"RED LINES" IN MANAGEMENT OF COMPLEX SYSTEMS

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Abstract: *The problem of optimizing a control system for an object created to fulfill several purposes is considered. Such a system has limited resources, determined during the administration process, based on the real capabilities of the developer. When optimizing the system, it is necessary to take these restrictions into account without violating them. This explains the presence of so-called "red lines", approaching which is undesirable or completely unacceptable. The optimization problem contains optimization arguments that deliver the extremum to the objective function. The objective function is based on the concept of a nonlinear trade-off scheme, for which the "away from restrictions" principle is satisfied. The optimization problem is solved formally, without the direct participation of the decision maker (DM).*

Keywords: *administration, restrictions, "red lines", multi-criteria, scalar convolution, tension of the situation, non-linear compromise scheme.*

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Introduction

We consider an object O, the state of which is determined by a set of values $x_1, x_2, ..., x_n$, from the admissible region *X* that make up the vector $x = \{x_i\}_{i=1}^{n} \in X$. An object pursues several goals, the degree of achievement of each of them is expressed by the corresponding criterion $y_k(x), k \in [1,...s]$. The criteria form a vector $y = \{y_k(x)\}_{k=1}^{s} \in M$. Area *M* is determined by restrictions $y_{k\min}(x) \le y_k(x) \le y_{k\max}(x)$ obtained during the administration process. This is the analytical expression of "red lines". The optimization problem is to determine the arguments $x_1, x_2, ..., x_n$ by extremizing the objective function $Y[y(x)]$. Essentially, this function is a scalar convolution of the criteria vector $y(x)$, reflecting the utility function of the decision maker (DM) when solving a specific optimization problem. Scalar convolution is the act of *composing* criteria. A criterion $y_k(x)$ is a measure of the quality of functioning of object O in relation to the achievement of the *k*-th goal. If “more” means “better,” then to improve quality this criterion must be maximized. Otherwise, the criterion is minimized. For definiteness, we consider the optimization problem with minimized quality criteria.

In the class of problems under consideration, system resources are limited. If resources can be selected in an open area, then there are no “red lines”. In an unconstrained problem, the principle of rational organization [Voronin, 2017] is applied and a consensus solution can be obtained. Limitations on resources give rise to “red lines” and the solution to the problem can only be a compromise.

An example of the tasks under consideration would be diplomatic negotiations. Here the parties can make concessions on some aspects, but only insofar as such a concession does not bring the solution to the problem closer to the corresponding “red line”. Negotiations are considered successful when the maximum possible distance of concessions from the “red lines” is achieved.

The objective function $Y[y(x)]$ connects the vector of quality criteria with the optimization arguments. With some reservations, the optimization problem is formulated as finding such a combination of arguments from the domain of their definition at which the objective function acquires an extreme value. If, without loss of generality, we assume that “better” means “less”, then

$$x^* = \arg\min_{x \in X} Y[y(x)].$$

Formalization

Restrictions can be imposed both on the optimization arguments $x \in X$ and on the criteria of the effectiveness of the solution $y \in M$. Even small changes in constraints can significantly affect the solution results [Lebedeva at al, 2020].

Moreover, the very concept of a decision-making *situation* is assessed by the measure of the dangerous approach of individual criteria to their maximum permissible values ("red lines"). Indeed, it is logical to consider the difference between the current value of the criterion and its maximum permissible value as a measure of the *tension of the situation*:

$$\rho_k(x) = y_{k\max} - y_k(x),\ \rho_k \in [0,\ldots y_{k\max}],\ k \in [1,\ldots s],$$

where $y_{\max} = \{y_{k\max}\}_{k=1}^{s}$ is the vector of maximum permissible minimized criteria.

Let's consider the logic of decision-makers in polar decision-making situations. If some criterion $y_p(x).\ p \in [1,\ldots s]$ is dangerously approaching its limit $y_{p\max}$, that is $\rho_p(x) \to 0$, then we call such a situation *tense*. In a tense situation, the decision-maker pays primary attention only to this, the most "unfavorable" criterion, trying to remove it from the dangerous "red line". In this case, with criteria of one dimension, the optimization problem is solved using the *minimax* (Chebyshev) model

$$x^* = \arg\min_{x\in X} Y[y(x)]_1 = \arg\min_{x\in X} \max_{k\in[1,\ldots s]} y_k(x).$$

In less tense situations, one should turn to satisfying other criteria, taking into account the contradictory unity of all interests and goals of the system. At the same time, the decision maker varies his assessment of winning according to some criteria and losing according to others, depending on the situation.

When $\rho_k(x) \to 1, k \in [1,\ldots s]$ the partial criteria are small, there is no threat of violating the restrictions. In a calm situation, the decision maker believes that a unit of deterioration in any of the criteria is fully compensated by an equivalent unit of improvement in any of the others. Here the optimization problem is solved using the *integral optimality* model

$$x^* = \arg\min_{x\in X} Y[y(x)]_2 = \arg\min_{x\in X} \sum_{k=1}^{s} y_k(x).$$

Thus, as a rule, the decision maker varies his choice from the integral optimality model in calm situations to the minimax model in tense situations. In intermediate cases, the decision maker selects optimality principles that provide different degrees of satisfaction of individual criteria, in accordance with the given situation. The type of objective function $Y[y(x)]$ depends on the chosen optimality principle and reflects the scheme of compromises adequate to the given situation.

Nonlinear Trade-off Scheme

Based on the above analysis of decision maker logic, it is advisable to formalize the multi-criteria decision procedure and obtain an algorithm that reflects this logic, but does not explicitly include elements of the human factor. From the standpoint of a systems approach, it is advisable to replace the task of *choosing* a compromise scheme with an equivalent task of synthesizing some *unified* scalar convolution of criteria, which in various situations would automatically express adequate principles of optimality. Separate models of trade-off schemes are combined into a single holistic model, the structure of which is adapted to the situation of making a multi-criteria decision.

The synthesized function $Y^*[y(x)]$ must be smooth and differentiable; in tense situations it should express the minimax principle $Y[y(x)]_1$; in calm conditions – the principle of integral optimality $Y[y(x)]_2$; in intermediate cases it should lead to Pareto-optimal solutions that provide various measures of partial satisfaction of the criteria.

Our principle of optimality “away from limits” expresses the desire in any situation to ensure the greatest possible distance from the “red lines”. Therefore, it is necessary to explicitly include a characteristic of the tension of the situation $\rho_k(x) = y_{k\max} - y_k(x),\ k \in [1,...s]$ in the expression for the desired scalar convolution. The simplest of them in the case of minimized criteria is scalar convolution

$$Y^*[y(x)] = \sum_{k=1}^{s} y_{k\max}\left[y_{k\max} - y_k(x)\right]^{-1}.$$

This objective function embodies a trade-off scheme that is adequate to the entire range of possible situations. Let's call the proposed scheme a nonlinear trade-off scheme (NTS). It corresponds to the vector optimization model

$$x^* = \arg\min_{x \in X} \sum_{k=1}^{s} y_{k\max} \left[y_{k\max} - y_k(x) \right]^{-1}.$$

Let's show how it works in different situations. If any of the criteria, for example $y_i(x)$, begins to approach its "red line" $y_{i\max}$, i.e. the situation becomes tense, then the corresponding term $Y_i = \dfrac{y_{i\max}}{y_{i\max} - y_i(x)}$ in the sum being minimized will increase so much that the problem of minimizing the entire sum will be reduced to minimizing only this worst term, i.e., ultimately, the criterion $y_i(x)$. This is equivalent to the action of the minimax model $Y[y(x)]_1$.

If the criteria are far from their limits, i.e. the situation is calm, then the model $Y^*[y(x)]$ acts equivalent to the integral optimality model $Y[y(x)]_2$. In intermediate situations, various degrees of partial alignment of criteria are obtained.

If "better" means "more", then for the criteria to be maximized the scalar convolution according to the nonlinear compromise scheme has the form

$$Y^*[y(x)] = \sum_{k=1}^{s} y_{k\min} [y_k(x) - y_{k\min}]^{-1},$$

where $y_{k\min}$ are the minimum acceptable values of the criteria to be maximized.

Note that the construction of scalar convolution $Y^*[y(x)]$ allows solving the optimization problem even in the case when the criteria have different dimensions.

A systematic approach to the problem of vector optimization made it possible to combine models of individual compromise schemes into a single holistic structure that adapts to the situation of making a multi-criteria decision.

The nonlinear compromise scheme has the property of *continuous adaptation* to the situation of making a multi-criteria decision. From this point of view, traditional trade-off schemes can be viewed as the result of the “linearization” of a nonlinear scheme at various “operating points” - situations. This, by the way, explains the name of the proposed *nonlinear* trade-off scheme, since in other respects it is no more “nonlinear” than other schemes considered in decision theory. We emphasize that the adaptation of a nonlinear scheme to the situation is carried out continuously, while the traditional choice of a compromise scheme is made discretely, which adds to the subjective errors the errors associated with the quantization of the compromise schemes.

Artificial Intelligence

The advantage of the concept of a nonlinear compromise scheme is the ability to make a multi-criteria decision formally without the direct participation of the decision maker, which is a distinctive feature of *artificial intelligence*. The apparatus of a nonlinear compromise scheme, developed as a formalized tool for studying systems with conflicting criteria, allows an artificial intelligence system to practically solve multi-criteria problems of a wide class.

Artificial intelligence (AI) systems are created in order to replace a person as a decision maker in a given situation [Savchenko, 2012]. AI systems such as robots, decision support systems, neural networks, etc. work in conditions that a person considers unfavorable for himself. Thus, a robot carrying out mine clearance operates in an environment that is dangerous for the sapper. Decision support systems are usually used under time pressure or in hostile environments. Neural network classifiers process volumes of information that exceed the capabilities of a human operator, etc. The implementation of the outlined stages of vector optimization allows, without the direct participation of the decision maker, to determine the architecture of the neural network classifier, in which conflicting criteria for the effectiveness of its functioning are systematically linked, and the resulting architecture itself is a compromise-optimal one.

The analytical solution to the optimization problem is represented as a solution to the system of equations

$$\frac{\partial Y^*[y(x)]}{\partial x_k} = 0,\ k \in [1,...,s].$$

If the analytical solution turns out to be difficult, then numerical methods or a computer program for multi-criteria optimization are used [Voronin, 2017].

Resource Allocation Problem

In the areas of management and economics, the actual task is to distribute the resources of a managed system between individual elements (objects), which ensures the most efficient functioning of the system in given circumstances. The problem of allocation of limited resources is the main problem of economics. They say that the correct distribution and redistribution of resources is the economy itself.

With a large number of objects and in critical cases, the cost of an error in management decisions increases sharply. It becomes necessary to develop *formalized* decision support methods for the competent distribution of limited resources between objects.

The most common case is that the total (global) resource of the system is limited and must be distributed among individual objects. In practical cases, restrictions are imposed not only on the global resource, but also on the partial resources allocated to individual objects. In this case, restrictions can be imposed both from below and from above.

An example of *lower* bounds is the distribution of fuel between aircraft when flying to different cities. For each flight there is a lower limit, below which it is pointless to allocate fuel; the plane simply will not reach its destination. This is the essence of the lower bound for each partial resource. If a given flight receives fuel in excess of a known lower limit, then it has the opportunity to freely maneuver through flight levels, bypass a thunderstorm front, go to an alternate airfield, etc.

On the other hand, it is also impossible to increase a partial resource unlimitedly; there is an *upper* limit for it. This is understandable, if only because each aircraft has a certain tank capacity, more than which it physically cannot take fuel on board.

Such restrictions (“red lines”) are either known in advance or are determined during administration by technical and economic calculations or expert assessment methods. Given a set of restrictions, it is required to distribute the global resource of the system between objects in such a way as to ensure the most efficient operation of the entire system as a whole. The problem is to construct an adequate objective function to optimize the process of resource allocation under conditions of their limitation.

In this paper, to solve the problem under consideration, a multi-criteria optimization approach is taken using a nonlinear trade-off scheme [Voronin, 2017].

Formulation of the Problem

The problem under consideration is relevant for various subject areas. Therefore, we present the statement of the problem in general form. The global resource R to be distributed is specified, as well as the $n \geq 2$ system elements (objects), each of which is allocated a partial resource r_i ; their totality constitutes a vector $r = \{r_i\}_{i=1}^{n}$. It is clear that $\sum_{i=1}^{n} r_i = R$.

For each object, the maximum permissible value of the allocated resource $r_{i\min}$ is known (or determined by the method of expert assessments), less than which the given object cannot function. Thus, a system of restrictions from below is specified

$$r_i \geq r_{i\min}, \sum_{i=1}^{n} r_{i\min} \leq R, i \in [1, n].$$

On the other hand, for each object a value $r_{i\max}$ is known, which the object’s resource cannot or should not exceed. The system of restrictions from above has the form

$$r_i \leq r_{i\max}, \sum_{i=1}^{n} r_{i\max} \geq R, i \in [1, n].$$

The task is posed: to determine such partial resources $r^* \in X_r$, at which the requirement $\sum_{i=1}^{n} r_i = R$. is met and the objective function $f(r)$, , the form of which is determined by a nonlinear compromise scheme, acquires an extreme value:

$$f(r) = \sum_{i=1}^{n} r_{i\min} (r_i - r_{i\min})^{-1} .$$

Based on the above, the problem of vector optimization of the distribution of limited resources, taking into account the isoperimetric constraint for the arguments, takes the form

$$r^* = \arg\min_{r \in X_r} f(r) = \arg\min_{r \in X_r} \sum_{i=1}^{n} r_{i\min} (r_i - r_{i\min})^{-1}, \sum_{i=1}^{n} r_i = R .$$

This problem can be solved both analytically, using the Lagrange method of indefinite multipliers, and by numerical methods if the analytical solution turns out to be difficult.

The analytical solution involves constructing the Lagrange function in the form

$$L(r,\lambda) = f(r) + \lambda(\sum_{i=1}^{n} r_i - R),$$

where λ is the indeterminate Lagrange multiplier, and the solution to the system of equations

$$\begin{aligned} \frac{\partial L(r,\lambda)}{\partial r_i} &= 0, i \in [1,n] \\ \frac{\partial L(r,\lambda)}{\partial \lambda} &= \sum_{i=1}^{n} r_i - R = 0 \end{aligned} .$$

Illustrative Example

To carry out two aviation flights (n=2), the airport has fuel with a total volume of R=12 tons (the figures are relative). The minimum requirement for the first flight is $r_1 \geq r_{1\min}$ = 2 tons, for the second flight $r_2 \geq r_{2\min}$ = 5 tons. These are lower bounds for partial resources. The tank capacity of the first plane $r_{1\max}$ = 7 tons, the second plane $r_{2\max}$ = 10 tons. These are restrictions from above.

The task is set: to obtain an analytical solution for the compromise-optimal distribution of fuel between flights.

Constructing the Lagrange function

$$L(r,\lambda) = r_{1\min}(r_1 - r_{1\min})^{-1} + r_{2\min}(r_2 - r_{2\min})^{-1} + \lambda(r_1 + r_2 - R)$$

we get a system of equations

$$\begin{aligned} &\frac{\partial L(r,\lambda)}{\partial r_1} = -r_{1\min}(r_1 - r_{1\min})^{-2} + \lambda = 0 \\ &\frac{\partial L(r,\lambda)}{\partial r_2} = -r_{2\min}(r_2 - r_{2\min})^{-2} + \lambda = 0\,. \\ &r_1 + r_2 - R = 0 \end{aligned}$$

Substituting numerical data

$$\begin{aligned} &-2(r_1 - 2)^{-2} + \lambda = 0 \\ &-5(r_2 - 5)^{-2} + \lambda = 0\,, \\ &r_1 + r_2 - 12 = 0 \end{aligned}$$

and solving this system by the Gaussian method (sequential elimination of variables), we obtain

$$r_1{*} = 3{,}94 \text{ tons}, \; r_2{*} = 8{,}06 \text{ tons}.$$

The apparatus of a nonlinear compromise scheme, developed as a formalized tool for studying systems with conflicting criteria, allows one to practically solve multi-criteria problems of a wide class.

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