COMPROMISE AND CONSENSUS IN MULTICRITERIA DECISION-MAKING

Albert Voronin, Alina Savchenko

Abstract. *Various approaches to solving multicriteria problems are considered, depending on the role of constraints in the problem statement. If the constraints are fixed and specified, then the calculation algorithm includes the preferences of the decision maker, and the corresponding multicriteria problem has a fundamentally compromise solution. If the values of the restrictions can be varied, then it becomes possible to obtain consensus decisions, and the calculation algorithm is free of heuristic elements.*

Keywords: *compromise, consensus, multicriteria problem, constraints, resources, calculation algorithm.*

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Introduction

To successfully achieve the goals set under the given operating conditions, each system has sufficient resource capabilities (financial, technical, etc.). These resources are usually limited and are called constraints in the theory of multicriteria optimization. Constraints play a fundamental role in solving multicriteria problems.

Constraints can be imposed both by the optimization arguments and by the criteria of the solution's effectiveness. Constraints imposed on the characteristics of a system under certain circumstances can often serve as a reason for introducing one or another criterion. To illustrate this point, consider an example. Under normal ground conditions, it is not customary to assess the quality of an ergatic system by the amount (or rate of consumption) of oxygen consumed by a human operator when performing a given job. It is quite another matter when the system operates without contact with the "unlimited" earth's atmosphere (in space, under water, etc.). In this case, the oxygen resources are limited and the economy of its consumption becomes a very important quality. A reflection of this requirement is the introduction of an appropriate criterion. In such cases, it can be said that the constraint generates a criterion.

Even small changes in the constraints can significantly affect the solution [Lebedeva at al, 2020]. And very serious consequences can be obtained by removing some restrictions and adding others. In 1956, Brazilian entomologists decided that bees were not producing enough honey. They crossed several European species and added a variety of African bees. Hybrid bees, indeed, produced more honey, were disease resistant, tolerated heat well, but at the same time they became incredibly aggressive and very poisonous. More than 150 people and hundreds of animals, both domestic and wild, have died from their bites in Brazil and the southern United States.

Therefore, there is a great danger in the formal optimization of complex systems, which was pointed out by N. Wiener in the very first publications on cybernetics. The fact is that, without setting all the necessary restrictions, we can simultaneously obtain unforeseen and undesirable side effects simultaneously with the extremization of the objective function.

In the theory of multi-criteria decision-making, two main approaches to solving problems of vector optimization of complex systems are considered. One of them consists in the formalization of multicriteria problems with given (fixed) constraints and resource capabilities of the system. The second approach shows what new perspectives open up for developers if they have the ability, within certain limits, to dispose of resource reserves (limitations) of multicriteria systems. Some results in this area are considered in [Glushkov, 1980] within the framework of the theory of system optimization.

Compromise

For the successful fulfillment of the set goals under the given operating conditions, each system has certain reserves and resources (in terms of strength, heat resistance, amount of fuel, etc.), which are limited. The concept of the first approach is characterized by the fact that the limitations and resource capabilities of the system are given and fixed.

Considering the fundamental role of constraints in solving multicriteria problems, we formulate, in contrast to the Charnes-Cooper concept, the following optimality principle: “away from constraints” [Voronin, 2009]. In accordance with this principle, a non-linear compromise scheme is determined, which allows one to obtain a compromise solution common to the criteria, which removes the partial criteria from their "red lines" (limitations) to the greatest extent.

Requirements for scalar convolution of criteria $Y[y(x)]$ by a nonlinear trade-off scheme:

1. It must "penalize" partial criteria for approaching its limitations.
2. It must be differentiable in its arguments.

Of the possible functions that meet the listed requirements, we will choose the simplest:

$$Y[y(x)] = \sum_{k=1}^{s} A_k [A_k - y_k(x)]^{-1},$$

where A_k are the upper constraints on the criteria to be minimized: $y_k \le A_k, k \in [1, s]$.

Minimization of the objective function leads to a compromise solution that removes the partial criteria from their limitations to the greatest extent:

$$x^* = \arg\min_{x \in X} \sum_{k=1}^{s} A_k [A_k - y_k(x)]^{-1}.$$

Scalar convolution by a nonlinear trade-off scheme penalizes partial criteria for approaching their limiting values. Indeed, let any criterion $y_m(x)$ from among them $[1, s]$ dangerously approach its limitation. This means that the difference $[A_m - y_m(x)] \to 0$ tends to zero and the corresponding term $\frac{A_m}{A_m - y_m(x)}$ in the minimized sum increases rapidly.

With a significant increase $y_m(x)$ and, consequently, the term $\frac{A_m}{A_m - y_m(x)}$, the minimization of the entire amount is reduced to the minimization of only this, the most "unfavorable" term. This means that a nonlinear trade-off scheme with a dangerous increase in one or several minimized partial criteria acts as a minimax (Chebyshev) optimization model

$$x^* = \arg\min_{x \in X} \max_{k \in [1,s]} \frac{y_k}{A_k}(x).$$

The situation in which partial criteria are dangerously close to their limitations is considered to be tense. On the contrary, if the criteria are far from limiting, the situation is calm. In a quiet situation, the nonlinear trade-off scheme acts as an integral optimization model:

$$x^* = \arg\min_{x \in X} \sum_{k=1}^{s} \frac{y_k(x)}{A_k}$$

In this way, the non-linear trade-off scheme adapts to the decision-making situation. The optimization model ranges from integral in calm situations to egalitarian (Chebyshev) in stressful situations. In intermediate situations, compromise schemes are obtained that satisfy partial criteria to the extent that they are removed from their limitations. This means that instead of choosing a trade-off scheme in different situations, we can apply a single universal non-linear trade-off scheme, which automatically gives a scheme that is adequate for a given specific situation.

Model example

Let some system be evaluated by two minimized criteria:

$$y_1 = x;$$
$$y_2 = (x-5)^2 + 2.$$

Restrictions are set: $y_1 \le A_1 = 6.5$ and $y_2 \le A_2 = 6$. It is required to find a compromise-optimal solution $x^* \in X^K \subset X$ according to a nonlinear compromise scheme. In line with the concept of a non-linear trade-off scheme, for this example

$$x^* = \arg\min_x \left[\frac{A_1 - y_1}{A_1} + \frac{A_2 - y_2}{A_2} \right] = \arg\min_x \left[\frac{6.5 - x}{6.5} + \frac{6 - (x-5)^2 - 2}{6} \right].$$

The necessary condition for the minimum of the function is

$$-\frac{1}{6.5} + \frac{-2(x-5)}{6} = 0.$$

From here $x^* = 4.08; y_1^* = 4.08; y_2^* = 2.85.$

A characteristic feature of the first approach is the presence of a compromise area $X^K \subset X$ (Pareto area, negotiation set), in which there is an optimal compromise solution x^*.

Consensus

The second approach is taken when the constraints on partial criteria are not fixed. How does this affect the choice of the trade-off scheme?

Each compromise scheme is a reflection of a well-defined useful property, which, according to the developers, should have a projected multicriteria system in a given situation [Sergienko at al,2000]. Thus, the egalitarian principle of uniformity presupposes the implementation of the idea of the most uniform change in the level of each of the partial criteria. As an example, we point out that a well-designed mechanism wears out evenly and at the end of the estimated operating time it breaks down simultaneously in all its links.

On the other hand, the application of the utilitarian principle of integral optimality suggests that developers focus on the efficiency of the total consumption of reserves and resources of the designed system. Example: the mentioned mechanism should last as long as possible.

The traditional formulation of the problem forces us to choose one, adequate to the given conditions, compromise scheme, ignoring the useful qualities contained in other principles of optimality.

Meanwhile, the practice of solving applied multicriteria problems shows that the solutions obtained using different compromise schemes are not always significantly different. In the best examples of multicriteria systems, they are close, and sometimes practically coincide. What does it depend on and what does it mean?

If the solutions obtained according to different compromise schemes coincide (or are close enough), this means that the resources and reserves of the designed system have been selected and used so successfully that, under the given operating conditions, the system simultaneously meets all the requirements laid down in different principles of optimality. Example: a rationally constructed mechanism lasts a long time, but at the end of its service life it turns out that all its parts are worn out to the same extent. If the solutions are significantly different, then the reserves and resources of the system are selected incorrectly, they are not balanced for the given operating conditions, and, therefore, the system is organized irrationally.

Thus, a sign of a rationally organized multicriteria system is the coincidence (proximity) of solutions obtained according to various principles of optimality, and the means of rational organization is the selection of reserves and resources on which the system's constraints depend. It is advisable not only to passively state that this system was designed irrationally (or rationally), but, when and to what extent it is possible, to include in the problem statement the purposeful selection (organization) of reserves and resources of the projected multicriteria system.

Let us consider such a design procedure when developers, within certain limits, can change the values of all or some of the reserves and resources of the system, harmoniously selecting a set of constraints adequate to the given conditions and thereby determining an adequate admissible range of solutions.

Let us formulate the principle of rational organization in multicriteria management problems: *In a rationally organized multicriteria system, under given operating conditions, limited resources and stocks are selected so that optimization of the efficiency vector according to various compromise schemes leads to coinciding (or close) solutions*.

The principle of rational organization is universal and represents a logical basis for formalizing the solution of multicriteria problems of various nature. If the solutions coincide, the problem of choosing a compromise scheme disappears, and the corresponding heuristic element is excluded from the methodology for solving a multicriteria problem. In this case, the vector optimization problem is completely and objectively reduced to a scalar one.

In the new formulation of the multicriteria problem, the constraints A are not given constants, but, along with the arguments x, are represented by independent variables. In this case, the result of solving the vector problem is to determine such vectors $x^{o} \in X$ and A^{o} for which the principle of rational organization is satisfied.

It can be argued that if the principle of rational organization is strictly observed, then the optimal solution x^{o}:

• belongs to the Pareto area;

• simultaneously satisfies all trade-off schemes leading to Pareto-optimal solutions;

• is the only one.

It follows that the mathematically rigorous implementation of the principle of rational organization is nothing more than the *contraction* by rational choice of the vector A^o of all Pareto solutions to a single point x^o, which is the desired optimal solution of the posed multicriteria problem.

The foregoing allows us to assert that when the principle of rational organization is fulfilled, polar compromise schemes are simultaneously satisfied, corresponding to the egalitarian principle of uniformity and the utilitarian principle of economy. The principle of uniformity is represented by the equality of relative criteria:

$$\frac{y_1(x)}{A_1} = \frac{y_2(x)}{A_2} = \ldots = \frac{y_s(x)}{A_s}, \qquad (1)$$

and the principle of economy requires a minimum of the function

$$Y[y(x)] = \sum_{k=1}^{s} \frac{y_k(x)}{A_k} = \min_x . \qquad (2)$$

Expanding expression (1), we get:

$$\frac{y_j(x)}{A_j} - \frac{y_{j+1}}{A_{j+1}} = 0,\ j \in [1, s-1]. \qquad (3)$$

Condition (2), under the assumption that the solution is attained within the admissible range of arguments, generates the system of equations

$$\frac{\partial Y}{\partial x_i} = \frac{\partial}{\partial x_i} \sum_{k=1}^{s} \frac{y_k(x)}{A_k} = 0, i \in [1, n], \tag{4}$$

where n is the dimension of the vector of arguments.

The principle of rational organization requires that equations (3) and (4) constitute a joint and complete set of equations. It can be seen, however, that this system is uncertain (one equation is missing). This is due to the fact that a solution according to the principle of rational organization can be obtained with an infinite number of combinations of absolute values of restrictions. Therefore, it is necessary to complete the task with an additional condition. Such a condition can be, for example, the equality of the l-th relative criterion (and automatically all relative criteria) to a specific value given for physical reasons:

$$\frac{y_l(x)}{A_l} = \mu, l \in [1, s], 0 < \mu \leq 1. \tag{5}$$

Thus, to solve a multicriteria problem according to the principle of rational organization, it is necessary to solve the system of equations (3) - (4) - (5). As a result, we obtain n required components of the solution vector x^o and s optimal components of the constraint vector A^o. The stated simple and objective technique can be applied if the problem of rational organization has an exact solution in a given field of arguments.

Among the solutions obtained according to the principle of rational organization, a special place is occupied by one in which the limited reserves and resources of the system are completely consumed. Such a solution is obtained by the intersection of conditions (3) and (4) at μ = 1. It is logical to call the obtained solution x^O a *global consensus*, since it is balanced according to all criteria, provided with completely expendable resource capabilities of the system, and is consistent.

Example 1. Let our system be evaluated by the same two minimized criteria:

$$y_1 = x;$$
$$y_2 = (x-5)^2 + 2.$$

Now let's change the problem so that the constraints A_1 and A_2 are not fixed and can be picked up from the open area. For physical reasons, the parameter $\mu = 0.6$ is selected. Required to find the values $x^o, A_1{}^o, A_2{}^o$, at which the principle of rational organization is satisfied. For this, we compose the system of equations (3) - (4) - (5):

$$\frac{x}{A_1} = \frac{(x-5)^2 + 2}{A_2};$$
$$\frac{\partial}{\partial x}\left[\frac{x}{A_1} + \frac{(x-5)^2 + 2}{A_2}\right] = 0;$$
$$\frac{x}{A_1} = \mu = 0.6.$$

The solution to this system leads to a quadratic equation with respect to the argument:

$$1.8x^2 - 12x + 16.2 = 0,$$

having two roots:

$x_1^o = 4.79,$ at which $A_{11}^o = 7.98; y_{11}^o = 4.79; A_{21}^o = 3.13;\ y_{21}^o = 2.05$ and $x_2^o = 1.88,$ at which $A_{21}^o = 3.41; y_{21}^o = 1.88; A_{22}^o = 19.56;\ y_{22}^o = 11.73.$

The first option seems to be preferable.

Example 2. Under the same conditions, select the parameter $\mu = 1.$ At this value, the solution of the system of equations (3) - (4) - (5) gives the values $x^O, A_1^O, A_2^O,$ corresponding to the global consensus.

In the process of solving, a quadratic equation is obtained

$$3x^2 - 20x + 27 = 0,$$

rooted:

$x_1^O = 4.79,$ at which $A_{11}^O = 4.79; y_{11}^O = 4.79; A_{21}^O = 2.05;\ y_{21}^O = 2.05$ and

$x_2^O = 1.88,$ at which

$A_{21}^O = 1.88; y_{21}^O = 1.88; A_{22}^O = 11.73;\ y_{22}^O = 11.73.$

Both options are acceptable.

Example 3. For a system evaluated by three minimized criteria:

$$
\begin{aligned}
&y_1 = x;\\
&y_2 = (x-5)^2 + 2;\\
&y_3 = 2 - 0.5x
\end{aligned}
$$

needs to define the terms of global consensus x^O, A_1^O, A_2^O, A_3^O.

Let's compose the system of equations (3) - (4) - (5):

$$
\begin{aligned}
&\frac{x}{A_1} = \frac{(x-5)^2 + 2}{A_2}; \frac{(x-5)^2 + 2}{A_2} = \frac{2-0.5x}{A_3};\\
&\frac{\partial}{\partial x}\left[\frac{x}{A_1} + \frac{(x-5)^2 + 2}{A_2} + \frac{2-0.5x}{A_3}\right] = 0;\\
&\frac{x}{A_1} = \mu = 1.
\end{aligned}
$$

In the process of solving this system, a cubic equation is obtained for the optimization argument:

$$2x^3 - 21x^2 + 67x - 54 = 0.$$

The solution of this equation according to the Vieta formula gives two complex roots and one real root, which is the required argument: Thus,

$x^O = 1.22,$ at which $A_1^O = 1{,}22; y_1^O = 1.22; A_2^O = 14.3; y_2^O = 14.3; A_3^O = 1.39; y_3^O = 1.39.$

Conclusion

Two approaches to the formalization of the solution of multicriteria problems are considered. One of them is applied for fixed values of constraints on partial criteria and consists in the fact that the desired compromise-optimal solution $x*$ is obtained as an argument for minimizing the scalar convolution of criteria according to a nonlinear trade-off scheme depending on the given constraints A [Voronin and Savchenko, 2020].

The second approach is used when the constraints A are not specified and it is possible to consider them as independent variables of the problem. In this case, the solution x^o, A^o is obtained according to the principle of rational organization, and if the resource opportunities are fully spent, then a solution x^O, A^O is obtained that expresses the global consensus of partial criteria.

Both approaches were used to solve specific problems of vector optimization of space control systems. Thus, the concept of a nonlinear scheme of compromises was used in the development of the law for the glide path descent control of the Buran spacecraft; when creating a regression model of expert assessments “reliability-cost” in the problems of insuring space objects; in the development of ergatic systems for aerospace purposes, etc.

The principle of rational organization makes it possible to most effectively solve the technical and economic tasks of performing complex technical and social projects [Voronin,2017], since it makes it possible to harmoniously coordinate the resource (financial, technical, ergonomic, etc.) capabilities of systems in the design and implementation of projects in conditions of stringent requirements.

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Authors' Information

Voronin Albert Nikolaevich *– professor, Dr.Sci (Eng), professor of the department of National aviation university of Ukraine, Lubomyr Husar Avenue 1, 03680, Kyiv, Ukraine;*

e-mail: alnv@ukr.net

Major Fields of Scientific Research: Multi-Criteria Decision Making; Man-Machine Complex Systems

Savchenko Alina Stanislavovna,

Candidate tech. Sci., Head of the Department of the National Aviation University, Kiev,

e-mail: a.s.savchenko@ukr.net.