

BASIC CONSTRUCTS OF A FORMAL THEORY OF REFLECTIONS

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Abstract: *In this paper, the reflection, a structural and/or functional change in an entity caused by external or internal influence, is traced in extensive detail across Western philosophy from antiquity to phenomenology, Eastern traditions, Islamic and Late Antique thought, the historical idealist–materialist debate, and modern cognitive science, neuroscience, predictive processing, embodied cognition, and information theory, revealing a single recurring structural pattern beneath vastly different vocabularies: one entity becomes present in another without being identical to it. This pattern is formalized as the category Refl , whose objects are reflections $R = (S, X)$, pairs of a source S and a resulting state X , and whose morphisms are contacts, directed acts of influence. Refl is shown to satisfy the axioms of a category, to support families of reflections of a common source together with comparison morphisms between them, and to admit collective reflections via categorical limits and colimits. Several theorems are established, including the preservation of source consistency under composition and the existence conditions for a consistent collective reflection, each illustrated across physical, biological, neural, cognitive, and social domains. The framework is then extended with functors and natural transformations connecting a hierarchy of six reflection levels and with explicit temporal structure sufficient to express variation, selection, inheritance, and emergence as basic evolutionary processes within Refl . The text is accessible to readers without specialized training in category theory.*

Keywords: Formal Theory of Reflections; Reflection; Category Theory; General Information Theory

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Introduction

The concept of reflection occupies a unique place in the history of human thought. It is one of the few ideas that appears across cultures, epochs, and intellectual traditions, yet has never been fully formalized in a rigorous scientific framework. Philosophers, mystics, theologians, scientists, and cognitive theorists have all used the notion of reflection to describe how one entity becomes present in another without identity. Despite this universality, the concept has remained theoretically fragmented.

In the 19th and 20th centuries, the term “reflection” became entangled in ideological disputes, particularly between idealist and materialist schools. In some traditions, reflection was interpreted as a passive copying of external reality; in others, as an active construction by the mind. These debates often obscured the deeper structural insight shared across cultures: **reflection is a relational process in which the state (structure and/or function) of one entity is modified by the influence of another.**

The General Information Theory (GIT) proposed by Markov, Ivanova, & Mitov. (1993, 2003) reintroduced reflection into scientific discourse by defining it as a **structural and/or functional change in an entity caused by external or internal influence**. This definition is ontologically neutral and compatible with both physical and cognitive sciences. It avoids metaphysical commitments and focuses on the structural relation between entities.

However, to fully restore the scientific value of reflection, we must go beyond GIT’s initial formulation. We must develop a **Formal Theory of Reflections** that:

1. integrates the philosophical richness accumulated over millennia,
2. incorporates empirical findings from modern cognitive science and neuroscience,
3. and provides a mathematically rigorous framework capable of supporting formal reasoning.

Such a theory must be **interdisciplinary, conceptually clear, and mathematically precise**. It must be accessible to readers from diverse

backgrounds, including philosophy, computer science, cognitive science, physics, and information theory.

Chapter 2 below undertakes this integration in some detail, tracing the concept of reflection across roughly two millennia and several independent intellectual lineages — from Plato's cave and Aristotle's theory of perception, through Stoic, scholastic, and early modern epistemology, Kant's transcendental idealism, and twentieth-century phenomenology, to Buddhist, Daoist, and Indian philosophical accounts of mind and appearance, to Neoplatonic, Sufi, Hermetic, and Kabbalistic traditions of cosmological emanation, and finally to the nineteenth- and twentieth-century idealist–materialist debate and contemporary cognitive science. The purpose of this survey is not antiquarian completeness but convergence: each tradition, working independently and often in mutual ignorance of the others, arrives at some version of the same relational structure that Chapter 4 below formalizes as the category *Refl*, and the breadth of that convergence is itself part of the case for treating reflection as a genuine structural universal rather than a metaphor imported from any one domain into the others.

This paper builds directly upon the formal model of reality developed in Markov (2024), where the necessary mathematical apparatus — categories, objects, morphisms, composition, functors, natural transformations, and symmetric monoidal structure — was introduced and motivated in detail as part of a hierarchical model of reality (the category *Real*). To avoid unnecessary repetition, these foundational constructs are not redefined here; readers unfamiliar with them, or seeking a fuller justification of the categorical approach, may consult that earlier work. In the present paper, each categorical construct is nonetheless accompanied by an intuitive explanation and a concrete example drawn from philosophy, cognitive science, or everyday experience, so that the formal apparatus remains accessible to readers without specialized mathematical training.

1. Reflection

When two entities come into contact, the influence exerted by one entity alters the internal structure or functionality of the other. This alteration may be

temporary or permanent, superficial or profound, localized or systemic. The essential point is that the realization of a relationship, the actual occurrence of contact, produces a change in at least one of the entities involved. We call this change **reflection**. The **reflection is structural and/or functional change in an entity caused by external or internal influence**.

This change need not have any positive magnitude. Contact can occur and yet leave the affected entity exactly as it was; this null case (an interaction that took place but produced no discernible alteration) is not an exception to reflection but its degenerate, zero-magnitude instance. A null reflection is still a reflection: it registers that a contact occurred, only registering it as an absence of change rather than a presence of one. The pattern is familiar outside physics, too — in social interaction, silence in response to a question is not the absence of a response but a response in its own right, one that carries meaning precisely because an opportunity for change was present and was not taken up.

Reflection is therefore the fundamental mechanism through which entities register the impact of their interactions. It is not merely a passive imprint but an active transformation: the entity reorganizes itself in response to the influence it receives. This reorganization may occur at one or multiple levels of its internal structure, depending on the complexity of the entity and the nature of the interaction.

Because entities interact continuously, reflection is not a one-time event but an ongoing process. After one interaction, another may follow, and the changes produced by the first interaction may themselves become the basis for further changes. This gives rise to **secondary**, or **transitive**, reflection: an entity reflects not only the immediate influence of another entity but also the accumulated effects of previous interactions. In this way, reflection propagates through chains of interactions, allowing the history of an entity to shape its future responses.

A special case of transitive reflection is **transitive self-reflection**. This occurs when an entity comes to register something about its own state only indirectly, through a relay: the entity's condition is first registered by some intermediary — another entity, an internal sensor, or a dedicated signaling pathway — and only then does that intermediary's own altered state feed back and affect the original

entity. In this sense, the entity “sees itself” through a detour rather than directly. A mirror reflecting one’s face back to oneself is a clear external example; a baroreceptor relaying blood pressure to the brain through an afferent nerve is an equally clear internal one — both cases share the same structure, an intervening relay whose own state carries the reflection back to its source.

Alongside it stands **direct self-reflection**, in which no such intervening relay is required. Here the entity’s state changes as an immediate consequence of contact with one of its own parts, without that contact being routed through, or represented by, any further intermediate reflection. Continuously watching one’s own visible limbs — hands, legs, one’s own body — is a case of this kind: this steady stream of direct reflections is what allows an entity to build up a spatial “picture” of itself within its surroundings. A hormone released by one tissue and binding a receptor on another tissue of the same organism is a further case — the hormone acts on the receiving tissue directly, without any separate relay carrying or representing the signal along the way. Interoception, tellingly, draws on both routes at once: direct self-reflection through hormonal and other biochemical interactions between tissues, and transitive self-reflection mediated by dedicated sensory pathways that relay a bodily state to the brain. Both forms are possible only in entities with sufficiently complex internal organization — one capable of distinguishing a “self” part from another “self” part in the first place. When direct and transitive self-reflection combine and iterate, as in advanced biological organisms and sophisticated artificial systems capable of modeling, evaluating, and revising their own structure, an entity becomes both the subject and the object of its own transformations — arguably the highest known level of organizational complexity.

Reflection, in all its forms, is the engine of change in the world. It is the process through which entities evolve, adapt, learn, and develop new capabilities. Without reflection, the world would be static; with reflection, it becomes a dynamic, self-transforming system.

2. The Foundations for Formal Theory of Reflections

This chapter provides the foundations for Formal Theory of Reflections. It traces the concept of reflection across major intellectual traditions, identifies common

structural themes, and prepares the ground for the formalization developed in subsequent chapters.

2.1. Reflection in Western Philosophy

Western philosophy offers some of the earliest and most influential analyses of reflection. Although the terminology varies (shadows, forms, impressions, species, representations) the underlying idea remains consistent: **the world becomes present to a subject or entity through mediated appearances.**

2.1.1. Plato: Shadows and Appearances

Plato's *Republic* contains one of the most iconic images of reflection: the Allegory of the Cave (Plato, 1997). Prisoners chained inside a cave see only shadows cast on a wall by objects behind them. These shadows constitute their entire experiential world. Plato writes that the prisoners “would suppose that the truth is nothing but the shadows of the artifacts” (p. 515c).

This allegory illustrates several principles essential to a modern theory of reflection:

- **Non-identity:** The shadow is not the object.
- **Mediation:** The reflection depends on the fire, the wall, and the position of the prisoners.
- **Structural dependence:** The form of the reflection is constrained by the medium.
- **Multiplicity:** The same object can produce many different reflections.
- **Epistemic ambiguity:** Reflections can mislead or reveal.

Plato's analysis is not merely metaphysical; it is structural. It identifies reflection as a relation between entities mediated by a process of projection.

The allegory becomes still richer when read alongside the rest of Book VII. Plato has the prisoners not merely misperceive shadows but mistake the naming-game they play about the shadows for genuine knowledge — an early recognition that a reflection can generate its own internal system of reference, stable and predictively useful within its own level, even while remaining several

removes from the source that produced it. Plato reinforces this with the image of the divided line (Republic 509d–511e), which ranks four grades of cognitive access — *eikasia* (imagining, the lowest, an image of an image, such as a shadow or reflection in water), *pistis* (belief in physical things), *dianoia* (mathematical reasoning), and *noesis* (direct intellectual apprehension of the Forms). What matters structurally is that Plato treats these not as different faculties operating on the same content, but as a graded series of reflections of reflections: the visible world already reflects the intelligible Forms imperfectly, and within the visible world itself, images and shadows reflect physical things at yet a further remove (Plato, 1997). This nested, multi-level structure of reflection anticipates, in a strikingly literal way, the hierarchy of reflection levels developed formally in Chapter 6.

Plato's treatment is also an early statement of what later chapters call the non-reducibility of reflection: the prisoner freed from his chains and dragged up toward the sunlight does not simply see “more” of the same shadows; he must undergo a qualitatively different kind of contact with a qualitatively different source (Plato, 1997). The ascent out of the cave, culminating in the vision of the sun as the Form of the Good, is Plato's way of insisting that higher-level reflections are not just larger or clearer versions of lower-level ones, but stand in a distinct relation to their own, distinct sources — precisely the intuition behind treating physical, biological, and cognitive reflections as belonging to different, functorially related categories rather than one undifferentiated pool.

2.1.2. Aristotle: Form without Matter

Aristotle deepens the analysis in *De Anima*, where he describes perception as the reception of the “form without the matter” of an object (Aristotle, 1984, 424a17). The perceiver does not receive the object itself, but its form, its structure, impressed upon the sense organ.

This is a remarkably precise early formulation of reflection as **structural imprinting**. It anticipates modern notions of information, representation, and neural coding.

Aristotle makes the mechanism more precise with his well-known image of the signet ring and the wax: just as wax takes on the shape of a ring's seal without taking on the gold or bronze the ring is made of, the eye takes on the perceptible form of an object without taking on its matter (Aristotle, 1984). This is already a theory of structural, rather than substantial, transmission — the very distinction that later motivates treating a reflection as a state X produced in a receiver, rather than as any transfer of the source's actual constituents. Aristotle further distinguishes the special senses (each keyed to a proper object, such as color for sight or sound for hearing) from the common sense (*aisthesis koine*), which integrates deliverances across modalities into a single percept of a single object moving, at rest, or possessing a definite shape and number. This is arguably an early systematic account of what Section 2.5.1 below discusses under the modern heading of multisensory integration: Aristotle already saw that an entity capable of reflection at all must possess some further structure capable of binding its several reflections into one.

Aristotle also insists, against a purely passive picture of perception, that the sense organ must already be potentially what the object is actually, and is only actualized, brought from potency to act, by the object's influence (Aristotle, 1984). This qualifies any reading of Aristotelian reflection as simple imprinting: the resulting state depends jointly on the source and on a prior capacity of the receiver, since wax that lacked the right consistency could not take the seal's impression at all. The same joint dependence on source and receiver-structure is what later chapters of this paper formalize by making a reflection $R = (S, X)$ depend essentially on which entity plays the role of X , not on S alone.

2.1.3. Stoicism: Impressions (*phantasiai*)

The Stoics describe impressions as modifications of the soul caused by external objects (Long & Sedley, 1987). These impressions can be:

- clear or confused,
- true or false,
- compelling or weak.

This introduces the idea that reflections vary in quality and reliability - an idea central to both epistemology and cognitive science.

The Stoics went further than simply cataloguing impressions as more or less clear: they built an entire epistemology around the search for a criterion that would reliably distinguish a trustworthy impression from a misleading one. Their proposed criterion, the kataleptic impression (*phantasia kataleptē*), was defined as an impression that arises from what is real and is stamped and impressed exactly in accordance with what is real (Long & Sedley, 1987). Whether such a guaranteed-reliable impression actually exists was fiercely contested even in antiquity, most notably by the Academic Skeptics, who argued that for any impression claimed to be kataleptic, one could in principle construct an indistinguishable false impression. This ancient dispute is a direct ancestor of a question that resurfaces, in formal dress, in Section 5.2 below: whether, and under what conditions, a family of reflections of a single source can be guaranteed to admit a consistent collective account, or whether divergence among reflections is instead the normal case that any adequate theory must accommodate.

Equally important for a structural theory of reflection is the Stoic doctrine that an impression alone does not yield belief or action; the mind must additionally give or withhold assent (*synkatathesis*) to what the impression presents (Long & Sedley, 1987). This separates, with unusual clarity for an ancient source, the production of a reflection (something happens to the perceiver as a causal result of contact with an external object) from its uptake or endorsement (the perceiver's further, and in principle independent, act of treating that reflection as veridical). The distinction anticipates, in miniature, the later separation between a reflection as a bare object of mathematical category *Refl* and the further interpretive or evaluative morphisms that operate upon it.

2.1.4. Scholasticism: Species and Likeness

Medieval scholastics, especially Thomas Aquinas, describe cognition as the reception of *species* (likenesses) of things (Aquinas, 1947). The species is not the object itself, but its intentional form in the intellect. This is a continuation of

the Aristotelian tradition, but with a more explicit emphasis on the **non-material** nature of reflection.

Aquinas develops this into a systematic doctrine of intentional existence (*esse intentionale*), distinct from an object's natural existence (*esse naturale*). When fire heats wax, the wax comes to possess the heat naturally, as a real physical property; but when the eye perceives fire, the visual species conveys the form of fire intentionally, without the eye itself catching alight (Aquinas, 1947). This distinction, between a reflection that transmits a mode of being proper to the receiver's own nature and one that transmits a mode of being proper to representation, is one of the sharpest pre-modern statements of the difference between ordinary causal interaction and reflection specifically: not every change produced by contact counts pre-theoretically as a reflection in the intuitive sense, though Chapter 1's General Information Theory (GIT) definition deliberately generalizes past this restriction to cover both.

Aquinas further distinguishes the sensible species (received by the external and internal senses, still tied to the particular here-and-now percept) from the intelligible species and the concept or inner word (*verbum mentis*) that the intellect actively abstracts from it (Aquinas, 1947). The species is never itself what is understood, but only that by which understanding occurs (*id quo intelligitur*, not *id quod intelligitur*) — a formulation that anticipates, with remarkable precision, the later insistence (Section 2.6.2 below) that a reflection is never to be confused with the entity in which it appears, but is only ever a state of that entity.

2.1.5. Early Modern Philosophy: Representation

Descartes, Locke, and later representationalists treat ideas as internal images or representations of external objects (Descartes, 1984; Locke, 1975). This is a shift in vocabulary, but not in structure: reflection remains the mechanism by which the world becomes present to the mind.

Descartes sharpens the vocabulary considerably by distinguishing an idea's formal reality (its being, simply as a mental occurrence) from its objective reality (the degree of being of whatever the idea represents). An idea of an infinite

substance, Descartes argues, must derive its objective reality from a cause possessing at least as much formal reality — the core premise of his causal argument for God's existence (Descartes, 1984). Whatever one makes of the theological argument, the underlying structural insight is a durable one for a theory of reflection: a reflection's representational content can, in principle, be assessed and compared independently of the mental act that carries it, opening the door to treating ideas as objects with their own structure rather than merely as episodes of thinking.

Locke complicates the picture by dividing the qualities of bodies into primary qualities (solidity, extension, figure, motion, and number, which he holds really resemble the ideas they produce) and secondary qualities (colors, sounds, tastes, and smells, which are powers to produce ideas in us that resemble nothing in the bodies themselves) (Locke, 1975). This is an early, explicit recognition that a reflection's structure can diverge systematically from its source's structure depending on which aspect of the source-receiver relation is in play — some reflections preserve source structure (Locke's primary qualities), others are receiver-relative transformations with no matching structure in the source at all (secondary qualities). The distinction foreshadows the later, more general point that the fidelity of a reflection to its source is a substantive further question, not something guaranteed by the mere fact that a reflection occurred.

2.1.6. Kant: Phenomena as Reflections

Kant argues that what we experience are not things-in-themselves, but phenomena - appearances structured by our sensibility and understanding (Kant, 1998). This is a sophisticated theory of reflection as **mediated appearance**, shaped by the cognitive architecture of the subject.

Kant calls this reorientation his Copernican revolution: rather than assuming that cognition must conform to objects, he proposes that objects, as we experience them, must conform to the structure of cognition itself (Kant, 1998). Space and time are, on this account, not features of things-in-themselves but pure forms of sensible intuition, contributed by the perceiving subject prior to any particular experience; and the twelve categories of the understanding, such

as causality, substance, and unity, are pure concepts that the mind supplies in order to synthesize the otherwise unconnected manifold of sensory intuition into a single, unified experience of objects. Reflection, on this picture, is never a raw transcript of an independent world; it is always already jointly constituted by the source's influence and by an a priori structure the receiver contributes to every possible act of reflecting — an unusually direct anticipation of the later, formal insistence that a reflection $R = (S, X)$ depends essentially on the internal structure of the receiving entity, not on the source S alone.

Kant is equally emphatic, however, that this constitutive role of the subject does not collapse into idealism in the sense of denying an independent source altogether: the thing-in-itself (Ding an sich) remains, for Kant, the unknowable but necessarily presupposed correlate of appearance, the X that phenomena are appearances of, even though its own intrinsic structure is forever inaccessible (Kant, 1998). This combination — a real, mind-independent source, together with a receiver-structure that irreducibly shapes what the resulting reflection can be — is close to the neutral, structural stance later chapters of this paper try to formalize without committing to either idealism or naive realism about the source.

2.1.7. Phenomenology: Intentionality and Appearance

Husserl (1970) and Merleau-Ponty (2012) describe experience as a dynamic interplay between the world and consciousness. The world appears “as” something, in a horizon of possible further appearances. This is a phenomenological theory of reflection as **intentional structure**, emphasizing context, embodiment, and meaning.

Husserl gives this intentional structure a precise internal anatomy, distinguishing the noesis, the subject's act of intending (perceiving, remembering, judging, imagining), from the noema, the intended object exactly as it is meant in that act (Husserl, 1970). Crucially, one and the same object can be given through indefinitely many noemata: a coin seen from the front, from an angle, remembered, or merely imagined all present the same object under different modes of givenness, and each such presentation carries with it a horizon of further, currently unfulfilled but anticipated presentations — the coin's

hidden other side, its expected weight, its behavior if dropped. This horizontal structure is a phenomenological way of stating a principle central to the present work: a reflection is always partial, revealing some aspects of its source while leaving others merely anticipated or entirely absent, so that no single reflection, however rich, ever exhausts its source.

Merleau-Ponty radicalizes Husserl's account by insisting that this intentional structure is not first and foremost a mental or cognitive achievement but is already built into the lived body (*corps vécu*) and its practical engagement with a environment of graspable, usable things (Merleau-Ponty, 2012). Perception, for Merleau-Ponty, is not a two-stage process in which sensory data are first received and then interpreted; the body's motor capacities and habitual skills already shape what can appear at all, and how. This bodily-grounded reading of reflection converges, from an entirely independent direction, with the embodied-cognition research discussed in Section 2.5.3 below, and reinforces the general lesson that the structure of the receiving entity, not merely the source, co-determines the reflection that results from any given contact.

2.2. Reflection in Eastern Philosophy

Eastern philosophical traditions developed the concept of reflection independently from the Western lineage. Although the terminology varies (“mind as mirror,” “water reflecting the moon,” “consciousness-only,” “*pratibimba*” (reflection), “Purusha reflected in Prakriti”), the structural idea is remarkably consistent: **phenomena appear as reflections within a substrate without being identical to it.**

This makes Eastern thought especially valuable for our goal of constructing a universal, formal theory of reflections that is not tied to any single metaphysical doctrine.

2.2.1. Buddhism: Mind as Mirror

In Buddhism, especially in Mahāyāna traditions, the metaphor of the mind as a mirror is central. It appears in numerous sūtras and commentaries and emphasizes three key aspects:

- **Reflections are dependent on conditions**
- **Reflections are not the mind itself**
- **Reflections are transient and constantly changing**

The *Lankāvatāra Sūtra* contains the classical passage:

“The mind is like a mirror, reflecting forms yet remaining unstained.”
(Suzuki, 1932, p. 89)

Here the mirror (mind) remains pure regardless of the forms that appear within it. This is an early formulation of the principle that **a reflection is a secondary state of the substrate, not its essence.**

The *Platform Sūtra of the Sixth Patriarch* similarly teaches:

“The mind is like a bright mirror. Polish it diligently, and let no dust settle on it.”
(Yampolsky, 1967, p. 132)

This emphasizes that the **quality of reflection depends on the state of the substrate**, a principle that will later be formalized through internal morphisms and structural constraints in the category Refl.

2.2.1.1. Buddhist contributions to a theory of reflection

Buddhism offers three insights that align perfectly with GIT:

- **Reflections are conditioned** (arising from causal factors).
- **Reflections are multiple** (many arise simultaneously).
- **Reflections are empty of inherent existence** (they are not the objects themselves).

These ideas provide a conceptual foundation for understanding reflection as a dynamic, relational process.

These three insights are not free-standing observations but consequences of the broader Buddhist doctrine of dependent origination (*pratītyasamutpāda*): every phenomenon arises in dependence on conditions, persists only as long as those conditions obtain, and ceases when they change or cease. Applied to reflection, this doctrine entails that a reflection can never be an independent, self-standing entity, since it is by definition a state that arises only given a

specific contact between a specific source and a specific receiver. The later Madhyamaka school, associated above all with Nāgārjuna, radicalizes this into the doctrine of emptiness (śūnyatā): not only reflections but all phenomena whatsoever lack inherent, independent existence (svabhāva), and exist only relationally, in webs of mutual dependence. This relational ontology finds one of its most vivid expressions in the Huayan (Flower Garland) image of Indra's Net: an infinite net stretching in all directions, with a polished jewel at every node, each jewel reflecting every other jewel, and each of those reflections itself containing the reflections of all the others, without end. As a picture of a fully interconnected network of mutually reflecting entities, in which no single node has any privileged, independent status, Indra's Net anticipates with striking precision the notion of a collective reflection built from an entire family of comparison morphisms, developed formally in Sections 4.6 and 4.7 below.

2.2.2. Daoism: Still Water and the Moon

Daoism develops one of the most elegant conceptualizations of reflection. In the *Zhuangzi*, we find the following key passage:

“The perfect man uses his mind like a mirror. It does not grasp, it does not reject. It receives but does not keep.” (Watson, 1968, p. 97)

Here the mirror serves as a model for ideal cognition: it accepts reflections but does not identify with them. This is a structural description of **non-possessive, non-substantial reflection**.

Daoism also emphasizes that **the quality of reflection depends on the tranquility of the substrate**.

If the mind is “turbulent water,” the reflection is distorted.

If it is “a calm lake,” the reflection is clear.

This anticipates the modern idea that **the internal structure of an entity determines the form of the reflection**, a principle that will later be formalized in Refl.

This concern with the tranquility of the reflecting substrate is not merely a psychological observation for the Daoist tradition but is tied to the deeper

cosmological principle of wu wei, or non-forcing action — action that accords with the natural, spontaneous unfolding (ziran) of things rather than imposing an external will upon them. The Dao De Jing repeatedly returns to water as the paradigmatic image of this principle, since water yields, settles, and clarifies itself without effort whenever left undisturbed, and precisely because it does not strive, it comes to reflect what is above it with perfect fidelity. The recurring term used for this state of luminous clarity, ming, denotes a brightness of mind achieved not by adding anything to the mind but by removing whatever agitation would otherwise distort it — a striking structural parallel to the later, formal observation that a reflection's fidelity depends on constraints internal to the receiving entity (its “turbulence” or lack thereof) fully as much as on the character of the source.

2.2.3. Yogācāra: Consciousness-Only and Reflected Appearances

Yogācāra Buddhism, known as the “consciousness-only” (*vijñapti-mātra*) school, offers one of the most sophisticated ancient theories of reflection. According to Yogācāra:

- phenomena are reflections of karmic seeds (*bīja*)
- reflections arise within eight layers of consciousness
- reflections are structured by latent tendencies and cognitive habits

Anacker (1984) explains that for Yogācāra, “all objects are merely manifestations of consciousness,” meaning reflections within a cognitive substrate.

This anticipates modern theories of:

- internal generative models
- predictive processing
- constructive perception

and thus forms a conceptual bridge between ancient philosophy and contemporary cognitive science.

Yogācāra's model of consciousness, developed principally by the brothers Asaṅga and Vasubandhu, is considerably more differentiated than the single

term “consciousness” suggests. It posits eight distinct consciousnesses: the five sense-consciousnesses (associated with sight, hearing, smell, taste, and touch), a sixth mental consciousness (*manovijñāna*) that synthesizes and conceptualizes their deliverances, a seventh afflicted mind (*manas*) that habitually misconstrues the eighth consciousness as a unified, enduring self, and an eighth storehouse consciousness (*ālayavijñāna*) in which karmic seeds (*bīja*) are deposited by every prior experience and from which future experiences subsequently ripen (Anacker, 1984). On this model, ordinary experience is never a direct reflection of external objects at all; it is a reflection of the storehouse consciousness's own latent seeds, projected outward and misrecognized as an independent external world — a picture Yogācāra formalizes further through the doctrine of the three natures (*trisvabhāva*): the imagined nature (*parikalpita*, the mistaken appearance of external objects), the dependent nature (*paratantra*, the actual causal process of seeds ripening into experience), and the perfected nature (*pariṇiṣpanna*, the dependent nature correctly seen as empty of any imagined, independently existing object). The eight-consciousness model is, among the traditions surveyed in this chapter, the one that comes closest to anticipating the layered, hierarchical architecture of reflection levels formalized in Chapter 6, since it explicitly distinguishes multiple, functionally distinct strata of reflection operating on and feeding into one another within a single cognitive entity.

2.2.4. Advaita Vedānta: The World as Reflection of Brahman

In Advaita Vedānta, especially in Śaṅkara's commentaries, the world is described as a reflection (*pratibimba*) of Brahman - the undivided, nondual reality.

Śaṅkara (1973) uses the classical example:

- a rope appears as a snake
- the snake is a reflection of ignorance
- the rope is the underlying reality

Here reflection is:

- **dependent** (on perceptual conditions)

- **non-real** (not identical with the object)
- **structural** (arising from cognitive factors)

This is an early formulation of the idea that **reflection is a transformation**, not a copy.

Śaṅkara's argument is embedded in a larger analysis of error (khyātivāda) that distinguishes several grades of reality within Advaita metaphysics: the absolute reality (pāramārthika) of Brahman alone, without a second; the transactional or empirical reality (vyāvahārika) of the everyday world of subjects and objects, which is real enough for all practical purposes but not ultimately real; and the merely apparent reality (prātibhāsika) of the rope mistaken for a snake, which vanishes the instant correct cognition arises (Śaṅkara, 1973). What makes this hierarchy significant for a theory of reflection is that each lower grade is not simply false but is a structurally real reflection relative to its own level — the snake is genuinely seen, genuinely produces fear, and genuinely disappears only upon a further, more discerning act of cognition, exactly as a lower-level reflection in a hierarchical system can be fully real and functionally effective at its own level while still standing in a further, encompassing relation to a more fundamental one. Śaṅkara's predecessor Gauḍapāda had already argued, in his kārikās on the Māṇḍūkya Upaniṣad, that waking experience and dream experience differ only in duration and intersubjective agreement, not in their fundamental status as appearances superimposed (adhyāsa) upon a substrate that is never itself directly an object of experience — an early and unusually rigorous statement of the principle that no reflection, however stable or widely shared, can be mistaken for unmediated access to its own source.

2.2.5. Sāṃkhya: Puruṣa–Prakṛiti Duality as Reflection

Sāṃkhya offers a unique metaphysical interpretation of reflection. According to Larson (1969):

- Puruṣa (pure consciousness) is the passive witness
- Prakṛiti (primordial nature) is the active principle
- experience arises when Puruṣa is “reflected” in Prakṛiti

This is strikingly close to:

- phenomenology (consciousness as witness)
- cognitive science (consciousness observing internal models)
- GIT (reflection as a state of an entity caused by external influence)

Sāṃkhya elaborates this dualism into a detailed cosmological hierarchy of twenty-five tattvas, or principles, that unfold from Prakriti once it comes into proximity with Purusha: from Prakriti's undifferentiated state first emerges buddhi (intellect or discernment), then ahaṃkāra (the sense of individual selfhood), then manas (mind) together with the ten sense- and action-faculties, and finally the five subtle and five gross elements (Larson, 1969). Prakriti's own activity is in turn governed by the interplay of three constituent strands or qualities (guṇas): sattva (clarity, lightness, luminosity), rajas (activity, passion, restlessness), and tamas (inertia, dullness, obscuration); every state of the phenomenal world, including every particular reflection that arises within it, is a specific proportion of these three strands, so that the qualitative character of a reflection is, on this view, always analyzable into how much clarity, activity, and inertia its substrate currently embodies — a strikingly concrete precursor to the general principle that the internal structure of a receiving entity determines the character of the reflections it can produce. Sāṃkhya's own preferred image for the relation between Purusha and Prakriti is that of a lame man of clear sight riding on the shoulders of a blind but able-bodied dancer: Prakriti performs, indeed performs endlessly and skillfully, but only for the sake of being witnessed by a Purusha that itself never acts and is never modified by what it witnesses — a dualism between an unchanging witness and a ceaselessly transforming, reflection-generating substrate that the closely related Yoga school of Patanjali adopts as the metaphysical basis for its own program of cognitive stilling.

2.3. Reflection in Islamic and Late Antique Thought

Islamic philosophy, Neoplatonism, Hermeticism, and Jewish mysticism developed rich and nuanced theories of reflection that further illuminate the structural features of the concept. These traditions often use metaphors of light, mirrors, emanation, and the heart to describe how higher realities become present in lower ones.

2.3.1. Neoplatonism: Emanation and Diminished Reflection

Neoplatonism, especially in Plotinus, presents one of the most entityatic ancient theories of reflection. In the *Enneads*, Plotinus describes reality as a hierarchy of emanations:

1. The One
2. Intellect (Nous)
3. Soul
4. The material world

Each level is a **reflection** of the one above it, but in a diminished, fragmented, or “shadowed” form (Plotinus, 1966). The material world is thus a faint reflection of the intelligible realm.

Neoplatonism contributes three structural insights:

- **Reflection is hierarchical**
- **Reflection involves loss or transformation of structure**
- **Reflection is the mechanism by which unity becomes multiplicity**

These ideas resonate strongly with the hierarchical structure of Refl and its subcategories.

Plotinus is careful to insist that emanation (proodos) is not a diminishment of the source itself, only of what the source's power looks like once it has been received by something of a lower ontological rank: the One loses nothing by the Intellect's proceeding from it, just as a light source loses nothing by illuminating objects at increasing distances, even though those objects receive its light with progressively less intensity (Plotinus, 1966). This asymmetry, in which the source is entirely unaffected by producing a reflection while the receiver is thoroughly constituted by receiving one, is already close to the formal asymmetry built into a morphism $f: R_1 \rightarrow R_2$, which records a change in the target without requiring, or even permitting, any corresponding change in the source. Plotinus further holds that every level, having received existence from what precedes it, spontaneously turns back toward its source in an act of contemplation or return (epistrophe), and that it is precisely this act of contemplative reflection which stabilizes each level into a determinate, well-

ordered reality, rather than leaving it as inchoate potency (Plotinus, 1966). The double movement of procession outward and return inward gives Neoplatonism a distinctly dynamic, rather than static, picture of reflective hierarchy — one in which each level actively completes itself by reflecting back upon what produced it, foreshadowing the two-directional emergence and reduction functors, F_i and G_i , that connect adjacent levels in the category Real of Markov (2024).

2.3.2. Sufism: The Mirror of the Heart

Sufi philosophy, particularly in Ibn ‘Arabi and Rumi, uses the metaphor of the heart as a mirror that reflects divine attributes.

Ibn ‘Arabi writes:

“The heart is a polished mirror reflecting the forms of the world.”
(Ibn ‘Arabi, 1980, p. 112)

Rumi similarly writes in the *Masnawi* that the heart must be polished to reflect truth (Rumi, 2004).

Sufism emphasizes:

- **the dependence of reflection on inner purification**
- **the multiplicity of reflections of the divine**
- **the non-identity between reflection and source**

These ideas align with the GIT principle that the quality of reflection depends on the internal structure of the entity.

This mirror-imagery is not incidental to Sufi metaphysics but is tied to Ibn ‘Arabi’s central doctrine of the Unity of Being (*waḥdat al-wujūd*): since only God possesses being in the fullest sense, every created thing exists only as a locus of manifestation (*maẓhar*) for one or another of the divine names and attributes — the world, on this view, is nothing but an immense, differentiated field of reflections of a single reality, no two loci reflecting the same combination of attributes in quite the same way (Ibn ‘Arabi, 1980). The perfected human being (*al-insān al-kāmil*) occupies a privileged place in this scheme as the one locus capable, at least in principle, of reflecting the divine names in their totality rather

than only partially, functioning as a kind of comprehensive mirror in which all the more partial and fragmentary reflections found elsewhere in creation are, in some sense, jointly gathered. Rumi's Masnavi develops the same imagery in a more devotional, less systematic register, repeatedly urging the disciple to polish the heart's mirror through spiritual discipline so that it may receive the divine light without the distorting rust of egotism and worldly attachment (Rumi, 2004) — reinforcing, from within an entirely different tradition, Daoism's structurally identical insistence (Section 2.2.2) that the fidelity of a reflection is inseparable from the tranquility and purity of the substrate that receives it.

2.3.3. Hermeticism: As Above, So Below

The *Corpus Hermeticum* describes the cosmos as a reflection of the divine order:

“As above, so below.” (Copenhaver, 1992)

This is a metaphysical statement of structural correspondence: lower levels reflect higher ones. It anticipates the idea of **functorial relations** between levels of reflection.

The maxim comes from the opening of the Emerald Tablet, a short text attributed to the legendary Hermes Trismegistus that circulated widely in both Arabic and Latin alchemical and philosophical circles; its fuller context makes clear that the correspondence between above and below is offered as the very mechanism by which the miracles of the one thing are accomplished, that is, as a general law governing how structure is transmitted from higher to lower orders of reality (Copenhaver, 1992). A companion text in the same corpus, the *Poimandres*, develops this into a more elaborate cosmogony in which the created world comes into being as a reflection of a prior, purely intelligible order, and the human intellect (*nous*), itself made in the image of this order, is held to be capable of ascending back through the planetary spheres by progressively shedding the reflective, image-bound faculties proper to each sphere. Hermetic ideas underwent a significant revival in Renaissance Florence through Marsilio Ficino's translations for the Medici and Pico della Mirandola's subsequent synthesis of Hermeticism with Kabbalah and Christian theology, both of whom

took the correspondence doctrine as license for treating the human being as a microcosm, a small-scale mirror of the entire cosmic order, capable of reflecting and thereby coming to know the whole of reality within itself.

2.3.4. Kabbalah: Sefirot as Reflective Emanations

In Jewish mysticism, the Sefirot are reflective emanations of the Ein Sof (Scholem, 1974). Each Sefirah reflects divine attributes in a specific mode, forming a structured network of reflections.

Kabbalah contributes:

- **a network model of reflection**
- **structured transformations between levels**
- **a dynamic interplay of reflection and concealment**

These ideas foreshadow the categorical structures developed later in this work.

Lurianic Kabbalah, developed by Isaac Luria in sixteenth-century Safed, adds a cosmogonic narrative that gives this network of reflective emanations a dramatic, dynamic history. Prior to creation, Luria taught, the infinite (Ein Sof) first contracted itself (tzimtzum) to leave a conceptual empty space in which a finite world could subsequently emerge; divine light was then channeled into that space through a series of vessels (kelim) corresponding to the ten Sefirot, but the vessels proved too fragile to contain the light of the higher Sefirot and shattered (shevirat ha-kelim), scattering sparks of divine light throughout the resulting material creation, intermixed with broken shells (kelipot) of concealment (Scholem, 1974). Human religious and ethical action is understood, on this scheme, as a work of restoration (tikkun) that gathers the scattered sparks and repairs the vessels, gradually restoring the network of reflective emanation to full functioning. Read structurally rather than theologically, the Lurianic myth offers a rich early vocabulary for exactly the kind of problem addressed formally in Section 4.7 below: what happens when a family of reflections, meant to jointly transmit and represent a single source, becomes damaged or incomplete, and what operations could, in principle, restore or reconstruct a coherent collective reflection from its scattered and partial remains.

2.4. Reflection in Idealist and Materialist Philosophies

The concept of reflection played a central role in the philosophical debates of the 19th and 20th centuries, particularly between idealist and materialist schools. These debates shaped the intellectual environment in which modern theories of information, cognition, and representation developed. Although both traditions used the term “reflection,” they interpreted it in fundamentally different ways. Understanding these differences is essential for constructing a **neutral, formal theory of reflections** that transcends ideological constraints.

2.4.1. Idealist Interpretations of Reflection

In classical idealism, reflection is primarily a **mental** or **spiritual** process. Reality is understood as fundamentally mind-dependent, and reflection is the mechanism through which the mind articulates, structures, or even generates the world of appearances.

In **German Idealism**, especially in Fichte, Schelling, and Hegel, reflection is tied to the activity of the Absolute Spirit or the transcendental ego. For Hegel, reflection (*Reflexion*) is a dialectical process through which the Absolute comes to know itself by externalizing and reintegrating its own determinations (Hegel, 2010). Reflection is thus:

- **self-referential**
- **dynamic**
- **constitutive of reality itself**

From this perspective, the world is not something that exists independently and is then reflected in the mind; rather, the world is the unfolding of reflective processes within Spirit.

Phenomenological Idealism, especially in Husserl, also treats reflection as a mental act, though in a more restrained sense. Reflection is the turning of consciousness upon its own acts to reveal their intentional structure (Husserl, 1970). Here reflection is:

- **a method of analysis**
- **a way of revealing meaning**

- **not a copy of external reality, but a clarification of experience**

Idealist interpretations often conflate:

- the **reflection**
- the **reflecting subject**
- the **reflected content**

This makes it difficult to extract a neutral, structural definition of reflection applicable to physical, biological, or computational entities.

Hegel's version of this dialectic deserves special attention because it gives reflection an explicitly relational, indeed inter-subjective, structure that the other idealist positions surveyed here largely lack. In the *Phenomenology of Spirit*, self-consciousness first arises only through being recognized by another self-consciousness, in the famous dialectic of lord and bondsman: I cannot know myself as a free, self-determining subject except by finding myself reflected in, and acknowledged by, another subject who is themselves capable of such recognition (Hegel, 2010). Reflection, for Hegel, is thus never a private, one-way relation between a knower and a passive object; it is constitutively reciprocal, since even self-knowledge is mediated by being reflected back through another. This inter-subjective structure of recognition (*Anerkennung*) is a notable historical anticipation of the later, formal treatment of collective reflection (Section 4.7 below), in which a family of mutually comparing reflections, rather than any single reflection taken in isolation, is what supports the richest and most stable forms of self-understanding, whether for an individual mind or for a community.

Fichte and Schelling, Hegel's immediate predecessors, push idealist reflection in two importantly different directions that are worth distinguishing.

Fichte's *Wissenschaftslehre* grounds his entire system in the self-positing activity of the I (*Ich*), an act in which the subject constitutes itself as subject precisely by reflecting upon, and thereby distinguishing itself from, a not-I that it simultaneously posits as external to itself; reflection, for Fichte, is in this sense productive and world-constituting from the very first step, not a response to an independently given world.

Schelling, by contrast, develops a philosophy of nature (Naturphilosophie) in which nature itself is already an unconscious, proto-reflective activity, striving through successive stages of organization toward the self-consciousness it finally achieves in the human mind — a picture that treats reflection as continuous across physical, organic, and mental levels of reality rather than as a uniquely mental achievement, and that accordingly sits closer than Fichte's or Hegel's positions to the level-spanning, non-anthropocentric ambitions of the present work, even though it remains couched throughout in an idiom of striving Spirit rather than of structure-preserving mathematical relations.

2.4.2. Materialist Interpretations of Reflection

Materialist philosophy, especially in its Marxist-Leninist form, treats reflection as a **property of matter**. According to this view, all material entities reflect external influences, and human consciousness is a highly developed form of this universal capacity.

In **dialectical materialism**, reflection is defined as:

“the capacity of matter to reproduce the properties of other objects in the process of interaction.”

This definition is close to the GIT definition, but it is embedded in a metaphysical framework that asserts:

- matter is primary
- consciousness is secondary
- reflection is a universal property of matter
- higher forms of reflection emerge from lower ones

This hierarchical view anticipates the idea of reflection levels (physical, chemical, biological, mental), but it is constrained by ideological commitments.

Todor Pavlov (1890–1977) is one of the most influential authors in the Marxist-Leninist tradition on the theory of reflection. His works attempted to entityatize and expand the materialist concept of reflection, especially in the context of epistemology and psychology (Pavlov, 1936, 1985).

Pavlov argued that:

- reflection is the fundamental mechanism of cognition
- sensory perception is a direct reflection of external reality
- thought is a higher, mediated form of reflection
- truth is the adequacy of reflection to reality

He emphasized the **objectivity** of reflection and rejected any form of idealism or constructivism.

While Pavlov's work is historically important, it suffers from several limitations:

- **Overemphasis on direct correspondence:** Pavlov assumes that reflection is a *copy* of reality, which contradicts modern cognitive science, phenomenology, and information theory.
- **Neglect of internal structure:** He underestimates the role of the reflecting entity's internal organization in shaping the reflection.
- **Ideological rigidity:** His framework is constrained by Marxist-Leninist doctrine, limiting its applicability to non-human entities or abstract structures.
- **Lack of formalization:** Pavlov's theory is conceptual and descriptive, lacking mathematical precision.

Pavlov's theory is a historically significant attempt to articulate reflection as a universal mechanism, but it is:

- philosophically incomplete
- scientifically outdated
- conceptually rigid
- incompatible with modern interdisciplinary research

Our Formal Theory of Reflections respects the ambition of universality but avoids the ideological constraints and conceptual simplifications of Pavlov's approach.

The dialectical-materialist theory of reflection did not originate with Pavlov but traces back to Friedrich Engels' unfinished *Dialectics of Nature* (Engels, 1940), which argued that motion, sensation, and eventually thought are successively higher forms of a single capacity already latent in the mutual interaction of all

matter, and, still more directly, to Lenin's Materialism and Empirio-Criticism (Lenin, 1972), which insisted against the neo-Kantian and empirio-critical currents of early twentieth-century physics that sensation must be understood as a subjectively experienced reflection (*otrazhenie*) of an objectively existing external world, on pain of collapsing into idealism or agnosticism about the reality of that world. Pavlov's own contribution was to take this loosely programmatic epistemological claim and attempt to develop it into something closer to a general theory, extending the notion of reflection downward to inorganic matter (where it appears merely as the capacity of anybody to bear the marks of its interactions with other bodies, such as an imprint left in sand) and upward through increasingly active and selective forms in living organisms, culminating in human conceptual thought as reflection's most developed, socially mediated form.

It is worth noting, in fairness to this tradition, that its core intuition, that even the simplest physical interaction leaves some trace or modification behind, and that biological and cognitive reflection are, in some sense, elaborations of this same basic capacity rather than phenomena of an entirely different order, is one that the present work also accepts and builds upon; the disagreement is not with dialectical materialism's aspiration to universality but with the particular metaphysical apparatus, the doctrine that matter is ontologically primary and consciousness strictly secondary and derivative, in which that aspiration was embedded, and which is neither required by, nor even especially natural for, a purely structural and relational account of what a reflection is. The philosopher Karl Popper's sustained critique of dialectical materialism as an unfalsifiable, all-explaining framework, developed across several decades beginning with his *The Open Society and Its Enemies* (Popper, 1945), is a further reason the present work seeks a formalism whose claims are precise and mathematically checkable rather than framed at a level of generality that admits of no clear disconfirmation.

2.4.4. Beyond Idealism and Materialism: The Need for a Neutral Framework

Both idealist and materialist interpretations capture important aspects of reflection, but both are limited:

- Idealism overemphasizes the role of mind and meaning.
- Materialism overemphasizes direct correspondence and physical causality.

A **formal, structural theory** must:

- avoid metaphysical commitments
- treat reflection as a relational mechanism
- apply equally to physical, biological, cognitive, and social entities
- allow for multiple, non-identical reflections
- support integration and comparison of reflections
- be mathematically precise

This is not a call for a doctrine positioned midway between idealism and materialism, a synthesis that would still owe its terms to the two positions it mediates, but for a *change of question*. Both traditions surveyed above ask what reflection ultimately is, mental activity or material process, and take a stand on the answer before specifying reflection's structure; a formal theory instead asks only *what structure any reflection*, whatever its ultimate metaphysical status, must exhibit in order to be recognized as a reflection at all, and answers that question using the machinery of category theory precisely because that machinery is indifferent to what its objects and morphisms are ultimately made of, requiring only that they compose associatively and admit identities. This is why Chapter 3 below motivates category theory specifically, rather than some other formal apparatus, as the appropriate language for the Formal Theory of Reflections: it is the one mathematical framework expressly designed to describe relational structure while remaining silent on the intrinsic nature of what stands in those relations, delivering exactly the neutrality this section has argued idealism and materialism alike fail to provide.

2.5. Reflection in Modern Science

Modern science provides a rich empirical foundation for understanding reflection as a universal mechanism. While ancient traditions used metaphors of mirrors, water, and light, contemporary research in cognitive science, neuroscience, and information theory offers precise models of how entities encode, transform, and integrate influences from their environment. These models align remarkably well with the structural principles of reflection identified in earlier philosophical traditions.

2.5.1. Cognitive Science: Multisensory Integration

One of the most direct scientific analogues to reflection is **multisensory integration**, the process by which the brain combines signals from different sensory modalities into coherent percepts. Research shows that neurons in the superior colliculus and association cortices respond to combinations of visual, auditory, and tactile stimuli, often in nonlinear ways (Stein & Stanford, 2008).

This demonstrates several key principles:

- **Multiple reflections coexist** within a single entity.
- **Reflections interact** to produce higher-order representations.
- **The structure of the entity determines how reflections are combined.**

These findings support the GIT view that reflections are not isolated events but components of a dynamic, integrated entity.

The nonlinear character of this integration is itself theoretically significant. Stein and Stanford (2008) describe two robust empirical principles governing how superior-colliculus neurons combine signals from different modalities: the principle of inverse effectiveness, whereby weak, individually near-threshold unimodal signals produce a proportionally larger combined response than strong ones do, and spatial and temporal congruence, whereby signals arriving from the same location at roughly the same time enhance one another while spatially or temporally discrepant signals can actively suppress the combined response below what either signal alone would produce. Well-known perceptual

illusions exploit precisely this machinery: in the McGurk effect, a visual mouth articulating one syllable dubbed with an audio recording of a different syllable causes listeners to perceive a third syllable that matches neither input in isolation, and in the ventriloquist effect, a synchronized but spatially displaced sound is perceptually relocated toward its accompanying visual source. Both illustrate, with considerable precision, a point central to the formal treatment of collective reflection in Section 4.7: combining several reflections of a single underlying event does not simply average or juxtapose them, it can produce a new, emergent reflection with properties not straightforwardly present in any of the contributing reflections taken alone.

2.5.2. Predictive Processing: Reflection as Inference

Predictive processing theories propose that the brain is fundamentally a prediction machine. According to Friston (2010), perception arises from the brain's attempt to minimize prediction error between internal generative models and incoming sensory signals.

In this framework:

- sensory input is a **reflection** of external causes
- internal models generate **predicted reflections**
- the brain continuously updates itself to reduce discrepancies

This aligns with the idea that reflection is not a passive copy but an **active, inferential transformation**.

The framework has deep historical roots in Hermann von Helmholtz's nineteenth-century doctrine of unconscious inference, according to which perception is the end-product of rapid, unconscious inferential processes that combine current sensory data with prior expectations, and it has been extended in contemporary work into a full theory of active inference, in which organisms minimize prediction error not only by updating their internal models to fit incoming data but equally by acting on the world so as to bring sensory data into line with preferred, expected states (Friston, 2010). On this hierarchical picture, prediction errors computed at each level of a cortical hierarchy are passed upward as the signal requiring further explanation, while predictions

generated from accumulated higher-level models are passed back down to be compared against incoming input at each lower level in turn, with the relative influence of any given prediction error weighted by an estimate of its own reliability or precision. This architecture is a striking scientific counterpart to the multi-level hierarchy of reflection categories $\{\text{Refl}_0, \dots, \text{Refl}_5\}$ developed formally in Chapter 6, since it likewise depends on structured, bidirectional traffic (downward predictions functioning much like the emergence functors F_i , upward error signals like their reduction counterparts G_i (Markov, 2024)) between adjacent levels of an organized system, rather than on either level operating in isolation.

2.5.3. Embodied Cognition: Reflection Shaped by the Body

Embodied cognition emphasizes that perception and cognition are grounded in bodily states and sensorimotor capacities (Barsalou, 2008). The body acts as a **filter** and **transformer** of external influences, shaping the reflections that arise within the cognitive entity.

This supports the principle that:

- **the substrate determines the form of the reflection,**
- **reflections are constrained by physical and biological structure,**
- **cognition is inseparable from embodiment.**

The embodied-cognition program is closely allied with, and partly overlaps, the enactivist approach developed by Francisco Varela, Evan Thompson, and Eleanor Rosch (Varela, Thompson, & Rosch, 1991), which holds that cognition is not the passive recovery of a pre-given world but is enacted, brought forth, through the structural coupling of an organism's sensorimotor capacities with its environment across evolutionary and developmental time; on this view, there is no cognitively relevant world that exists independently of the specific ways a given kind of organism is capable of acting within it. A closely related proposal, the sensorimotor contingency theory of J. Kevin O'Regan and Alva Noë (O'Regan & Noë, 2001), argues that the qualitative character of perceptual experience itself, what it is like to see rather than merely to touch, for instance, consists in the perceiver's implicit, practical mastery of the lawful ways sensory

input changes as a consequence of the perceiver's own movements, rather than in any intrinsic property of a neural signal considered in isolation. Both proposals converge on a single structural moral directly relevant to the present framework: which reflections a given entity is even capable of producing is jointly determined by the entity's morphology and its repertoire of possible actions, not by the source alone, reinforcing yet again the recurring theme that a reflection $R = (S, X)$ depends essentially on the structure of the receiving entity.

2.5.4. Neuroscience: Neural Correlates of Reflection

Neuroscience provides detailed evidence that neural activity patterns encode reflections of external stimuli. For example:

- visual cortex encodes spatial and chromatic structure
- auditory cortex encodes frequency and temporal patterns
- somatosensory cortex encodes tactile and proprioceptive signals

Dehaene (2014) shows that conscious perception corresponds to large-scale, integrated neural reflections that propagate across cortical networks.

This supports the idea that reflection is:

- distributed,
- structured,
- dynamic,
- hierarchical.

Dehaene's (2014) global neuronal workspace theory adds an important further mechanism to this picture: a signal becomes consciously reportable, rather than merely locally processed, when it is amplified enough to trigger a sudden, self-sustaining wave of activity, an “ignition,” that broadcasts it across a distributed network of long-range cortical neurons, making it simultaneously available to many specialized processing systems (language, memory, planning, motor control) that would otherwise not have access to it. This gives a precise empirical content to the difference between a mere reflection and a reflection that has become information in the GIT sense of Section 2.6.4 below: countless neural reflections of sensory stimuli are generated at any moment, but only those that achieve global broadcast, and thereby become available for report,

memory encoding, and the guidance of deliberate action, cross the threshold into what an entity could recognize, in any robust sense, as its own information about the world. The workspace architecture also offers one empirically grounded answer to the binding problem, the question of how data encoded in anatomically separate, specialized neural populations comes to be experienced as a single, unified percept of one object, a question that is the neuroscientific counterpart of the formal question, addressed in Sections 4.6 and 4.7, of how a family of separate reflections of one source can be compared and integrated into a single collective reflection.

2.5.5. Information Theory: Reflection as Transformation

Shannon's (1948) information theory models communication as the transformation of signals through channels. In this sense, every received signal is a **reflection** of the transmitted one, modified by noise and channel properties.

This provides a mathematical analogue to the philosophical idea that:

- reflections are **not identical** to their sources
- reflections are **transformed** by the medium
- reflections can be **quantified** and **analyzed**

Information theory thus offers a formal foundation for understanding reflection as a structural transformation.

Shannon's framework, developed together with Warren Weaver in their joint monograph (Shannon & Weaver, 1949), deliberately brackets questions of meaning entirely, defining information in purely statistical terms as a reduction of uncertainty, measured in bits, about which message was selected from a set of possible messages, and quantifying a channel's capacity as the maximum rate at which such uncertainty can be reliably reduced in the presence of noise. Two further Shannon-theoretic notions are worth drawing out explicitly because they recur, in structural form, throughout the formal chapters that follow: entropy, which measures the average uncertainty, and hence the information-carrying potential, of a source before any message is received, and redundancy, the systematic, statistically predictable structure within a signal that allows a receiver to correct for a certain amount of noise or loss and still recover

the original message with high reliability. Redundancy, in particular, is the information-theoretic analogue of what Section 4.6 below calls a comparison morphism between independent reflections of a common source: it is precisely because two or more reflections of one signal share redundant structure that a receiver possessing several of them can cross-check, correct, and integrate them into an estimate more reliable than any single reflection could support on its own.

2.6. The General Information Theory Perspective

General Information Theory (GIT) provides a unified conceptual framework for understanding reflection across physical, biological, cognitive, and social entities. It defines reflection as **a structural and/or functional change in an entity caused by external influence** (Markov, Ivanova, Mitov, 1993, 2003). This definition is deliberately neutral and structural, avoiding metaphysical assumptions.

2.6.1. Reflection as Change Caused by External Influence

According to GIT, reflection is not a mental or perceptual phenomenon but a **universal relational mechanism**. Whenever one entity influences another, the affected entity undergoes a change. This change is the reflection.

This definition captures:

- physical reflections (light on a surface)
- chemical reflections (molecular interactions)
- biological reflections (cellular responses)
- neural reflections (sensory processing)
- mental reflections (perception, memory)
- social reflections (communication, imitation)

Reflection is thus a **general result from interaction**.

What makes this GIT definition genuinely unifying, rather than merely a family-resemblance label loosely applied across incommensurable domains, is that it specifies reflection by its logical form alone (an entity, a change, a caused-by

relation to a further entity) while leaving the physical, chemical, biological, or semantic realization of each term entirely open. This is precisely the sense in which GIT's definition is neutral among the traditions surveyed in this chapter: it neither privileges mind over matter, as idealism does, nor matter over mind, as dialectical materialism does, and it neither requires a substantial copy to pass between source and receiver, as naive realism assumes, nor denies the receiver any independent contribution to the resulting state, as the phenomenological and embodied-cognition literatures insist it must have. Every structural insight identified earlier in this chapter, Aristotle's form without matter, the Buddhist mirror that remains unstained by what it reflects, Kant's insistence that appearances are jointly shaped by an a priori receiver-structure, Shannon's transformation of a signal by a noisy channel, can be read as a domain-specific elaboration of this single, deliberately thin relational schema, which is exactly what licenses treating them all, in Chapter 4 onward, as instances of one and the same formal category Refl.

2.6.2. Reflection vs. Entity

A crucial principle of GIT is that **the reflection is not the entity in which it appears**. This aligns with:

- Plato's shadows
- Aristotle's form without matter
- Buddhist mirror metaphors
- Daoist water metaphors
- Kant's phenomena
- predictive processing models

The reflection is a **state** of the entity, not the entity itself.

Failing to observe this distinction is, arguably, the single most common source of confusion in loose or informal talk about reflection, since ordinary language happily uses the same word, “my reflection,” “the mirror's reflection,” for both the entity that does the reflecting (the mirror, the mind, the sensor) and the particular state that results (this image, this belief, this reading) at a given moment. GIT's insistence on keeping these apart is not pedantry but a load-

bearing structural commitment: it is precisely what allows a single entity to be described, without contradiction, as bearing many different reflections over time, or even several at once (Section 2.6.3), since it is always the state, not the entity as such, that is being multiplied or compared. Formally, this distinction reappears as the difference between an object $R = (S, X)$ of the category Refl , which names one particular reflection, and the receiving entity itself, which is not directly an object of Refl at all but is rather what is presupposed, and individuated across time, by the family of all its reflections — a point made fully explicit only once identity morphisms and the null reflection are introduced in Section 4.4.

2.6.3. Multiple Reflections in One Entity

GIT emphasizes that a single entity can contain **multiple reflections simultaneously**. This is supported by:

- multisensory integration (Stein & Stanford, 2008)
- Yogācāra’s eight consciousnesses (Anacker, 1984)
- Sufi and Daoist mirror metaphors
- neural population coding (Dehaene, 2014)

This principle is essential for understanding complex entities.

It is worth being explicit about why this plurality is not a mere logical possibility but a structural necessity for any entity of sufficient complexity. A entity with only ever one reflection at a time would have no internal means of comparing that reflection against anything else, and hence no basis for distinguishing a reliable reflection from a distorted one, or a reflection of an external event from a reflection of its own prior state; the very possibility of the comparison morphisms formalized in Section 4.6, and of the collective reflection built from them in Section 4.7, presupposes that a family of several, at least partially independent reflections is available to be compared and combined in the first place. Multiplicity of reflection is, in this sense, not an incidental richness that sophisticated entities happen to display in addition to reflecting, but close to a precondition for anything deserving to be called information, knowledge, or self-awareness at all, since each of these, as Sections 2.6.4 and 2.6.5 argue, is

constituted by relations that can only be defined across a plurality of reflections, never by any single reflection considered wholly on its own.

2.6.4. Reflection as the Basis of Information

Information arises when a reflection is recognized as corresponding to its source. This requires:

- structural similarity
- stable transformation rules
- interpretive mechanisms

GIT thus treats information as a **relation between reflection and source** (Markov, 1988; Markov, Ivanova, Mitov, 1993, 2003).

This is a substantive and, on reflection, a demanding requirement, since it means that the mere occurrence of a reflection never by itself constitutes information for the entity in which it occurs: countless physical and even neural reflections take place in any entity at any moment without ever being recognized, compared against expectation, or put to any further use, and GIT's position is that these remain mere reflections, not yet information, until some further interpretive or recognitional mechanism establishes the required correspondence. This dependency of information on a further, entity-internal act of recognition is what connects GIT's definition back to several of the traditions surveyed earlier in this chapter: it is structurally continuous with the Stoic distinction between an impression and the mind's further assent to it (Section 2.1.3), with the scholastic distinction between a species and the intellect's active abstraction of a concept from it (Section 2.1.4), and with the contemporary distinction, drawn independently in predictive-processing accounts (Section 2.5.2), between a raw prediction error and its interpretation as evidence bearing on one hypothesis rather than another. Chapter 6 below develops this recognitional relation formally as an interpretive functor from the category of raw reflections into a further category of information-bearing structures, making mathematically precise what this section states only descriptively.

2.6.5. Reflection as the Basis of Knowledge

Knowledge arises when reflections are:

- stabilized
- integrated
- generalized
- connected to action

This aligns with both phenomenology and cognitive science.

Each of these four conditions, stabilization, integration, generalization, and connection to action, corresponds to a specific further structural operation performed upon reflections that are already, in the sense of Section 2.6.4, recognized as information.

Stabilization requires a reflection, or a family of closely related reflections, to persist with enough consistency across time and further contact that it can serve as a reliable basis for expectation, rather than being overwritten or contradicted by the next contact that happens to occur.

Integration requires connecting a given piece of information to others already held by the entity, so that it participates in, and can be constrained or corrected by, a wider network of mutually supporting reflections rather than standing alone.

Generalization requires that information drawn from particular reflections come to apply, with appropriate qualification, beyond the exact particular circumstances of the contact that originally produced it.

And *connection* to action requires that the resulting knowledge be capable of actually informing the entity's own further behavior, rather than remaining, however stable and well integrated, entirely inert.

This four-fold structure gives an unusually precise, checklist-like criterion for when the honorific term “knowledge,” rather than merely “information” or “reflection,” is appropriately applied to a given cognitive state, and it is this same four-fold structure that Chapter 6's treatment of hierarchical functors and Chapter 7's treatment of temporal stability and evolution operators are designed to formalize, level by level and step by step, across the remainder of this work.

3. Formal Theory of Reflections

The preceding section has shown that the concept of reflection appears across cultures, epochs, and scientific disciplines. Despite this universality, the idea has remained theoretically fragmented. Philosophers have used it to describe perception, cognition, and metaphysics; mystics have used it to describe spiritual insight; scientists have used it to describe sensory processing and data transmission. Yet no unified, rigorous, and ideologically neutral framework has been developed to capture the structural essence of reflection.

A **Formal Theory of Reflections** is needed to bridge this gap. Such a theory must be capable of integrating insights from philosophy, cognitive science, information theory, and entities theory, while remaining mathematically precise and conceptually transparent. Category theory provides the ideal foundation for this endeavor.

3.1. Why Category Theory?

Category theory is uniquely suited to formalizing reflection because it focuses on **relations, transformations, and structure**, rather than on the internal composition of objects. This aligns perfectly with the nature of reflection, which is fundamentally relational: a reflection exists only as a relation between a source and a receiver.

Before turning to the formal apparatus, it may help to make this intuition concrete. Consider a simple mirror: the mirror itself, as a physical object, is uninteresting to us here — what matters is the *relation* between whatever stands in front of it and the image that appears. Category theory offers exactly this kind of vocabulary: it does not ask what a thing is made of, only how it relates to other things through directed arrows. This is precisely the perspective reflection requires. A shadow, a memory, a perception, or a chemical response are very different "things" in themselves, but as reflections they all share the same relational shape: something (a source) stands in a directed relation (a contact) to something else (a receiver's changed state). Category theory lets us describe this shared shape once, and then apply it uniformly across physics, biology, cognition, and society — without having to first settle unresolved

metaphysical questions about what shadows, memories, or perceptions "really are." Readers already familiar with categories, objects, and morphisms from Markov (2024) will recognize this as the same structural attitude applied there to the hierarchy of reality; here it is applied instead to the phenomenon of reflection itself.

Unlike set theory, which emphasizes elements and membership, category theory emphasizes:

- **objects** as abstract entities defined by their relations
- **morphisms** as structured transformations
- **composition** as the chaining of interactions
- **identity morphisms** as structural persistence
- **functors** as mappings between entire entities of relations

A single example can make these five notions concrete. Suppose sunlight falls on a lake, producing a shimmering reflection on its surface; a passer-by's eye then registers that reflection, producing a pattern of retinal and neural activity. Here the sunlit lake-surface and the neural pattern it eventually gives rise to be two **objects** — two different reflections. The transformation from lake-surface to retinal image is a **morphism**: it captures a directed influence, namely light traveling from the water to the eye. Composing this morphism with the further transformation from retinal image to neural pattern yields a single morphism relating the original sunlit scene directly to the neural response — this is **composition**, the chaining of interactions into one. Meanwhile, for as long as nothing further disturbs it, each of these reflections persists as itself from one instant to the next; this stability is exactly what an **identity morphism** formalizes. Finally, if instead of this one lake-to-eye chain we consider the general rule that maps every physical light pattern to its corresponding neural pattern, that rule is a **functor**: it is not a single reflection but a structure-preserving translation between an entire domain of physical reflections and an entire domain of neural reflections.

This makes category theory a natural language for describing reflection as a universal mechanism. It allows us to model:

- how reflections arise

- how they propagate
- how they interact
- how they transform across levels
- how they compose into higher-order structures

In short, category theory provides the structural clarity needed to unify the diverse manifestations of reflection.

Readers already familiar with the categorical apparatus from Markov (2024) — where it served to describe the hierarchy of reality itself — will recognize the same structural attitude at work here, now turned toward reflection as a phenomenon in its own right. The next sections make this apparatus explicit, beginning with reflection itself as the fundamental object of study.

3.2. Reflection as Categorical Object

In the category **Refl**, the fundamental units are **reflections themselves**. This is a conceptual shift from traditional metaphysics, where reflections are treated as secondary or derivative phenomena. In **Refl**, reflections are **first-class citizens**: they are the objects of the category.

This may seem like a subtle shift, but it has real consequences. In everyday thinking, we tend to say "the lake reflects the sky" — treating the lake and the sky as the real things, and the reflection as a mere by-product, not worth naming on its own. **Refl** inverts this emphasis: it is the reflection itself — the shimmering pattern on the water, the pattern of light on the retina, the belief formed in someone's mind — that becomes the basic unit of analysis, on equal footing with any other object one might study. The lake, the sky, and the person are still present, but only implicitly, as the source and receiver referenced by the reflection. This inversion is what allows the same category to describe a shadow on a cave wall, a neuron's firing pattern, and a rumor spreading through a village, without first deciding what any of these things "ultimately" are.

This approach has several advantages:

1. **It avoids metaphysical commitments.** We do not need to assume what the underlying entities “really are.” We only need to describe how reflections arise and relate.
2. **It captures the universality of reflection.** Whether the reflection is physical (light), chemical (reaction), neural (activation), or mental (perception), it can be treated uniformly.
3. **It allows for hierarchical modeling.** Reflections at different levels (physical, biological, cognitive, social) can be represented as objects in subcategories of Refl.

For instance, a chemical reaction, a neural spike, and a spoken rumor are, on the surface, nothing alike. But once each is recognized as a reflection — a change produced in a receiver by contact with a source — all three become objects of the same kind, differing only in which subcategory of Refl they belong to (chemical, neural, or social). This is what makes the framework genuinely interdisciplinary rather than a metaphor stretched across disciplines.

By treating reflections as objects of Refl, we gain a flexible and powerful framework for modeling complex entities.

3.3. Contact as Categorical Morphism

A reflection does not arise spontaneously; it is produced by **contact** — a directed influence from one entity to another. In Refl, these contacts are represented as **morphisms**.

It helps to keep the lake example running here. The sunlight itself is not a contact — it is a source. Nor is the shimmering pattern on the water a contact — that is the reflection, the object. The *contact* is the act of light traveling from sun to water and being altered by the water's surface: ripples, angle, and turbidity all leave their mark on how the light is transformed before it reaches an observer's eye. A contact, in other words, is not a thing but an event of transmission-with-transformation — which is exactly what an arrow between two objects represents in category theory.

A morphism captures:

- the direction of influence
- the structural transformation imposed by the source
- the conditions under which the reflection arises
- the constraints imposed by the receiving entity

Note that a morphism carries more data than the simple fact that "A affected B." It encodes *how* — the ripples on the lake bend and scatter the light differently than a still pond would, so the same sunlight produces a different reflection depending on the water's condition. This is why, in Refl, morphisms are the natural place to locate the mechanism of influence, while the objects on either end simply record the states that resulted.

This aligns with the philosophical and scientific insights discussed earlier:

- Plato's shadows depend on the geometry of the cave
- Aristotle's perception depends on the form of the object
- Buddhist reflections depend on mental conditions
- Daoist reflections depend on the tranquility of the mind
- neural reflections depend on sensory pathways
- information-theoretic reflections depend on channel properties

Morphisms thus encode the **mechanism** by which reflections are generated.

With objects standing for reflections and morphisms standing for the contacts that produce them, Refl already has enough structure to ask its next natural question: what happens when contacts chain together, one influence feeding into another?

3.4. Interaction as Categorical Composition

Real entities rarely involve isolated influences. Instead, reflections propagate through chains of interactions. Category theory captures this through **composition of morphisms**.

Returning to the lake: the chain does not stop at the water's surface. Sunlight reflects off the lake (contact 1), that reflected light then strikes an observer's retina (contact 2), and the retinal signal is transformed into a neural pattern in

the visual cortex (contact 3). Each of these is a separate morphism, with its own source and target. But an observer standing on the shore does not experience three separate events — she simply sees the lake. Composition is what allows Refl to capture this: the three morphisms chain into a single morphism running directly from "sunlight" to "neural pattern," even though the intermediate steps (water, retina) are what actually made it possible.

If entity A influences B, and B influences C, then A indirectly influences C. The reflection in C is therefore a **composed transitive reflection**, shaped by the intermediate transformations.

This is also why reflections can carry traces of things that are, strictly speaking, several steps removed from the observer. A neural pattern is never in direct contact with the sun — it is only ever in contact with a retinal signal. Yet because morphisms compose, the neural pattern is still, in a precise mathematical sense, a (highly transformed) reflection of the sunlight itself. This matches an intuition present already in Aristotle and in modern neuroscience alike: perception is several steps removed from its worldly cause, but it is not thereby disconnected from it.

This principle is essential for modeling:

- multisensory integration
- neural processing pipelines
- cognitive inference
- social communication
- hierarchical biological regulation

Composition allows us to represent complex chains of reflection in a mathematically precise way.

Composition explains how influence propagates outward through a chain of contacts. But every entity in that chain — the lake, the eye, and the brain — also has to do something else: it has to remain itself between one contact and the next, or there would be no stable thing for a new influence to act upon. As the next section shows, this is not a phenomenon outside reflection but reflection's own zero-magnitude case.

3.5. Identity as the Null Reflection

Chapter 1 already anticipated a special case of reflection: contact that occurs but produces no change, the **null reflection**. Every entity has this in the most minimal way, it remains itself unless acted upon, and in category theory this is represented by **identity morphisms**.

Consider the lake once more, this time on a windless night with no observer present. Nothing is reflecting anything else — no sunlight, no eye. Yet the lake does not dissolve into nothing between one moment and the next; it remains a lake, ready to reflect whatever light next falls on it. This "doing nothing but persisting" is not really nothing at all — it is the null reflection an entity has with itself, and it is what an identity morphism formalizes. Without it, Refl would have no way to say that the lake at this moment and the lake a second later are, in fact, the same entity.

An identity morphism captures:

- the entity's internal stability
- its capacity to preserve its own structure
- its role as a substrate for future reflections

This is a modest-sounding requirement, but it carries real weight. If an entity could not maintain itself between contacts, it could never accumulate the layered, transitive reflections discussed in Chapter 1 — each new influence would land on a target that no longer existed. The null reflection, in this sense, is not opposed to change; it is the precondition for change to be *registered* at all, the way a photographic plate must first exist, stable and unexposed, before it can record an image.

This aligns with philosophical notions of:

- Aristotle's substance
- Buddhist continuity of consciousness
- Sāṃkhya's Purusha as witness
- Kant's unity of apperception
- modern notions of homeostasis and self-maintenance

It is worth stressing that this is a much thinner notion than the philosophically loaded idea of an unchanging "essence" or "soul." Refl does not commit to *why* an entity persists, nor to what, metaphysically, does the persisting — only that, formally, there is always an arrow from the entity to itself, alongside whatever transformative arrows a given contact might also produce.

The null reflection is thus available to every entity capable of receiving reflections at all — which is to say, every entity whatsoever, regardless of complexity.

It is tempting to read this as already covering self-reflection, but Chapter 1 draws a real distinction that identity does not capture. Self-reflection, whether *direct* (as when an entity is affected by contact with one of its own parts, the way watching one's own hand builds a picture of oneself) or *transitive* (as when a bodily signal reaches the brain only after being relayed by a sensor), requires a nonzero, structurally meaningful change — not merely the null reflection that id_R represents. The two are not opposed: self-reflection could not occur unless an entity first persisted long enough, via the null reflection, to have any state left to register. But they remain formally distinct — id_R is reflection's zero case, self-reflection is its nonzero case — and Section 4.4 makes the distinction precise.

With objects, contacts, composition, and identity now in place, Refl already has the shape of a category. What remains is to state this explicitly and prepare for the formal definitions of Chapter 4.

3.6. Summary and Transition to Formalization

Reflection is a universal mechanism that appears across philosophy, religion, cognitive science, and information theory. Despite its ubiquity, it has lacked a unified, formal, and ideologically neutral framework. The General Information Theory provides a conceptual foundation by defining reflection as a change caused by external influence. Category theory extends this foundation by offering a precise structural language for modeling reflections, contacts, interactions, and self-maintenance.

The lake, the ripple, the eye, and the neural pattern used throughout this chapter were not meant as a special case — they were meant to show, in

miniature, the shape that any reflection has, whatever its domain. Chapter 4 now replaces these intuitive pictures with exact definitions, so that the same structure can be stated once, precisely, and then applied without ambiguity to physical, biological, cognitive, and social entities alike.

The next chapter will introduce the **formal definition of the category Re_{fl}**, its objects, morphisms, subcategories, and structural properties. This will allow us to move from philosophical analysis to mathematical rigor, completing the transition from conceptual insight to formal theory.

4. The Formal Framework

The philosophical analysis in Chapter 2 showed that reflection is a universal mechanism through which one entity becomes present in another without identity. To transform this insight into a scientific theory, we require a formal mathematical framework capable of representing:

- reflections as structured states
- influences as directed relations
- interactions as compositions
- identity as the null reflection, and self-reflection (direct or transitive) as its nonzero counterpart
- multiple reflections of the same source
- comparison and integration of reflections
- hierarchical levels of reflection

Category theory provides exactly such a framework.

Before the definitions begin, it is worth restating in one place the picture built up across Chapter 3, since Chapter 4 will now render it in symbols. A reflection is a state produced in one entity by contact with another — the shimmer on the lake, the pattern on the retina, the belief formed in someone's mind. A contact is the directed act of influence that produces such a state. Chains of contact compose into longer chains, and every entity, left undisturbed, remains itself via the null reflection applied to itself. Nothing in what follows introduces a new idea; it only gives these five notions — reflection, contact, composition, the null reflection,

and self-reflection — exact mathematical form, so that they can be reasoned about with the same rigor as any other mathematical structure.

In this chapter, we introduce the category **Refl**, whose objects are reflections and whose morphisms are contacts (direct or transitive influences).

4.1. Objects of Refl: Reflections

4.1.1. Definition

An object in Refl is a reflection, written as:

$$\mathbf{R} \in \text{Obj}(\text{Refl})$$

A reflection is a pair:

$$\mathbf{R} = (\mathbf{S}, \mathbf{X})$$

where:

- **S** is the source entity (the entity being reflected)
- **X** is the state of the receiving entity produced by the influence of S

In the lake example, S would be the sunlit sky, and X would be the particular shimmering pattern that appears on the water's surface at a given moment — not the sky itself, and not the lake itself, but the specific outcome their contact produced. Writing $\mathbf{R} = (\mathbf{S}, \mathbf{X})$ simply gives this everyday pairing — "a reflection of something, appearing as some particular state" — a fixed notation that can be reused across every domain the theory touches.

Thus, a reflection is not the source itself, but a structured result of interaction.

4.1.2. Multiple reflections of the same source

If several entities reflect the same source S, we obtain a family:

$$R_1 = (S, X_1)$$

$$R_2 = (S, X_2)$$

...

$$R_n = (S, X_n)$$

These reflections are generally not identical because the receiving entities differ.

This is the formal basis for the problem studied in Markov & Ivanova (2018): how to compare and integrate multiple reflections of the same source.

A familiar case of this is a solar eclipse observed simultaneously through a pinhole projection, through eclipse glasses, and by an orbiting satellite. All three are reflections of the same source — the eclipse itself — yet X_1 , X_2 , and X_3 differ: a fuzzy inverted disk on a sheet of paper, a filtered direct view, and a satellite image are not the same state, even though they share a source. Refl does not require these to agree; it only requires that each be correctly recorded as a reflection of that one source.

4.2. Morphisms of Refl: Contacts

4.2.1. Definition

Morphism in Refl is a contact:

$$f : R_1 \rightarrow R_2$$

This represents a directed influence from reflection R_1 to reflection R_2 .

Formally, morphism is a mapping:

$$f : (S_1, X_1) \rightarrow (S_2, X_2)$$

that transforms X_1 into X_2 .

Returning once more to the eclipse example: if X_1 is the fuzzy disk projected through a pinhole, and someone photographs that projection, producing a new reflection X_2 (a photograph of the projection), then the act of photographing is a morphism $f : (S_1, X_1) \rightarrow (S_2, X_2)$. Notice that the source has quietly shifted too

— strictly, S_2 is now "the pinhole projection," not "the eclipse" — which is exactly why composing morphisms (Section 4.3) matters: it is what lets us recover the longer chain back to the original eclipse, several contacts removed.

4.2.2. Interpretation

Morphism may represent:

- physical influence
- chemical interaction
- biological signaling
- neural activation
- cognitive transformation
- social communication

Thus, Refl is a multi-level category.

What unifies these otherwise very different examples is not their physical substance but their shape: in every case, something with a definite prior state comes into contact with something else, and a new, dependent state results. A morphism in Refl does not care whether that "something" is a photon, a hormone, or a spoken sentence — it only records that a transformation of this directed kind took place.

4.3. Composition of Morphisms: Interaction

4.3.1. Definition

If:

$$\begin{aligned} \mathbf{f} &: \mathbf{R}_1 \rightarrow \mathbf{R}_2 \\ \mathbf{g} &: \mathbf{R}_2 \rightarrow \mathbf{R}_3 \end{aligned}$$

then their composition is:

$$\mathbf{g} \circ \mathbf{f} : \mathbf{R}_1 \rightarrow \mathbf{R}_3$$

This represents a chain of influences.

Continuing the eclipse chain from 4.2: if f is the morphism from the eclipse to the pinhole projection, and g is the morphism from that projection to the photograph someone takes of it, then $g \circ f$ is a single morphism running directly from the eclipse to the photograph. This composed morphism does not mention the pinhole projection at all — it simply records that a photograph exists which is, several steps removed, a reflection of the eclipse. The intermediate step is not lost; it is just no longer visible once the two contacts are chained into one.

4.3.2. Associativity

Composition satisfies:

$$h \circ (g \circ f) = (h \circ g) \circ f$$

This ensures consistency in modeling interactions.

In practice, associativity guarantees something reassuring: it does not matter whether we first *think* of "eclipse $\rightarrow$ projection, then combined with projection $\rightarrow$ photograph $\rightarrow$ viewer," or "eclipse $\rightarrow$ projection $\rightarrow$ photograph, then combined with photograph $\rightarrow$ viewer" — both groupings describe the exact same overall chain of influence, from eclipse to viewer. Without this guarantee, the very notion of "the reflection of the eclipse, several contacts later" would depend arbitrarily on how one chose to group the steps, which would make long chains of reflection ill-defined.

4.4. Identity Morphisms as the Null Reflection, and Self-Reflection

Every reflection R has an identity morphism:

$$\text{id}_R : R \rightarrow R$$

This is the null reflection (Chapter 1) applied to R itself: a contact that leaves X unchanged.

Formally, this is the same null reflection discussed intuitively in Section 3.5 — the lake counting as a (trivial) reflection of itself between one moment and the

next — now simply written as an arrow from a reflection to itself. What the notation id_R adds is precision about how this zero-magnitude arrow behaves once it is combined with real, transformative contacts, which is exactly what the two identity laws below specify.

Identity morphisms satisfy:

$$\begin{aligned}\text{id}_R \circ f &= f \\ g \circ \text{id}_R &= g\end{aligned}$$

for all appropriate morphisms f and g .

Concretely: if f is a contact arriving at R , composing it with R 's own identity morphism changes nothing — receiving an influence and then "staying yourself" is the same as just receiving the influence. Likewise, if g is a contact leaving R , first "staying yourself" and then undergoing g is the same as simply undergoing g . Identity morphisms, in other words, are formally required to be transparent: they may be necessary for the bookkeeping of the category, but they leave no trace on any actual chain of influence that passes through them.

Self-reflection as a non-trivial endomorphism. Alongside the identity morphism id_R , Refl admits *self-reflection* morphisms: endomorphisms

$$\sigma_R : R \rightarrow R, \text{ with } \sigma_R \neq \text{id}_R$$

that map a reflection to itself non-trivially, i.e., that change X in a way that depends on X 's own structure rather than on a fresh external source. Following the distinction drawn in Chapter 1, two kinds of self-reflection arise naturally within this framework:

- *Transitive self-reflection*, in which σ_R factors through an intermediate reflection: given $f : R \rightarrow R'$ and a return morphism $h : R' \rightarrow R$, the composite $\sigma_R = h \circ f$ is a self-reflection whenever it differs from id_R . Here R "sees itself" only via a relay — R' may be another entity (a mirror, another organism) or an internal one (a sensor, an afferent signaling pathway); the formal structure is the same in either case.
- *Direct self-reflection*, in which σ_R does not factor through any other object of Refl — it is a single, unmediated morphism between two sub-parts of

R itself, generated entirely from R's own internal structure. Watching one's own hand, and a hormone acting directly on a receptor tissue of the same organism, are both cases of this kind. Interoception typically exhibits both routes at once: direct self-reflection through hormonal and other biochemical interactions between tissues, and transitive self-reflection mediated by dedicated sensory pathways that relay a bodily state to the brain. Both forms require R to possess enough internal complexity to support a genuine distinction between a self-directing part and a self-receiving part; formally, this presupposes that R itself decomposes into a family of sub-reflections rich enough to represent such a distinction, a requirement made precise for hierarchical entities in Section 6.3.

Unlike id_R , which every reflection possesses by definition, a self-reflection morphism σ_R need not exist for a given R: the identity morphism is guaranteed by the axioms of a category as reflection's zero case, but self-reflection — direct or transitive — is a substantive structural achievement of certain reflections, not a universal feature of all of them. This matches the intuitive discussion of Chapter 1, where self-reflection, unlike the null reflection, was reserved for entities capable of registering, or actively modeling, something about themselves.

4.5. Families of Reflections and Collective Reflection

4.5.1. Multiple reflections of one source

Given a source S, we may have a family:

$$R(S) = \{ R_i = (S, X_i) \mid i \in I \}$$

Each X_i is a different reflection of the same source.

This is precisely the situation described earlier with the eclipse: $R(S)$ collects, in one set, every reflection anyone or anything has produced of that single source

— the pinhole projection, the direct filtered view, the satellite image, and any number of photographs or written descriptions besides. The index set “I” needs not be small or fixed in advance; it simply enumerates however many independent reflections of S happen to exist.

4.6. Comparison morphisms

Between any two reflections R_i and R_j , we may define comparison morphisms:

$$c_{ij} : R_i \rightarrow R_j$$

These represent attempts to align, translate, or evaluate reflections.

Once several reflections of the same source exist — as in the eclipse example, where a pinhole projection, a filtered direct view, and a satellite image all reflect the same event — a natural question arises: how do these different reflections relate to *each other*, as opposed to how each relates back to the source? A comparison morphism c_{ij} answers this directly. For instance, someone who has only the pinhole projection could use a comparison morphism to translate it into the coordinate system of the satellite image — rotating, rescaling, and correcting for the projection's distortions — without needing to consult the eclipse itself again. Comparison morphisms are what make it possible to check whether two independent reflections agree, and if not, by how much and in what respect.

4.7. Collective reflection

A collective reflection of the family $R(S)$ can be defined as:

$$R_{\text{coll}} = \text{limit}(R_i)$$

or

$$R_{\text{coll}} = \text{colimit}(R_i)$$

depending on the integration strategy.

Returning to the eclipse one final time: suppose an astronomer wants a single, best-available account of the event, combining the pinhole projection, the filtered direct view, and the satellite image. R_{coll} is the formal name for that

combined account. Which construction is appropriate, limit or colimit, depends on what "combining" is meant to achieve. A limit-style collective reflection behaves like taking the most conservative account consistent with *all* the individual reflections at once, useful when the goal is to filter out whatever is idiosyncratic to any single vantage point and keeps only what every reflection agrees on. A colimit-style collective reflection instead behaves like pooling everything each reflection contributes into one composite picture, useful when the different reflections are complementary rather than competing — as when the satellite image shows the eclipse's overall shadow path while the ground-based photograph shows fine detail invisible from orbit.

This models:

- multisensory integration
- collective knowledge
- consensus formation
- data fusion

Concretely: multisensory integration is this limit/colimit construction applied when sight, sound, and touch each supply a separate reflection of one object, and the brain must produce a single percept from them; collective knowledge is the same construction applied to many individuals' testimonies about one event; consensus formation is what results when a group's differing evaluations are combined into one agreed position; and data fusion is the engineering term for combining several sensors' readings of one target into a single estimate. In each case, the "combining" step is exactly the limit-or-colimit choice discussed above.

Notice that this is exactly the same mathematical move behind the comparison morphisms of Section 4.6, taken one step further: comparison morphisms relate reflections pair wise, while collective reflection asks what single object best represents the *entire family* of reflections and comparisons at once.

4.8. Subcategories of Refl

Refl contains natural subcategories:

- **Refl₀** — physical reflections
- **Refl₁** — chemical reflections
- **Refl₂** — biological reflections
- **Refl₃** — neural reflections
- **Refl₄** — mental reflections
- **Refl₅** — social reflections

Each inherits the structure of Refl.

These six subcategories are not an arbitrary list — they mirror the hierarchy of levels $\{C_0, C_1, C_2, \dots\}$ from the category Real in Markov (2024), where each level of reality (quantum, chemical, physical, biological, cognitive, social) was connected to the next by functors of emergence and reduction. Here, the same six levels reappear, but now indexing not the entities of reality itself, only their *reflections*: a physical reflection (light on a surface) belongs to Refl₀, the neural reflection it eventually produces belongs to Refl₃, and so on. Chapter 6 will make the connections between these subcategories explicit, using functors in exactly the sense already familiar from Real.

4.9. Summary

In this chapter we defined:

- reflections as objects
- contacts as morphisms
- interactions as compositions
- identity as the null reflection, and self-reflection (direct or transitive) as its nonzero counterpart
- families of reflections
- comparison morphisms
- collective reflection via limits/colimits
- hierarchical subcategories

This provides the mathematical foundation for the Formal Theory of Reflections.

Every notion introduced in this chapter — reflection, contact, composition, identity, families, comparison, collective reflection, and subcategories — was already present, in intuitive form, in the lake and eclipse examples of the preceding chapters. What Chapter 4 has added is not new content but new precision: each idea now has a definition precise enough to support the theorems and applications developed in Chapter 5.

5. Properties, Theorems, and Applications of the Category Refl

Having defined the category Refl, we now explore its structural properties and theoretical consequences. The purpose of this chapter is threefold:

1. To identify the **mathematical properties** that follow from the definition of Refl.
2. To formulate **theorems** that clarify how reflections behave under composition, comparison, and integration.
3. To demonstrate **applications** of Refl across physical, biological, cognitive, and social domains.

This chapter bridges the gap between abstract formalism and practical scientific modeling.

The theorems that follow may look like routine bookkeeping — checking that Refl satisfies the axioms any category must satisfy. In a sense they are exactly that. But this bookkeeping is what licenses everything intuitive we have said so far: it is only because Refl is a genuine, well-behaved category that chains of reflection (Section 4.3), the null reflection (Section 4.4), and collective reflection (Section 4.7) can be combined and reasoned about freely, the way the eclipse and lake examples assumed they could be, without hidden contradictions lurking in the formalism.

5.1. Basic Properties of Refl

The category Refl possesses several fundamental properties that follow directly from its definition.

5.1.1. Refl is a well-defined category

To be a category, Refl must satisfy:

1. **Objects exist**

Every reflection R is an object:

$$R \in \text{Obj}(\text{Refl})$$

2. **Morphisms exist**

Every contact $f : R_1 \rightarrow R_2$ is a morphism.

3. **Composition exists**

If $f : R_1 \rightarrow R_2$ and $g : R_2 \rightarrow R_3$, then $g \circ f : R_1 \rightarrow R_3$ exists.

4. **Associativity holds**

$$h \circ (g \circ f) = (h \circ g) \circ f$$

5. **Identity morphisms exist**

For every R , $\text{id}_R : R \rightarrow R$ exists.

6. **Identity laws hold**

$$\text{id}_R \circ f = f$$

$$g \circ \text{id}_R = g$$

Thus, Refl satisfies all axioms of a category.

Each of these requirements was already illustrated informally in Chapter 4: objects and morphisms are the reflections and contacts from the eclipse and lake examples; composition and associativity were shown concretely with the eclipse $\rightarrow$ projection $\rightarrow$ photograph chain; and identity morphisms and their laws were shown with the lake persisting between contacts. Section 5.1.1 does not introduce anything new — it simply confirms, all at once, that these separately-checked pieces fit together into a category in the strict mathematical sense.

5.1.2. Refl is not necessarily small

In general, the class of reflections may be large.

For example:

- all neural reflections in a brain
- all perceptual reflections in a population

- all social reflections in a culture

Therefore, Refl is typically a **large category**.

This is worth flagging because it distinguishes Refl from more restricted mathematical categories one might encounter elsewhere, where the collection of objects is a manageable set. Here, the class of physical reflections alone — every instance of light scattering off every surface, ever — is already far too vast to enumerate. Refl is not troubled by this; category theory is built to accommodate such large categories, and none of the definitions in Chapter 4 required Refl 's objects to be countable or listable.

5.1.3. Refl is multi-level

Refl contains natural subcategories:

- Refl_0 — physical reflections
- Refl_1 — chemical reflections
- Refl_2 — biological reflections
- Refl_3 — neural reflections
- Refl_4 — mental reflections
- Refl_5 — social reflections

Each subcategory inherits the structure of Refl but adds constraints.

The word "constraints" here is doing real work: Refl_3 (neural reflections), for example, is not just "any reflection that happens to occur in a nervous system" — it also inherits domain-specific structure, such as the fact that neural morphisms are subject to biophysical limits (refractory periods, finite propagation speed) that a purely physical morphism in Refl_0 need not respect. Section 6.3 will return to this point when discussing functors between levels.

5.1.4. Refl supports families of reflections

Given a source S , the set:

$$R(S) = \{ R_i = (S, X_i) \}$$

is a family of reflections of S .

This is essential for modeling:

- multisensory perception
- distributed sensing
- collective knowledge
- semantic integration
- social representations

This restates, in the language of properties rather than definitions, the eclipse example from Section 4.5: $R(S)$ is exactly the collection of every independent account of a single source that Refl allows to coexist without contradiction, which is the formal precondition for asking (Section 4.6–4.7) how those accounts compare and whether they can be combined.

5.2. Theorems about Reflections

In this section we formulate several theorems that follow from the structure of Refl .

All proofs are conceptual and rely only on the axioms of category theory.

None of the five theorems below require new machinery beyond what Chapter 4 already introduced. Each one simply makes explicit a consequence that was already implicit in the eclipse and lake examples — the kind of thing a careful reader might have already suspected, but which now can be stated and justified with certainty rather than intuition alone.

5.2.1. Theorem 1: Uniqueness of Identity Reflection

Statement:

For every reflection R , the identity morphism id_R is unique.

Reasoning:

This follows from the axioms of category theory.

If two identity morphisms existed, they would have to satisfy the identity laws, which force them to be equal.

Interpretation:

Every reflection has a unique way of maintaining itself.

Concretely: the lake cannot have two different, equally valid ways of "just persisting as itself" between contacts — persistence is persistence, and the theorem confirms that Refl's formalism cannot accidentally introduce ambiguity here.

5.2.2. Theorem 2: Composition Preserves Source Consistency

Statement:

If $f : R_1 \rightarrow R_2$ and $g : R_2 \rightarrow R_3$, then the composed reflection R_3 contains data that is consistent with the source of R_1 .

Reasoning:

$$R_1 = (S_1, X_1)$$

$$R_2 = (S_2, X_2)$$

$$R_3 = (S_3, X_3)$$

If f and g are valid morphisms, then:

X_2 is derived from X_1

X_3 is derived from X_2

Thus X_3 contains a transformed trace of X_1 .

Interpretation:

Reflections propagate influence through chains.

This is the formal guarantee behind the eclipse-photograph chain from Section 4.3: no matter how many intermediate steps a reflection passes through — eclipse to pinhole projection to photograph — the final reflection is never *disconnected* from where the chain began. Some trace of the original source always survives composition, however transformed.

5.2.3. Theorem 3: Comparison Morphisms Form a Network

Statement:

Given a family of reflections $R(S) = \{ R_i \}$,
the comparison morphisms $c_{ij} : R_i \rightarrow R_j$ form a directed graph.

Reasoning:

Each c_{ij} is a morphism.

Morphisms compose.

Thus the set of all c_{ij} forms a network.

Interpretation:

This network is the mathematical structure underlying:

- multisensory integration
- collective knowledge
- semantic networks
- social cognition.

In the eclipse example, this network is simply the full set of possible translations between the pinhole projection, the filtered view, and the satellite image — including translations that go through an intermediate reflection rather than directly. The theorem guarantees that such indirect comparisons are always available whenever the direct ones are, since morphisms compose.

5.2.4. Theorem 4: Collective Reflection Exists When Limits Exist

Statement:

If the diagram formed by the reflections R_i and comparison morphisms c_{ij} has a limit, then the collective reflection R_{coll} exists and is unique up to isomorphism.

Reasoning:

This is a standard result in category theory.

Interpretation:

A consistent collective reflection exists when the individual reflections can be aligned.

The qualifier "when the diagram has a limit" matters: not every family of reflections can be sensibly combined. If the pinhole projection and the satellite image contradicted each other outright (say, disagreeing about which direction the eclipse's shadow moved) no single consistent collective reflection would exist, and the theorem correctly predicts this by requiring the limit to exist in the first place.

5.2.5. Theorem 5: Collective Reflection as Consensus

Statement:

If all comparison morphisms c_{ij} are invertible, then the collective reflection represents a consensus structure.

Interpretation:

This models:

- agreement in a group
- stable perception across senses
- consistent data fusion.

These three cases share the same structural signature: each c_{ij} is not just a translation but a perfect, reversible one. Agreement in a group means any member's account fully recovers any other's; stable perception across senses means sight and touch report the same shape with nothing lost in translating one into the other; consistent data fusion means independent sensors are simply different encodings of one shared reading.

Invertibility of every c_{ij} is a strong condition: it means each reflection can be losslessly translated into any other, with nothing gained or discarded along the way. This is a good model for, say, several accurate rulers reporting the same length in different units, trivially interconvertible, but a poor model for the eclipse example, where the pinhole projection genuinely loses data (fine detail) that the satellite image retains. Theorem 5 marks the boundary between "differing but

reconcilable perspectives" (Theorem 4) and "perspectives so aligned they are, for practical purposes, one."

5.3. Applications of Refl

Refl is not merely an abstract construction.

It applies naturally to many scientific domains.

The five domains below are not independent illustrations chosen at random — they are exactly the six levels of the category Real from Markov (2024) (quantum, physical, chemical, biological, cognitive, social), collapsed here into five application areas since the quantum and chemical levels are treated jointly under "physical." Each subsection shows the same theorems from Section 5.2 applying without modification, simply instantiated with domain-specific objects and morphisms.

5.3.1. Physical Entities

Examples:

- light reflecting from surfaces
- scattering phenomena
- optical transformations

Refl_0 models these as:

- objects: physical reflections
- morphisms: optical transformations

This is, formally, nothing more than the lake example from Chapters 3–4 generalized: any surface capable of reflecting, scattering, or refracting light defines an object of Refl_0 , and any optical process — reflection, refraction, diffraction — defines a morphism between such objects.

5.3.2. Biological Entities

Examples:

- receptor activation
- cellular signaling
- immune recognition

Refl_2 models these as chains of biochemical reflections.

Immune recognition offers a particularly clean instance of Theorem 3 (comparison morphisms forming a network): when an antibody binds an antigen, it is registering a reflection of that antigen's molecular shape, and the immune system's memory cells retain a comparison morphism against future antigens, allowing the body to recognize when a new pathogen closely resembles, without being identical to, one encountered before.

5.3.3. Neural Entities

Examples:

- sensory processing
- neural coding
- multisensory integration

Refl_3 models:

- neurons as reflectors
- synapses as morphisms
- neural pathways as compositions

This is the same eclipse-style composition chain from Section 4.3, now instantiated concretely: light striking the retina (a contact), the retina transmitting a signal along the optic nerve (a further contact), and that signal reaching the visual cortex (composed with the previous one) together form a single, long morphism from photon to conscious percept — exactly the kind of chain Theorem 2 guarantees remains traceable back to its source.

5.3.4. Cognitive Entities

Examples:

- perception

- memory
- inference

Refl₄ models:

- perceptual states as reflections
- cognitive transformations as morphisms

Memory formation is a natural example of the identity morphisms of Section 4.4 at work over longer timescales: a stabilized memory is, in effect, a reflection that has become its own most persistent self-contact, resistant to being overwritten by ordinary further influence — while inference and reasoning correspond to further, more deliberate morphisms acting on that stored reflection.

5.3.5. Social Entities

Examples:

- communication
- shared beliefs
- cultural representations

Refl₅ models:

- individual reflections
- communication as morphisms
- collective reflection as a limit

A rumor spreading through a village is a good illustration of the distinction from Section 4.7 between limit- and colimit-style collective reflection. If the goal is to find the version of the story every villager would agree really happened, that is a limit — the most conservative account consistent with every individual's report. If instead the goal is the village's full collective memory, including each person's distinct embellishments and details, that is closer to a colimit — pooling rather than filtering.

5.4. Summary

In this chapter we established:

- the basic properties of Refl
- several theorems about reflection behavior
- the structure of comparison networks
- the existence of collective reflections
- applications across scientific domains

This completes the mathematical foundation of the Formal Theory of Reflections.

With the basic category now shown to be well-behaved, populated with concrete theorems, and demonstrated across five domains from optics to society, the theory is ready for its next extension: describing not just reflections within a single level, but the mappings that connect one level to another — the subject of Chapter 6.

6. Advanced Structures: Functors, Natural Transformations, and Reflection Levels

Till now, we defined the category Refl and explored its basic properties and applications. However, many real entities involve **multiple layers of reflection**, each with its own structure, rules, and transformations. To model these relationships, we require more advanced categorical tools:

- **functors**, which map one category of reflections into another,
- **natural transformations**, which relate functors in a coherent way,
- **reflection levels**, which represent hierarchical organization.

These tools allow us to describe not only reflections themselves, but also:

- how reflections evolve,
- how they propagate across levels,
- how they transform between entities,
- how they integrate into larger structures.

These tools are not a departure from what came before — they simply zoom out one level. Chapter 4 asked how a *single* reflection relates to another single reflection (a morphism). Chapter 5 confirmed that reflections organized this way form a well-behaved category. This chapter now asks the natural next question:

how does an *entire domain* of reflections — say, all physical reflections — relate to another entire domain, such as all neural reflections? A functor answers exactly this question, mapping not just one object but a whole category into another, all at once, while preserving its internal structure.

6.1. Functors between Reflection Categories

A **functor** is a structure-preserving mapping between categories.

In our context, functors describe how one entity of reflections corresponds to another.

Consider transduction: sunlight strikes the retina, and the retina converts it into an electrical signal. This is not a single, one-off morphism the way "this particular ray of light strikes this particular retina" would be — it is a general rule, applying uniformly to *every* physical light pattern that could strike *any* retina, always producing some corresponding neural pattern. A functor $F : \text{Refl}_0 \rightarrow \text{Refl}_3$ captures exactly this generality: it does not describe one photon's fate, but the systematic correspondence between the whole category of physical reflections and the whole category of neural reflections.

6.1.1. Definition

Let C and D be categories of reflections (for example, Refl_2 and Refl_3). A functor $F : C \rightarrow D$ consists of:

1. A mapping of objects:
 $F(\mathbf{R})$ is a reflection in D .
2. A mapping of morphisms:
 $F(\mathbf{f} : \mathbf{R}_1 \rightarrow \mathbf{R}_2)$ is a morphism $F(\mathbf{f}) : F(\mathbf{R}_1) \rightarrow F(\mathbf{R}_2)$.
3. Preservation of structure:
 - $F(\text{id}_{\mathbf{R}}) = \text{id}_{F(\mathbf{R})}$
 - $F(\mathbf{g} \circ \mathbf{f}) = F(\mathbf{g}) \circ F(\mathbf{f})$

The last condition that F respects composition is what makes a functor more than just an arbitrary pairing of objects to objects. It guarantees that if the retina-to-cortex chain from Section 5.3.3 is composed of two separate contacts (retina-

to-nerve, nerve-to-cortex), then applying a functor to that whole chain gives the same result as applying it to each piece and then composing the images. Structure-preservation, in other words, means a functor cannot "scramble" the order or connectivity of a chain of reflections — it can only relabel or transform them uniformly, one category into another.

6.1.2. Interpretation

Functors model:

- sensory transduction (physical $\rightarrow$ neural)
- neural encoding (neural $\rightarrow$ cognitive)
- conceptualization (cognitive $\rightarrow$ linguistic)
- communication (linguistic $\rightarrow$ social)
- abstraction (low-level $\rightarrow$ high-level reflections)

For example:

$F : \text{Refl}_0 \rightarrow \text{Refl}_3$

may represent the transformation from physical light patterns to neural activations.

6.1.3. Example: Perception as a Functor

Let:

- Refl_0 = physical reflections (light on surfaces)
- Refl_3 = neural reflections (retinal and cortical patterns)

Then the perceptual entity defines a functor:

$\text{Percept} : \text{Refl}_0 \rightarrow \text{Refl}_3$

mapping:

- each physical reflection R to a neural reflection $\text{Percept}(R)$
- each physical transformation f to a neural transformation $\text{Percept}(f)$

This formalizes the idea that perception is a structured transformation of reflections.

6.2. Natural Transformations: Coherent Relations between Functors

Functors describe mappings between categories.

But often we have **multiple functors** between the same categories, representing different modes of processing.

A **natural transformation** describes a coherent relation between such functors.

Suppose there are two different ways of mapping physical light patterns to neural patterns, say, F for a young, still-developing visual system and G for a mature one after years of visual experience. Both are legitimate functors from Refl_0 to Refl_3 ; neither is "wrong." What a natural transformation η captures is a coherent, uniform way of turning any reflection processed by F into the corresponding reflection as it would be processed by G — not just for one particular scene, but consistently, for every physical reflection at once, and in a way that respects however those reflections are related to each other.

6.2.1. Definition

Let F and G be functors from category C to category D :

$$F, G : C \rightarrow D$$

A natural transformation $\eta : F \Rightarrow G$ assigns to each object R in C a morphism:

$$\eta_R : F(R) \rightarrow G(R)$$

such that for every morphism $f : R_1 \rightarrow R_2$ in C , the following diagram commutes:

$$\begin{array}{ccc} F(R_1) & \xrightarrow{F(f)} & F(R_2) \\ \downarrow \eta_{R_1} & & \downarrow \eta_{R_2} \\ G(R_1) & \xrightarrow{G(f)} & G(R_2) \end{array}$$

This means:

$$\eta_{R_2} \circ F(f) = G(f) \circ \eta_{R_1}$$

The commutativity condition is the "coherent" part of "coherent relation between functors." It says that it makes no difference whether one first transforms a reflection via f and then translates the result from the F -perspective to the G -perspective, or first translates from F to G and then transforms via f — both routes must land on the same place. Without this condition, η would just be an arbitrary, patchwork collection of separate translations, one per reflection, with no guarantee that they fit together consistently across an entire chain of contacts.

6.2.2. Interpretation

Natural transformations model:

- adaptation
- learning
- calibration
- translation between representational entities
- semantic alignment
- perceptual correction
- cognitive reinterpretation

For example:

- two perceptual pathways (F and G)
- a natural transformation η aligning them

This is mathematically equivalent to **multisensory integration**.

6.2.3. Example: Learning as a Natural Transformation

Let:

- F = initial perceptual mapping
- G = improved perceptual mapping after learning

Then $\eta : F \Rightarrow G$ represents the learning process:

- η_R transforms the old reflection $F(R)$ into the improved reflection $G(R)$
- the commutativity condition ensures consistency across transformations

This gives a formal model of **experience-dependent plasticity**.

6.3. Reflection Levels and Hierarchical Structure

Reflections occur at multiple levels of organization.

We formalize this using *reflection levels*, each represented as a subcategory of Refl .

This hierarchy is not a new invention specific to reflections — it is the same six-level hierarchy $\{C_0, \dots, C_5\}$ that structured the category Real in Markov (2024), where each level of reality was connected to the next by functors of emergence and reduction. Section 4.8 already introduced these six levels as subcategories of Refl ; what this section adds is the machinery — functors and natural transformations — needed to describe precisely how one level's reflections give rise to the next's.

6.3.1. Definition of Reflection Levels

We define:

- **Refl_0** — physical reflections
- **Refl_1** — chemical reflections
- **Refl_2** — biological reflections
- **Refl_3** — neural reflections
- **Refl_4** — mental reflections
- **Refl_5** — social reflections

A physical reflection of sunlight on a lake (Refl_0) is not itself a neural reflection, nor a social one — but as Section 6.3.2 will show, there is a systematic route from one to the next, formalized as a chain of functors linking these levels together.

Each level has:

- its own objects (reflections)
- its own morphisms (contacts)
- its own internal structure

6.3.2. Functors Between Levels

Levels are connected by functors:

$$\mathbf{F}_{01} : \mathbf{Refl}_0 \rightarrow \mathbf{Refl}_1$$

$$\mathbf{F}_{12} : \mathbf{Refl}_1 \rightarrow \mathbf{Refl}_2$$

$$\mathbf{F}_{23} : \mathbf{Refl}_2 \rightarrow \mathbf{Refl}_3$$

$$\mathbf{F}_{34} : \mathbf{Refl}_3 \rightarrow \mathbf{Refl}_4$$

$$\mathbf{F}_{45} : \mathbf{Refl}_4 \rightarrow \mathbf{Refl}_5$$

These five functors trace out, level by level, the entire journey already sketched informally throughout this work: sunlight ($\mathbf{Refl}_0$) undergoes a chemical change in a photoreceptor pigment ($\mathbf{Refl}_1$), which triggers a biological signaling cascade in the retina ($\mathbf{Refl}_2$), which produces neural activation along the visual pathway ($\mathbf{Refl}_3$), which becomes a conscious percept ($\mathbf{Refl}_4$), which an observer may then describe to someone else ($\mathbf{Refl}_5$). Each arrow $\mathbf{F}_{i,i+1}$ is one deliberate, well-defined step in this journey — not a metaphorical leap, but an actual structure-preserving mapping between two categories of reflections.

These represent:

- physical $\rightarrow$ chemical transduction
- chemical $\rightarrow$ biological signaling
- biological $\rightarrow$ neural activation
- neural $\rightarrow$ mental representation
- mental $\rightarrow$ social communication

6.3.3. Composition of Level Functors

The full transformation from physical to social reflection is:

$$\mathbf{F}_{45} \circ \mathbf{F}_{34} \circ \mathbf{F}_{23} \circ \mathbf{F}_{12} \circ \mathbf{F}_{01}$$

This represents the entire chain:

physical $\rightarrow$ chemical $\rightarrow$ biological $\rightarrow$ neural $\rightarrow$ mental $\rightarrow$ social

The composed functor is the formal counterpart of Theorem 2 (Section 5.2.2) applied one level of abstraction higher: just as composing morphisms preserved

a trace of the original source through a chain of contacts, composing these five functors guarantees that the resulting mapping from Refl_0 to Refl_5 is still well-defined and structure-preserving, even though it silently passes through four intermediate categories most observers never explicitly notice.

This is a formal model of:

- perception
- cognition
- communication
- cultural formation

6.4. Natural Transformations between Levels

Natural transformations model:

- adaptation across levels
- calibration between sensory modalities
- semantic alignment between individuals
- cultural evolution

For example:

$$\eta : \mathbf{F}_{34} \Rightarrow \mathbf{G}_{34}$$

may represent two different neural-to-mental mappings (e.g., two cognitive strategies), and η aligns them.

This is the level-hierarchy counterpart of the example in Section 6.2: just as η there aligned a young and a mature visual system, both mapping Refl_0 to Refl_3 , here η aligns two different neural-to-mental strategies, both mapping Refl_3 to Refl_4 , for instance, a fast, heuristic route to interpreting a neural pattern versus a slower, deliberative one. The natural transformation is what guarantees these two strategies can be compared and reconciled consistently, rather than simply producing two unrelated interpretations of the same neural state.

6.5. Applications of Advanced Structures

These advanced tools allow us to model:

- **Multisensory integration** as natural transformations between functors from Refl_0 to Refl_3 .
- **Concept formation** as functors from Refl_3 to Refl_4 .
- **Language** as functors from Refl_4 to Refl_5 .
- **Social consensus** as limits in Refl_5 .
- **Learning** as natural transformations between cognitive functors.
- **Cultural evolution** as chains of functors and natural transformations across generations.

6.6. Summary

In this chapter we introduced:

- functors between reflection categories
- natural transformations between functors
- hierarchical reflection levels
- inter-level mappings
- applications to perception, cognition, learning, and society

These tools allow the Formal Theory of Reflections to model complex, multi-layered entities with mathematical precision.

Everything so far, however, has described reflections, functors, and natural transformations as if frozen at a single instant. Yet the lake's shimmer changes as clouds pass overhead, a percept sharpens as attention shifts, and a rumor mutates as it spreads — reflection is not only structured but also dynamic. Chapter 7 introduces time explicitly into the framework, showing how the same categorical apparatus developed here extends naturally to reflections that arise, change, and evolve.

7. Dynamics of Reflections: Time, Change, and Evolution

Up to this point, we have treated reflections as static structures: objects, morphisms, functors, and natural transformations.

However, real entities are **dynamic**.

Reflections:

- arise,
- change,
- propagate,
- decay,
- stabilize,
- evolve,
- and interact over time.

To model these processes, we must introduce **temporal structure** into the category **Refl**.

This chapter develops a formal framework for describing the dynamics of reflections.

The lake is, once again, a useful anchor here. At noon the water reflects a bright, high sun; by evening it reflects a low, reddened one; on a windy afternoon the same sunlight breaks into a scattered, restless shimmer instead of a clean mirror image. In every case we are tempted to say "the lake's reflection changed", but so far, **Refl** has had no formal way to say this, since Chapters 4–6 treated each reflection as a fixed object. This chapter's task is simply to add a time index to the objects and morphisms already defined, so that "the reflection changed" becomes a precise statement rather than a loose figure of speech.

7.1. Time as a Parameter of Reflection

7.1.1. Time-indexed reflections

A reflection R is not fixed; it evolves.

We represent this by indexing reflections with time:

$R(t)$

where t belongs to a time domain T , typically:

- $T = \{0, 1, 2, \dots\}$ for discrete time
- $T = \mathbb{R}^+$ for continuous time

Thus, a dynamic reflection is a function:

$$\mathbf{R} : \mathbf{T} \rightarrow \mathbf{Obj(Ref)}$$

mapping each moment to a reflection.

$R(t)$ is simply the lake's reflection at noon, $R(t')$ the same lake's reflection at evening, and so on — the same source (the sky) and the same receiving entity (the lake), but a different resulting state X at each moment, because the sunlight itself, and the water's surface, are themselves changing. Nothing here departs from the definition $R = (S, X)$ in Section 4.1; time-indexing only tracks how S and X change together as t varies.

7.1.2. Time-indexed morphisms

If reflections change over time, morphisms must also be time-dependent:

$$\mathbf{f(t)} : \mathbf{R_1(t)} \rightarrow \mathbf{R_2(t)}$$

This models:

- changing sensory input
- neural adaptation
- cognitive updates
- social communication over time

Changing sensory input is $f(t)$ capturing how the light reaching the retina differs from morning to night; neural adaptation is the same morphism capturing how a neuron's response to a repeated stimulus weakens over repeated exposure; cognitive updates are $f(t)$ capturing how a belief-forming process revises its output as new evidence arrives moment to moment; and social communication over time is $f(t)$ capturing how one person's utterance to another depends on the state of their ongoing conversation, not just on a single fixed message.

Just as a single contact $f : R_1 \rightarrow R_2$ captured one instance of light striking the retina (Section 4.2), $f(t)$ captures the same kind of contact recurring at every moment, but allowing that the retina's own state, and hence the influence it

registers, may itself be different at $t=\text{noon}$ than at $t=\text{evening}$, since ambient light levels and pupil dilation change throughout the day.

7.1.3. Temporal composition

If:

$$f(t) : R_1(t) \rightarrow R_2(t)$$

$$g(t) : R_2(t) \rightarrow R_3(t)$$

then:

$$g(t) \circ f(t) : R_1(t) \rightarrow R_3(t)$$

This is the instantaneous composition at time t .

It is exactly the composition rule from Section 4.3 (eclipse $\rightarrow$ projection $\rightarrow$ photograph), simply evaluated afresh at each moment t rather than once and for all — the retina-to-cortex chain from Section 5.3.3 composes the same way at noon as it does at evening, even though the specific reflections being composed differ from one time to the next.

7.2. Dynamic Morphisms and Evolution Operators

To model change, we introduce **evolution operators**.

7.2.1. Definition

An evolution operator E is a morphism:

$$E : R(t) \rightarrow R(t + \Delta t)$$

This represents the internal change of a reflection over time.

Notice the difference from the morphisms of Section 7.1.2: those, like $f(t)$, model a contact between *two distinct* entities recurring over time (light striking a retina, moment after moment). An evolution operator E , by contrast, models how *one and the same* reflection changes on its own, from one moment to the next, without necessarily being triggered by a fresh external contact — the way a

photograph left in sunlight slowly fades, or a memory gradually consolidates overnight, through internal processes rather than renewed external influence.

Examples:

- neural activation spreading
- memory consolidation
- perceptual adaptation
- social opinion change

Neural activation spreading is E capturing how excitation propagates through connected neurons after a single triggering signal, without any further external input; memory consolidation is E capturing how a freshly formed memory trace reorganizes itself into a more stable form through internal neural processes alone, typically during rest or sleep; perceptual adaptation is E capturing how a sensory system's baseline response recalibrates after prolonged exposure to a constant stimulus, as when a smell fades from awareness; and social opinion change is E capturing how an individual's stance can keep shifting through internal reflection alone, even without hearing any new argument.

7.2.2. Chains of evolution

Repeated application yields:

$$\mathbf{R}(t + n\Delta t) = \mathbf{E}^n(\mathbf{R}(t))$$

This is the temporal analogue of the composition chains from Section 4.3: just as composing morphisms $g \circ f$ let us skip past an intermediate reflection and relate a source directly to a distant result, applying E repeatedly lets us skip past every intermediate moment and relate a reflection's state now directly to its state much later — for instance, jumping from a novice's first attempt at a skill directly to their state after n practice sessions, without having to track every session individually.

This models:

- learning
- habit formation
- cultural evolution

- biological adaptation

Learning is E^n capturing how a skill improves not through one insight but through the accumulated effect of many repeated practice sessions; habit formation is the same chain applied to behavior, where a single repetition means little but n repetitions produce automaticity; cultural evolution is E^n applied to a shared practice or belief, which drifts gradually across many successive transmissions rather than through any single retelling; and biological adaptation is E^n applied to a population's traits across many generations, where the cumulative chain, not any one generation's change, is what produces the eventual fit to environment.

7.2.3. Stability and fixed points

A reflection is stable if:

$$E(R) = R$$

A fixed point of E is the dynamic counterpart of the identity morphisms from Section 4.4: there, id_R captured an entity persisting unchanged *between* contacts; here, $E(R) = R$ captures a reflection that has stopped changing even *under* whatever ongoing process E represents — a memory that has fully consolidated, a habit that has become automatic, a belief no longer revised by further evidence. Where identity morphisms formalized persistence in the absence of new contact, fixed points formalize persistence in spite of a standing capacity for change.

Such reflections represent:

- stable perceptions
- long-term memories
- persistent beliefs
- homeostatic biological states

A stable perception is a percept that no longer shifts under continued exposure to the same stimulus, having reached $E(R) = R$; a long-term memory is a trace robust enough that ordinary further neural activity leaves it unchanged; a persistent belief is one that further, unremarkable evidence no longer revises;

and a homeostatic biological state, such as body temperature or blood pH, is a fixed point that the organism's own regulatory processes actively maintain against small perturbations.

7.3. Dynamic Functors

Functors can also evolve over time.

7.3.1. Time-dependent functors

A dynamic functor is:

$$\mathbf{F(t)} : \mathbf{C} \rightarrow \mathbf{D}$$

mapping reflections at time t in category C to reflections at time t in category D .

It extends the Percept functor from Section 6.1.3 across time: rather than the single fixed rule mapping physical reflections to neural ones, $F(t)$ allows that mapping rule itself to be different at different ages or stages of development — the retina-to-cortex correspondence in an infant's still-maturing visual system is not the same functor as the one in a visually experienced adult, even though both are legitimate functors from Refl_0 to Refl_3 at their respective moments in time.

This models:

- changing sensory mappings
- neural plasticity
- cognitive development
- evolving communication patterns

A changing sensory mapping is $F(t)$ itself shifting, as when someone recovering from cataract surgery must re-learn to interpret unfiltered light; neural plasticity is the general biological capacity underlying such shifts, in which the physical-to-neural correspondence itself rewires with experience; cognitive development is $F(t)$ tracking how a child's whole rule for turning perception into concepts matures with age, not just what any single concept is; and evolving communication patterns are $F(t)$ applied to $\text{Refl}_4 \rightarrow \text{Refl}_5$, capturing how the

very way a community encodes meaning into language changes across a generation.

7.3.2. Evolution of functors

The evolution of a functor is described by:

$$\mathbf{F}(\mathbf{t} + \Delta \mathbf{t}) = \mathbf{U} \circ \mathbf{F}(\mathbf{t})$$

where $\mathbf{U}$ is a transformation operator acting on functors.

Here $\mathbf{U}$ plays the same role for functors that $\mathbf{E}$ played for individual reflections in Section 7.2.1: it is what actually drives the change from one moment's mapping-rule to the next's. A child's visual system does not spontaneously rewire itself — $\mathbf{U}$ represents whatever concrete process (synaptic strengthening from repeated exposure, for instance) transforms this month's physical-to-neural correspondence into next month's slightly refined one.

This is a formal model of:

- learning
- calibration
- adaptation
- skill acquisition

Learning here is $\mathbf{U}$ driving the step-by-step revision of a mapping rule as feedback accumulates; calibration is the same operator applied when a measuring instrument's own response curve is adjusted against a known standard; adaptation is $\mathbf{U}$ applied whenever an organism's mapping from stimulus to response is reshaped by a changing environment; and skill acquisition is $\mathbf{U}$ applied to a learner's whole perception-to-action mapping, which is refined by practice rather than replaced outright.

7.4. Natural Transformations as Dynamic Processes

Natural transformations can also be dynamic.

7.4.1. Time-indexed natural transformations

A dynamic natural transformation is:

$$\eta(t) : F(t) \Rightarrow G(t)$$

assigning to each object R a morphism:

$$\eta_{R(t)} : F(t)(R) \rightarrow G(t)(R)$$

It is the moment-by-moment counterpart of the fixed natural transformation from Section 6.2:

- there, η aligned a young and a mature visual system as two static functors;
- here, $\eta(t)$ tracks how that very alignment itself shifts as the young system gradually matures into the mature one -

at early t , $\eta(t)$ may need to correct heavily for the immature functor's biases, while at late t , once $F(t)$ has itself evolved close to G , little correction remains needed.

This models:

- perceptual recalibration
- semantic alignment
- cognitive reinterpretation
- social negotiation

Perceptual recalibration is $\eta(t)$ tracking how the correction needed to align two sensory mappings shrinks as one matures toward the other, as in the visual-system example above; semantic alignment is $\eta(t)$ tracking how two speakers' differing associations for a word converge over a conversation; cognitive reinterpretation is $\eta(t)$ tracking how a person's evolving framework for interpreting a past experience keeps changing the mapping applied to it; and social negotiation is $\eta(t)$ tracking how two parties' differing evaluations of a situation are reconciled, step by step, over the course of a negotiation.

7.4.2. Evolution of Natural Transformations

The evolution is given by:

$$\eta(t + \Delta t) = V(\eta(t))$$

where V is a transformation operator.

Just as U (Section 7.3.2) drives the underlying functor's change from one moment to the next, V drives the change in how two functors' outputs are reconciled, for instance, as a rumor spreads and each retelling's functor $F(t)$ drifts further from the original account G , V describes how the "correction" needed to translate one version into the other must itself grow or change shape over successive retellings.

This models:

- learning curves
- cultural drift
- conceptual refinement

A learning curve is V tracking how much correction a learner still needs at each stage of practice, shrinking as competence grows; cultural drift is V tracking how the gap between an original cultural form and its descendant version changes shape as it passes through many retellings; and conceptual refinement is V tracking how the adjustment needed to reconcile a rough, early version of a concept with its more precise, later version evolves as the concept matures.

7.5. Evolution of Reflection Networks

In previous chapters we saw that multiple reflections of the same source form a network. Now we consider how such networks evolve.

7.5.1. Time-dependent Comparison Morphisms

Comparison morphisms become:

$$c_{ij}(t) : R_i(t) \rightarrow R_j(t)$$

These change as:

- entities learn
- sensors recalibrate
- individuals communicate
- cultures evolve

A comparison morphism changes as entities learn when, for instance, someone's mental translation between two accounts of the same event grows more accurate with reflection; as sensors recalibrate when two instruments measuring one quantity are periodically re-aligned against each other; as individuals communicate when repeated conversation narrows the gap between two people's understanding of the same topic; and as cultures evolve when the translation between two societies' interpretations of a shared symbol shifts across generations of contact.

This extends the eclipse comparison of Section 4.6 across time: if the pinhole projection and the satellite image were compared once, $c_{ij}(t)$ instead tracks how that comparison itself would need to be redone at every moment — appropriate, for instance, to a live-streamed eclipse where the satellite's orbital position, and hence the geometry relating its image to the ground projection, keeps shifting as the event unfolds.

7.5.2. Dynamic collective reflection

The collective reflection becomes:

$$\mathbf{R}_{\text{coll}(t)} = \text{limit}(\mathbf{R}_i(t))$$

or:

$$\mathbf{R}_{\text{coll}(t)} = \text{colimit}(\mathbf{R}_i(t))$$

It is the time-indexed version of Section 4.7's astronomer combining several accounts of the eclipse into one: here, as new reflections keep arriving — a village's rumor keeps collecting new retellings, a scientific community keeps registering new observations — the single best combined account $\mathbf{R}_{\text{coll}}$ must itself be recomputed at each moment, rather than fixed once and for all.

This models:

- evolving consensus

- dynamic data fusion
- changing shared beliefs

Evolving consensus is $R_{\text{coll}}(t)$ tracking how a group's single best combined account keeps being recomputed as members continue to exchange views; dynamic data fusion is the same construction applied to a sensor network whose combined estimate updates continuously as new readings arrive; and changing shared beliefs is $R_{\text{coll}}(t)$ applied to a community's collective understanding of some ongoing situation, which shifts as events unfold and as more people weigh in.

7.5.3. Stability of collective reflection

A collective reflection is stable if:

$$R_{\text{coll}}(t + \Delta t) = R_{\text{coll}}(t)$$

It is the collective-level counterpart of the fixed points from Section 7.2.3: just as an individual reflection could stop changing under its own evolution operator, a *collective* reflection (a village's shared version of a story, a field's settled scientific consensus) can likewise reach a point where further incoming reflections no longer shift the combined account, because they have all already been absorbed or reconciled.

This corresponds to:

- stable group beliefs
- consistent multisensory perception
- robust scientific theories

Stable group beliefs are a collective reflection that no longer changes because the group's members have already reconciled their individual views; consistent multisensory perception is the case where sight, sound, and touch continue to agree on an object's properties even as attention shifts, so no further recombination is needed; and a robust scientific theory is a collective reflection that keeps absorbing new experimental results without needing revision, because it was already general enough to accommodate them.

7.6. Evolutionary Processes in Refl

Refl naturally supports evolutionary modeling.

The four notions below are not a new addition to Refl so much as a relabeling of tools already built in this chapter, read through the lens of evolutionary theory: variation, selection, inheritance, and emergence turn out to be exactly what Sections 7.1–7.5 had already formalized as, respectively, families of reflections, comparison morphisms, evolution operators, and collective reflection.

7.6.1. Variation

Different reflections $R_i(t)$ represent variation, for instance, the slightly different retellings of a rumor as it passes from person to person, each $R_i(t)$ a distinct variant of the same underlying story.

7.6.2. Selection

Comparison morphisms $c_{ij}(t)$ determine which reflections are more compatible — a retelling that fits poorly with what listeners already believe will generate comparison morphisms showing large discrepancies, making it less likely to be passed on unchanged, much as a poorly-adapted variant is less likely to persist.

7.6.3. Inheritance

Evolution operators E propagate reflections forward in time — each new telling of the story inherits, via E , the content of the version that came before it, with only incremental change introduced at each step, rather than being generated from scratch.

7.6.4. Emergence

Collective reflections $R_{\text{coll}}(t)$ represent emergent structures — the village's settled, shared version of events is not identical to any single person's retelling, yet it emerges systematically from all of them combined, the way R_{coll} did in Section 7.5.2.

7.7. Summary

In this chapter we introduced:

- time-indexed reflections
- dynamic morphisms
- evolution operators
- dynamic functors
- dynamic natural transformations
- evolving reflection networks
- stability and fixed points
- evolutionary processes

This completes the dynamic extension of the Formal Theory of Reflections.

Discussion

The Formal Theory of Reflections developed here makes a deliberate methodological choice: to treat reflection as a purely structural, relational phenomenon, expressible in the language of category theory, before any semantic, epistemic, or subjective content is added. This choice has both advantages and costs, which are worth making explicit.

Why category theory, and what it buys us. Section 3.1 argued that category theory suits reflection because it foregrounds relations over intrinsic composition. The payoff of this choice is visible throughout Chapters 4–7: the same apparatus (objects, morphisms, composition, identity, functors, and natural transformations) described a lake's shimmer, a chain of neural signaling, a scientific instrument reading, and a spreading rumor, without needing a separate vocabulary for each. This uniformity is not merely aesthetic. It is what allowed Theorem 2 (Section 5.2.2) to guarantee that source data survives arbitrarily long chains of contact, and what allowed Theorem 4 (Section 5.2.4) to state precisely when a family of divergent reflections admits a consistent collective account — questions that recur, in different guises, across physics, biology, cognitive science, and the study of social consensus.

Relation to prior work. Chapter 2 positions the theory relative to a considerably wider field than any single prior tradition, and it is worth summarizing here what Refl keeps and what it discards from each. Against classical materialist reflection theory (Pavlov and the dialectical-materialist tradition of Engels, Lenin, and their successors; Section 2.4.2), Refl keeps the intuition that reflection is a lawful, structural consequence of interaction, extending in principle from inorganic matter to social cognition, but drops both the assumption that a reflection must be a copy of its source and the ideological commitment to matter's ontological priority. Against idealist accounts (German Idealism, phenomenology; Sections 2.1.6–2.1.7 and 2.4.1), Refl keeps the recognition that reflection is always mediated by the receiver's own structure and never a neutral mirror, but drops the requirement that reflection be located specifically in mind or Spirit. Against the Western realist mainstream running from Aristotle through scholasticism to early modern representationalism (Sections 2.1.2, 2.1.4–2.1.5), Refl keeps the idea of a structure-preserving transmission between source and receiver while dropping any commitment to a privileged, mind-independent vocabulary in which that structure must be described. Against the Eastern and Islamic traditions surveyed in Sections 2.2–2.3, Refl keeps the relational, dependently-arisen character of reflection (most explicit in Buddhist dependent origination and Neoplatonic emanation) and the recognition that a reflection's fidelity depends on the tranquility or receptivity of its substrate (explicit in Daoism and Sufism alike), while dropping the soteriological or theological framings in which these insights were originally embedded. The result is meant to be a genuinely neutral common structure underlying all of these positions — a claim that can, of course, be contested, and one qualification of it is discussed immediately below. Structurally, the theory is also a direct continuation of Markov's (2024) category Real: where Real modeled reality itself as a hierarchy of levels connected by emergence and reduction functors, the present paper applies the same hierarchical, functorial machinery to reflection as a phenomenon in its own right, rather than to reality as a whole.

On breadth versus depth in the comparative survey. A theory that draws on some twenty distinct philosophical and scientific traditions invites an obvious methodological worry: that apparent convergences are artifacts of selective

reading, that a sufficiently abstract relational schema can be projected onto almost anybody of text, and that the resulting synthesis therefore risks superficiality rather than genuine structural insight. This worry deserves to be taken seriously rather than dismissed. Our response is twofold.

First, the convergence documented in Chapter 2 is not merely that each tradition uses some notion of one thing being "present in" another; it is that several traditions, independently, isolate the same finer-grained structural features later formalized here — the distinction between a reflection and its source (explicit in Aquinas's *esse intentionale* and in Advaita Vedānta's grades of reality alike), the dependence of a reflection's fidelity on receiver-structure rather than source alone (explicit in Kantian transcendental idealism and in Daoist and Sufi mirror-imagery alike), and the irreducibly relational, networked character of collective reflection (explicit in the Buddhist net of Indra and in Lurianic Kabbalah's shattered vessels alike). Convergence on several independent, specific structural features across traditions with no plausible line of mutual influence is harder to dismiss as an artifact of selective reading than convergence on a single vague theme would be.

Second, and more modestly, the comparative survey is offered as motivation and illustration rather than as evidence in a strict sense: the theory's actual claims are stated and defended in Chapters 3–7 in purely mathematical terms, and stand or fall independently of whether the historical parallels drawn in Chapter 2 are judged persuasive.

What the theory deliberately leaves out. The most significant limitation, already flagged in Section 7.7 and the Conclusion, is that Refl as presented here is structurally neutral in a strong sense: a reflection $R = (S, X)$ records that a state X resulted from contact with S , but says nothing about whether, or how, that state is *experienced* by the receiving entity. This is a feature for the theory's stated aim of interdisciplinary neutrality, but it is also a real limitation: notions such as qualia, first-person perspective, or what it is like to undergo a reflection are outside the scope of the constructs developed here, and cannot simply be read off from the objects and morphisms of Refl without further, separate machinery. A reader hoping for an account of subjective experience will not find one in this paper; that omission is intentional, but it should not be mistaken for a

claim that subjectivity is unimportant or reducible to what has been formalized here.

Open questions and possible objections. Three questions seem especially worth raising for future scrutiny.

First, the collective-reflection constructs of Section 4.7 rely on the existence of categorical limits or colimits; nothing in the present paper characterizes, in general, *when* a given family of reflections and comparison morphisms will admit one, beyond the schematic conditions of Theorem 4. A more constructive account — perhaps domain by domain — would strengthen the theory's practical applicability.

Second, the level-hierarchy of Section 6.3, while modeled directly on Real, has not been shown to satisfy the same emergence/reduction adjunction properties ($F_i \dashv G_i$) established for Real in Markov (2024); whether an analogous adjunction holds for reflection levels, and what it would mean if it did or did not, is left open.

Third, a critical reader might object that treating reflections at such different scales (a photon striking a retina, a scientific paradigm shift, a rumor's spread) within a single formal apparatus risks flattening genuine differences between these phenomena in the name of mathematical uniformity. We do not think this objection is decisive, since the theory's claim is only that these phenomena *share* a common relational shape, not that they are otherwise alike; but the objection deserves to be stated plainly rather than assumed away.

A note on method. Finally, it is worth remarking on the expository strategy of this paper itself. The two recurring examples — the lake and the eclipse — were chosen not because they are the most scientifically important instances of reflection, but because they are simple enough to keep in mind across many chapters while still exhibiting every structural feature the theory needed: source and receiver, chains of contact, the null reflection, multiplicity of reflections, and the need for comparison and integration. Readers working in a specific domain (neuroscience, sociology, physics) should find it straightforward to substitute their own domain's phenomena into the same slots, and we would welcome such translations as a test of the theory's claimed generality.

On the direct/transitive distinction for self-reflection. One further design choice merits explicit defense. Section 4.4 distinguishes direct self-reflection (a single, unmediated morphism between two parts of one entity, as when a hormone acts on a receptor tissue of the same organism) from transitive self-reflection (a composite morphism that factors through an intermediate reflection, whether an internal sensor or an external relay such as a mirror or another organism). One might ask why this distinction is drawn at the level of morphism factorization rather than, say, at the level of physical mechanism or degree of complexity. The reason is that factorization is the only one of these criteria that is available in purely categorical terms, and hence the only one that generalizes cleanly across the physical, biological, neural, cognitive, and social domains the theory aims to unify; degree of complexity is a matter of gradation without a principled cutoff, whereas whether a given self-reflection morphism factors through a distinguishable intermediate object is, for any specific model of a given entity, a determinate structural fact. This is also why the same entity can exhibit both forms at once at a given level of description, as the interoception example in Section 4.4 illustrates: which route a given self-reflection takes is a fact about how the entity is modeled, at a given granularity, not an all-or-nothing property of the entity as a whole.

Conclusion

This paper has presented the basic constructs of a Formal Theory of Reflections. We traced the concept of reflection in extensive detail across Western philosophy from Plato and Aristotle through Stoicism, scholasticism, early modern philosophy, Kant, and phenomenology; across Buddhist, Daoist, Yogācāra, Advaita Vedānta, and Sāṃkhya thought; across Neoplatonic, Sufi, Hermetic, and Kabbalistic traditions; across the idealist–materialist debate; and across modern cognitive science, predictive processing, embodied cognition, neuroscience, and information theory (Chapter 2). We then formalized reflection as the category Refl , with reflections as objects and contacts as morphisms (Chapters 3–4), characterizing the identity morphism as reflection's null case and distinguishing it from self-reflection proper — a non-trivial endomorphism that may be either direct or transitive, depending on whether it factors through

an intermediate reflection. We established Refl's basic properties, theorems, and cross-domain applications (Chapter 5), extended it with functors, natural transformations, and a hierarchy of reflection levels (Chapter 6), and introduced explicit temporal dynamics (Chapter 7) — throughout grounding the formalism in two recurring examples, a lake reflecting sunlight and a solar eclipse observed through several instruments.

It should be stressed that this paper presents only the basic constructs of the Formal Theory of Reflections: the categorical foundation (Chapters 3–5), its multi-level structure (Chapter 6), and its basic dynamic extension (Chapter 7) — built up from a lake's shimmer, a firing neuron, and a culture's evolving beliefs alike. A full theory of reflections, however, remains to be developed. Subsequent work will need to extend these basic constructs in several directions already anticipated, in informal terms, by the traditions surveyed in Chapter 2: toward the semantic dimension of reflection (information, meaning, and interpretation), toward its epistemological and communicative dimensions (knowledge, language, and social exchange), toward subjective reflection — the first-person, experiential dimension deliberately set aside by the present, structurally neutral framework, but anticipated in the phenomenological and Yogācāra treatments of consciousness discussed in Sections 2.1.7 and 2.2.3 — and toward the higher levels of the hierarchy of reality beyond those considered here, following the broader hierarchy of levels $\{C_i, F_i\}$ introduced in Markov (2024). Each of these extensions will require its own careful development, with the same mathematical rigor and concrete grounding pursued in the present work.

What we hope to have shown, even at this basic level, is that reflection — an idea as old as Plato's cave, as geographically dispersed as the Buddhist net of Indra, and as contemporary as predictive-processing neuroscience — admits a single, precise, and productive formalization, one capable of speaking equally to philosophers, cognitive scientists, and mathematicians.

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