DIDACTICAL MAPPING IN TOPIC-ORIENTED DIDACTICAL UNITS

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Abstract*: Didactic map is analogue to concept map when an educational process is under consideration. Two quantitative characteristics of a didactical map are introduced: weight and distance. These characteristics are used to evaluate the 'didactic price' of didactical map modifications. A complex structure of the map links is proposed aiming to perform a more coherent image of the connections between the entities of the map. The implementation of such complex structure in a topic-oriented didactical unit potentially facilitates the execution of a particular integrated didactic scenario. The theoretical frame is operationalized by a sample instruction for didactical map design. The scope of the study includes the innovative schools in Bulgaria, which are adopting integrated approach as an upgrade of the compulsory subject-oriented curricula.*

Keywords: didactic map weight, didactic map distance, integrated approach, processing matrix, topic-oriented education.

ITHEA Keywords: K.3 COMPUTERS AND EDUCATION

Introduction

The new Bulgarian legislation provides the innovative schools to arrange partially the school plan with introducing didactical approaches to ensure an effective teaching-learning process [MON, 2015]. Among them are project-oriented initiatives for upgrading the subject-oriented curricula through topic-oriented didactic scenarios in which the integrated approach is crucial [Lazarov, 2018]. The integrated approach supposes that any included subject in a topic-oriented didactical unit (TODU) comes with its own methodology and the subject teachers prepare the separate parts of the TODU scenario at their own discretion, having in mind some specific educational goals. Development of a concept map materially facilitates the analytical constituents of the ingredients of the topic under consideration [Giouvanakis et al., 2016]. Routine procedures are usually applied at this stage, though rethought in a new context. But at the synthesis level, in most cases, there are certain problems with the multifaceted presentation of the topic. Then it is necessary to concert the concepts and to merge analytical conceptual maps. Such a technical procedure has its "didactic price", which will be discussed below.

Theoretical Base

The concept map matrix shown in Figure 1 (based on [Kitano et al., 2008]) represents a modification of the concept map key introduced by Novak and Gowin [1984].

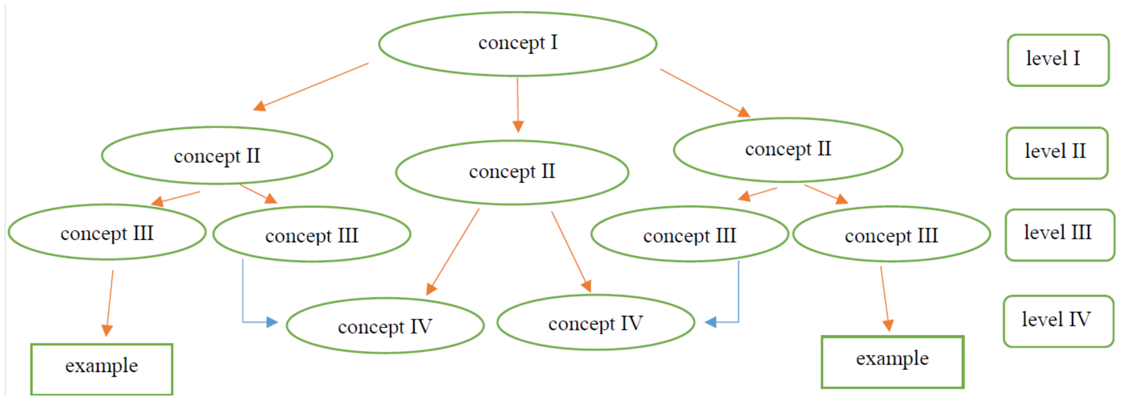

Figure 1. Concept map matrix

Here the concepts are accommodated in several hierarchic levels and a concept of higher level correlates with some concepts of lower levels (*higher* means smaller level number). The relationships between the concepts can be of two types: links (straight arrows in the figure) or crosslinks (broken arrows). The score of this particular map is calculated as

$$11 \cdot 1 + 4 \cdot 5 + 2 \cdot 10 + 2 \cdot 1 = 53.$$

This score corresponds to 11 links $(11 \cdot 1)$, 4 levels $(4 \cdot 5)$, 2 crosslinks $(2 \cdot 10)$ and 2 examples $(2 \cdot 1)$ [Kitano et al., 2008]. The values of the entities are commensurate with that of one link. In our opinion, the scoring formula is incomplete. It does not take into account the role of the concepts themselves. Simply expanding the formula with the addend (number-times-value) does not read the contribution of concepts from given level to the complexity of the map. The value 10 for a crosslink is also overestimated.

The relationships in Figure 1 constitutes an ontology and the classical Aristoteles’ genus-species hierarchy is easily recognizable in it. An *similarity index* could be appropriated to any two concepts in such an ontology [Giouvanakis et al., 2016]. The authors give the definition via an example:

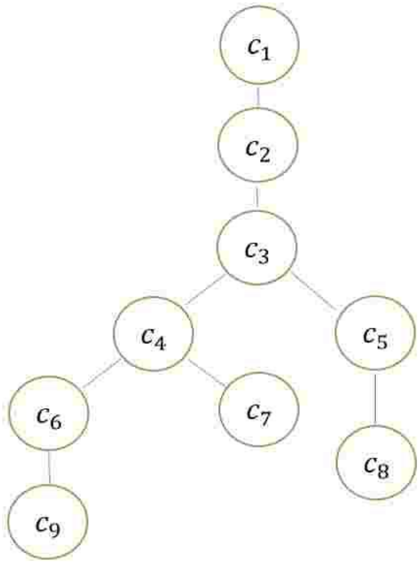

Figure 2. Ontology graph-tree

Following the notations in Figure 2, let:

- d_1 be the number of the links from the concepts c_6 to the common concept c_4;
- d_2 be the number of the links from c_7 to c_4;
- d_3 be the number of the links from c_4 to the genus concept c_2.

The similarity index for the concepts c_6 and c_7 is defined by the formula

$$\mathrm{sim}(c_6, c_7) = \frac{2d_3}{2d_3 + d_1 + d_2}.$$

About the Integrated Approach in Secondary School

Our standing point on the implementation of integrated approaches in secondary school is described more comprehensive in [Lazarov, 2018]. The fundamental idea is to amalgamate two (or more) methodical units from different subjects in one common methodical unit keeping the relative independence of any subject methodic. We speak for relatively independent inclusion of the methodic because the analytical knowledge developed by any methodic is expected to be synthesized in the frame of the common unit. The synthesis supposes transferability and multifunktionality of the complex *knowledge-skills-attitude* (KSA) into another context that differs from the ones, in which the analytical KSAs were developed. So, a kind of synchronization of the subject methodic should be orchestrated.

The *processing matrix* is a didactical tool that makes the implementation of the integrated approach via TODU more technological. It includes:

(1) definitions of the objectives, indicators of progress and benchmarks;

(2) selection of educational content to be examined;

(3) schedule and methodic for didactical support of the students;

(4) methodic for data collecting and processing the data collected;

(5) template for presenting information and frame for presentation design;

(6) assessment system;

(7) resources provided to the students and organizing a school conference;

(8) analysis of the outcomes of the initiative and drawing conclusions.

The processing matrix allows reconsidering the KSA developed in the traditional curriculum topics through the perspective of their potential application in a new context. Its proper design allows obtaining synergetic effect from the TODU, i.e. the resulting effect of the integrated teaching-learning process to exceed the effect of the expected one of any subject if it acts separately [ibid.]. But amalgamating methodic of different curriculum subjects is tricky business and some antinomies arise especially in points (2), (3) and (6) of the processing matrix. Resolving them could help a proper concept mapping.

Tuning the Concept Map Matrix

Basically, the structure of the concept map matrix in Figure 1 could be applied to a TODU for simple synchronization of concepts. However, a teaching methodic operates with more complex entities and relations than subject concepts and links between them. This is why we are going to conform the general concept map matrix to the purposes of the integrated approach. We call *didactical map* (DM) an analogue of the concept map for TODU with the modifications as follow.

First, we count the levels down up, i.e. the DM is supposed to be designed from the bottom to the top (bigger level number means higher level as in Figure 3). Next, let us define the nodes of the DM as *atoms*, which stands for ‘atomic didactical components of the subject methodic’. An atom could be a theorem, ethic category, historical fact etc. We call *beams* the edges of the concept map. This is to emphasize the multicomponent type of the relations between the atoms. The reformulation is not just a technical act but also a meaningful reconsideration of the didactical process of concept mapping. E.g. a beam that links the atoms (Hristo Botev) [TheFamousPeople, 2019] and (Bulgarian National Revival) in a DM could contain the knowledge in different levels about: the role of the hero in several stages of the revolutionary movement, some momentous of the historical period, his education etc. Any beam component reflects some specifics of the educational context in which the particular TODU takes place. In a DM, a beam can be either direct (d-beam), or crossing (c-beam).

In this connection, let us list several areas from a topic-oriented process of education in which the didactical mapping is potentially useful. It:

- illustrates the hierarchy and relationships between different elements of a topic;
- facilitates the perception and memorization of key elements (bases and beams) as fewer characters achieve greater information saturation;
- allows easy modification and restructuring in already constructed map fragments;

- allows for ideas to be shared, formalized quickly and accurately, and correctly displayed when preparing documentation;
- helps to clarify the logical consistency of presentation in the separate elements;
- allows evaluating more precisely the complexity of a topic, to define its complex structure, to decompose separate elements.

Developing the Theory

We are going to make more technological the advantages listed in section 4 by introducing two quantitative attributes related to the new atom-beam structure of the concept map matrix: distance and weight. We consider the educational objectives of a TODU in relation with some components of a synthetic competence that is developed via integrated approach [Lazarov, 2013]. Thus, the attributes of a beam should reflect the observed indicators for progress of these components of the synthetic competence. E.g. let the qualitative indicators I_k be the attributes of a d-beam, $k = 1,2, \dots ,5$. Let us assign the benchmark i_k to I_k for the transition from the origin atom to the beam-connected one. We take the value $J = \sum_{k=1}^{5} i_k$ as a quantitative measure of the transition calling it beam-value. We take the beam-value instead of the corresponding link from the definition of the similarity index sim. When stating the indicators, it is important to consider the direction of the educational process. This means that any indicator reflects some elements of the progress in the transition from an atom to the succeeding one, thus the beams should be thought as vectors.

Distance in DM

We are going to invert the ontological similarity taking into account the structure of the concept map matrix in Figure 1 but with down up counting of the levels. Let C' and C'' be two atoms in the first (bottom) level both connected with an atom C^* (it could be also C' or C'') and having a genus root C (it could be C^*); let δ be the level number of C and let d' and d'' equal the sums of the beam-values J for the beams connecting C^* with C' and C'' respectively.

I. Suppose the DM is an ontology graph-tree, i.e. there are no c-beams. In this case, we define *ontological distance* as:

$$D_C(C', C'') = \frac{d' + d''}{\delta} \ \& \ \forall C: (D_C(C', C'') = 0 \ \Leftrightarrow C' \equiv C'') \, .$$

Indeed, the values of sim are fractions (decimals) between 0 and 1, which makes difficult to feel the similarity. On the contrary, the ontological distance shows quite clear the relation between atoms. Here it is a kind of motivation to introduce the concept of ontological distance:

- the root C (the topic under consideration) is in the area of knowledge for which the concept map is intended;

- a hierarchical chain of atoms, which are common for C' and C'', is formed up from C^* to C;

- down the atom C^* there is a split into two different subject branches, hierarchically arranged to any of the atoms C' and C'';

- the farther away C' and C'' are from C^*, the larger the distance;

- the higher the root C in the map, the shorter the distance.

The consideration is that a long distance reflects some differences in fundamental details of the atoms and vice versa – small distance means conceptual proximity. Another feature of D is its root sensitivity. This means that $D_C(C', C'')$ changes when taking the genus root C in different levels of the map. Figuratively, the distance decreases when upgrading the ontological tree "up". Let us clarify this by an example. Consider C' stands for ‘pekinese’, C'' stands for ‘bulldog’. Now taking $C \equiv C^*$ to be ‘dogs’ $D_C(C', C'')$ seems to be large, but taking C to be ‘pets’ (keeping C^* for ‘dogs’) the distance become smaller. Mathematically, larger level of the genus root increases the denominator of D, hence diminish the fraction. Didactically, it is crucial for the effectiveness of a teaching-learning the atoms of the educational process to be immersed in a deeper origin.

II. Now suppose that there are c-beams in the DM. The ontological distance in the case “there exists a c-beam between C' and C'' with the beam-value J” we define as

$$D_C(C', C'') = \frac{J}{\delta}.$$

III. In both cases I and II, if C' and C'' are in different levels, we consider δ as the relative level with respect to the lowest one among the levels of C' and C''.

The ontological distance could be extended on cases that are more sophisticated. For instance, when “C' and C'' are indirectly connected with c-beams” or “C' and C'' are indirectly connected with c-beams and d-beams”. In these cases, one should combine the ideas in stating $D_C(C', C'')$ in the two basic cases of DM presented in this section.

Weight of DM

We follow the Novak and Govin's ideas for the score of a concept map outlined above by modifying the algebraic sum. We will form the characteristic *weight of the DM* by considering levels, bases and beams with specific coefficients that depend on the structure of the respective component.

Let us evaluate the level structure of a graph-tree. In binary branching, each level doubles the number of atoms in the above level. If there is an atom in the top level then there are two atoms in the level below, four atoms in the level, which is two levels below and so on. The total number of atoms in the DM of a binary-graph-tree type with m levels equals $2^m - 1$. Thus the weight of the map receives $L = 2^m - 1$ points from the level-map-characteristic. We adopt this simplest binary structure as a medium model. Indeed, the graph-trees that are more sophisticated could have different type branching in any particular knot, e.g., single or triple, which in average give binary branching.

An atom t participates with an individual coefficient B_t. Suppose there are k_t elements of already studied material and n_t elements new material, which should be studied. The material of the atom-structure could be of rather different type: experimental work, learning a poem by heart, drawing a figure etc. We assume that the points, which this particular atom contributes to the map weight are $B_t = k_t + 2n_t$ in total. Motivation: In an atom, the usage of concepts requires an appropriate time resource and complicates the synthesis in the same time. Moreover, introduction of a new concept requires additional technological time for operationalization.

A beam t will contribute its beam-value J_t to the map weight. Here we make no formal difference between d-beams and c-beams. However, the complexity of the beam could be taken into account by stating the benchmarks i_k.

Thus, we define *weight W of a DM* having p atoms and q beams as

$$W = L + \sum_{t=1}^{p} B_t + \sum_{t=1}^{q} J_t.$$

Weight is a measure of the permissibility of a particular didactic scheme described by the DM. It is clear that when considering the values L, B, J, some interdependence and overlap are allowed. For example, concepts at one atom affect the indicators of both the incoming beam and the outgoing one. We do not consider this as a drawback, but rather as a collateral feature of the integrated approach.

Editing DM

A need for editing could appear when two independently designed DM are formally combined into one. E.g., if the total weight is too big or appear too long distances. The following operations are possible:

- adding an atom;
- removing an atom;
- replacement of an atom;
- adding a beam;
- removing a beam;
- adding an attribute to a beam;
- removing an attribute from a beam.

Any of these operations has a didactic price. It can reflect directly the corresponding action, but it can also take into account factors, which are determined by the educational context. The price can be in the direction of gain (usually associated with weight or distance reduction) or loss (increase in performance values).

Sample Didactical Mapping Technology for TODU

DM structurizes the didactic process: the hierarchy of concepts is clarified, and hence the order of their introduction; links are clarified, cross-domain links are rethought. The technology of DM design could be as follows:

1) a team of teachers (usually two subject teachers and a coordinator, preferably an ICT teacher) traces the topic for integrated consideration;
2) the subject teachers independently highlight the topic from their methodology, any one separately clarifies the educational goal and indicators of progress;
3) the subject teachers prepare a preliminary list of atoms, taking into account the concepts learned, the already developed students' knowledge and skills, as well as the need of introducing new concepts, building additional skills, forming the positive attitude;
4) atoms are ranked in levels, only d-beams are defined;
5) the subject teachers design concept maps independently of each other, where the topic is at the top;
6) the team jointly draws up a DM, uniting and editing the two DMs into one;
7) the weight of the DM is calculated and, if necessary, ways of reducing it are sought;
8) when drawing up the DM, the distance between the atoms is taken into account and, if appropriate, the DM is edited, possibly by adding c-beams.

Example

In this section, we present the didactical mapping in a TODU that could be titled ‘Hristo Botev – translator and propagandist’ (it was dedicated to the 140th anniversary of the Bulgarian hero Hristo Botev) in the experimental classes on mathematics at the 125th Secondary School – Sofia. Botev cannot be fully studied in neither history nor literature, since Hristo Botev's multifaceted appearances does not fall within the narrow scope of any particular school subject. Figure 3 gives us an idea of Botev's hypostases. The left branch of Botev’s subtree is subject of history, the right branch is studied in literature school course. (The examples in the rectangles on the bottom of this DM could be different and they do not form a separate level in the DM.)

The activities of the translator and the propagandist do not easily fit together, insofar as the translators are tasked with retelling some source, and the mission of the propagandists is to persuade through (their own) ideas. Thus, there is no place in the presented DM for a little known side in Botev's activity: the translation of the textbook "Lessons for the first four arithmetic rules" by Mikhailov (a detailed review of this translation was made in [Penkov and Chobanov, 1958]). Here, Botev's genius manages to synthesize a brilliant translation with propaganda, which gave us a reason to organize a corresponding TODU devoted to the celebration of his anniversary.

Figure 3. Ontology graph-tree of Botev's hypostases

Let us calculate the ontological distance between the atoms C' (publicist) and C'' (poet) taking $J = 5$ for all beams (a motivation for this value is given in the next paragraph). We have C^*(Botev), C (Bulgarian National Liberation Movement), $d' = d'' = 10,\ \delta = 4$. Thus $D_C(C', C'') = \frac{10+10}{4} = 5$. Let us note that if extract the branch 'Botev' from the Bulgarian-National-Liberation-Movement didactic map as an autonomous DM for which $C \equiv C^*$ (hence $\delta = 3$), then the distance increases: $D_{C^*}(C', C'') = \frac{20}{3} \approx 6.7$.

Another (third) hypostasis, namely 'Hristo Botev – translator and propagandist', cannot be find under the atom C^* (Botev) in Figure 3. It needs two new atoms to be added: (translator) in level III and (mathematician) in level IV. The transition (translator) → (mathematician) requires studying a fragment of Botev's biography by Zahariy Stoyanov [1888]. The bottom row of this branch will be supplemented by the example [Lessons for the first four arithmetic rules]-rectangle. Let us consider the beam from the atom (translator) to the atom (mathematician) with attributes (indicators) I_k , $k = 1,2,\dots,5$, as follows: I_1 – knowledge, I_2 – skills, I_3 – attitude, I_4 – transferability of KSA, I_5 – multifunctionality of KSA. Let the benchmark of any indicator takes value from the set $\{0; 1; 2\}$. We state: $i_1 = 2$ (fundamental knowledge in Bulgarian history for that period); $i_2 = 1$ (basic math skills); $i_3 = 1$ (moderate positive attitude); $i_4 = 0$ (no requirements); $i_5 = 1$ (basic). Thus $J = 5$.

The introduction of the new atoms allows the atoms (revolutionary) and (enlightener) to be connected in a natural way through intersecting beams with beam-values $J_{1,2}$. Taking the genus root (Bulgarian National Liberation Movement), the distance in the chain

$$\{(\text{publicist}) \overset{J_1}{\leftrightarrow} (\text{mathematician}) \overset{J_2}{\leftrightarrow} (\text{poet})\}$$

calculated via c-beams equals $\frac{J_1}{\delta} + \frac{J_2}{\delta} = \frac{5}{4} + \frac{5}{4} = 2.5$. Comparing with the distance $D_C(C', C'') = 5$, which we obtained above, this is twice less. Such shortage of the distance could be done with a minimal didactic price: preparatory independent work for the students to study the text [Stoyanov, 1888], one additional lesson as well as monitoring the transferability and multifunctionality of the knowledge. In the Appendix, we give the test-item from the term test that checks these transferability and multifunctionality.

The shortage of the distance $D_C(C', C'')$ derives increasing the DM weight. Suppose the weight of the DM in Figure 3 is W (it could vary depending on the structure of the level III). Let W' be the weight of the new DM, which supplements the initial one with the branch

$$\{(\text{Botev}) \overset{J_4}{\leftrightarrow} (\text{translator}) \overset{J_3}{\leftrightarrow} (\text{mathematician})\}$$

and the c-beams that were defined above. Thus

$$W' = W + \sum_{t=1}^{2} B_t + \sum_{t=1}^{4} J_t = W + 26.$$

Skipping the motivation, here $B_1 = 2$ stands for the atom (translator), $B_2 = 4$ stands for the atom (mathematician). We take $J_3 = J_4 = 5$ analogously to the calculated above beam-values for $J_{1,2}$. Fortunately, the increase of the weight does not affect the didactic price in this particular case.

Conclusions

The didactical mapping we presented is still a theoretical model in an experimental phase. We tried to recruit some teachers for implementing the didactic map in their pedagogical practice but we met their tacit resistance. Such attitude is understandable on this stage of our research. However, our plans include elaborating map-shell that automatically calculates the weight and the ontological distance when stating the benchmark values. In our opinion, the technology and the model for didactical mapping could help in deeper reconsideration of didactic process of integrated approach in topic-oriented educational units.

The positive results reported in [Giouvanakis et al., 2016] encourage us to develop more TODU that integrate different school subjects, providing a priory calculated quantitative characteristics of the particular TODU. Further clarifications of the parameters are expected to be done by taking into account the educational context in which the teachers are going to implement the integrated approach.

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Appendix

Item 10 from the term test. Hristo Botev is the author of the following problem:

'Assen and Peter liberated Bulgaria from the yoke of the Greeks in 1190. After 206 years, Bulgaria fell under Turkish rule. In what year did our country lose its freedom?'

Write down the years in which the Bulgarian kingdoms fell under Turkish slavery, and the names of the respective Bulgarian rulers in which this occurred.

The Kingdom of Tarnovo:

Year .... King ....

Kingdom of Vidin:

Year .... King ....

Which kingdom does Botev have in mind? ....

Answers. Tarnovo Kingdom: 1393 Tsar Ivan Shishman. Kingdom of Vidin: year 1396 Tsar Ivan Sratsimir. Botev refers to the kingdom of Vidin.

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