution

$$Y(\alpha, y_0) = \sum_{k=1}^{s} \alpha_k [1 - y_{0k}(x)]^{-1}; \alpha_k \geq 0, \sum_{k=1}^{s} \alpha_k = 1,$$

where α_k=const are the formal parameters defined on the simplex and having a double physical meaning. On the one hand, these are the coefficients that express the preferences of the decision-maker for certain criteria. On the other hand, these are the coefficients of regression of a meaningful regression model based on the concept of a nonlinear scheme of compromises.

Thus, the nonlinear scheme of compromises is considered as the basic one, to which corresponds the model of vector optimization, which explicitly depends on the characteristics of the tension of the situation:

$$x^* = \arg\min_{x \in X} \sum_{k=1}^{s} \alpha_k \left[1 - y_{0k}(x)\right]^{-1}. \qquad (3)$$

From this expression it is clear that if any of the relative partial criteria, for example, $y_{0i}(x)$, approaches closely to its limit (unit), i.e. the situation becomes strained, then the corresponding term $Y_i=\alpha_i/[1-y_{0i}(x)]$ in the minimized sum increases so much that the problem of minimizing the entire sum is reduced to minimizing only the given worst term, i.e., ultimately, the criterion $y_{0i}(x)$. This is equivalent to the action of the minimax model (1). If the relative partial criteria are far from unit, i.e. the situation is calm, then the model (3) acts equivalent to the model of integral optimality (2). In intermediate situations, different degrees of partial alignment of the criteria are obtained.

This means that the nonlinear compromise scheme has the property of continuous adaptation to a multi-criteria decision making situation. From this point of view, traditional schemes of compromises can be considered as a result of the "linearization" of a nonlinear scheme at various "work points" - situations. This, by the way, explains the name of the proposed *nonlinear* scheme of compromises, since in other respects it is no more "nonlinear" than other schemes considered in decision theory. We emphasize that the adaptation of the nonlinear scheme to the situation is carried out *continuously*, while the traditional choice of the compromise scheme is done discretely, which adds to the subjective errors also the errors associated with the quantization of the compromise schemes.

We have repeatedly stressed above that the choice of a compromise scheme is a person’s prerogative, a reflection of his subjective utility function when solving a particular multicriteria task. Nevertheless, we managed to identify some regularities and, on this objective basis, construct a scalar convolution of criteria, the form of which follows from meaningful ideas about the essence of the phenomenon under study. The phenomenon of individual preferences of the DM is formally represented by the presence of the vector α in the structure of the meaningful model (3).

The Pareto optimality questions of the nonlinear scheme of compromises and its axiomatics were investigated in [4,6].

Unification

Various assessments of the role of subjective factors in the solution of multicriteria problems are possible. Subjectivity is permissible and even desirable if such a task is solved in the interests of a particular person. Therefore, the mechanism of individual preferences is rather intensively applied in the practice of solving multicriteria problems.

However, subjectivity in their decision is permissible and desirable only as long as the result is intended for specific decision-makers or narrow collectives of people with similar preferences. If it is intended for general use, then it must be completely objective, unified.

When the result of solving a multicriteria problem is intended for wide use, it is unified and individual preferences are leveled by statistics. If there is no a priori information about the differentness of the criteria, then the principle of the insufficient foundation of Bernoulli-Laplace says that in this case we must accept all the weight coefficients in expression (3) *equal to each other*. It follows from the normalization on the simplex that $\alpha_k \equiv 1/s$, $\forall k \in [1,s]$. Then

$$Y(\alpha, y_0) = \frac{1}{s}\sum_{k=1}^{s}\left[1 - y_{0k}(x)\right]^{-1}.$$

Taking into account that multiplication by $1/s$ is a monotonic transformation, which, by the theorem of Hermeyer, does not change the results of the comparison, we pass to the unified (without weight coefficients) expression for the scalar convolution of the criteria

$$Y(y_0) = \sum_{k=1}^{s}\left[1 - y_{0k}(x)\right]^{-1}. \qquad (4)$$

This formula is recommended to be applied in all cases when a multicriteria problem is solved not in the interests of one particular DM, but for wide use.

The unified scalar convolution by the nonlinear scheme has the form (4) or, in an equivalent form,

$$Y(y) = \sum_{k=1}^{s} A_k\left[A_k - y_k(x)\right]^{-1},$$

i.e., without preliminary normalization of partial criteria. The concept of a non-linear scheme of compromises corresponds to the principle "away from restrictions".

For the criteria being maximized, the unified scalar convolution has the form

$$Y(y) = \sum_{k=1}^{s} B_k [y_k(x) - B_k]^{-1},$$

where $B_k, k \in [1, s]$ is the minimum permissible values of the criteria to be maximized.

The Dual Method

If a multicriteria problem is solved in the interests of a particular decision maker, it is recommended first to obtain a unified (basic) solution and present it to the person. And only if this solution does not satisfy him and correction is required, it is necessary to proceed to the determination of weight coefficients reflecting his individual preferences. It is important that the search process does not start from an arbitrary point in the criteria space, but from a unified, basic solution.

The practice of solving multicriteria problems shows that the assumption that there is a ready and stable (at least implicitly) utility function of the decision maker is not always fair. Solving a multi-criteria task, the decision maker compares sets of specific criteria values with different alternatives, makes trial steps, makes mistakes and interprets the relationship between his needs and the possibilities of meeting them with a given object in a given situation.

With contradictory criteria, this ratio is by its nature a compromise, however, a decision maker does not have a consciously a priori scheme of compromises, or so far it is only in its infancy. Usually, the idea of a compromise scheme that is necessary to solve a problem arises and is gradually improved only as a result of attempts by the decision maker to improve a multi-criteria solution in a series of test steps. It is clear that the presence of interactive computer technology is implied. "In kind" such a procedure is usually impossible.

Thus, simultaneously and interdependent, on the one hand, a person adapts to the multicriteria problem being solved, carrying out the structuring of preferences and improving his understanding of the utility function, and on the other, consistently finds a series of solutions optimal in the sense of the current utility function. The mutually conditioned processes of adaptation of the decision maker to the task and finding the best result are of a dual nature and are, in principle, part of the methodology of the human-machine solution of multicriteria problems.

As noted, in the initial stage of the decision process, the DM practically lacks not only an analytic description of the utility function, but also a ready a priori idea of it. Therefore, the interactive procedure should be organized as dual, and the search optimization method should allow dialog programming in ordinal scales and use minimal information about the utility function. This method, based on the comparison of preferences with specially calculated alternatives, is an ordinal analogue of the simplex-planning method [4,6].

An important factor contributing to the effectiveness of the method is that the starting point of the search is chosen not as an arbitrary point in the Pareto set, but as an axiomatically grounded basic solution that should only be adjusted in accordance with the informal preferences of a particular decision maker. The process of adjustment provides mutual adaptation: a person adapts to this particular multicriteria task, and the model of a non-linear scheme of compromises becomes a reflection of the individual preferences of the person.

The fundamental difference between convolution in a nonlinear scheme and other known scalar convolutions is the organic connection with the situation of a multi-criteria decision. In fact, the proposed convolution is a non-linear regression function (linear in parameters), chosen for physical reasons and therefore effective. The coefficients α in the expression for the nonlinear scalar convolution have the meaning of the parameters of the nonlinear meaningful regression function, therefore, when found, they do not change from situation to situation, as in the case of linear and other known convolutions that do not adapt to the situation.

The problem of determining the coefficients α in a dual procedure can be considered as the problem of synthesizing a decision rule, which, when applied formally, adequately reflects the logic of a particular decision maker in any possible situation. Such a problem arises, for example, when a multi-criteria system operates in the mode of the operator's advisor in the conditions of time deficit. Here, it is desirable that the system in any situation quickly made the same decision as this operator, if he had the opportunity to calmly think. Similar problems have to be solved in the development of a decisive system for an intelligent robot that functions in changing and uncertain dynamic environments, if you want it to act in the same way a person who trained it would act in its place, etc.

Conclusion

As a result of a systemic approach, a multi-criteria optimization model is obtained, which allows an object to achieve all of its goals in the whole range of possible situations. A systemic approach to the problem of multi-criteria optimization allowed us to combine the models of individual compromise schemes into a single holistic structure, adapting to the situation of multi-criteria decision making. The advantage of the concept of a non-linear compromise scheme is the possibility of making a multi-criteria decision formally, without direct human participation. At the same time, on a single ideological basis, both tasks that are important for general use and those which main content essence is the satisfaction of individual preferences of decision makers are solved. The apparatus of the non-linear compromise scheme, developed as a formalized tool for the study of control systems with conflicting criteria, allows us to practically solve multi-criteria tasks of a wide class.

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