14. Shaffer S. A. Test of Competition in Canadian Banking // Journal of Money, Credit and Banking. – 1993. – Vol. 25, № 1. – P. 37–56.
15. Akhrimenko A.A., Zhelezko B.A., Ksenovich D.V., Morozevich A.N. Methods of Multilevel Aggregated Estimate and Forecasting of Enterprises Financial Position // http://sedok.narod.ru/scgroup.html.

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AN ALGORITHM FOR FRESNEL DIFFRACTION COMPUTING BASED ON FRACTIONAL FOURIER TRANSFORM

Georgi Stoilov

Abstract: The fractional Fourier transform (FrFT) is used for the solution of the diffraction integral in optics. A scanning approach is proposed for finding the optimal FrFT order. In this way, the process of diffraction computing is speeded up. The basic algorithm and the intermediate results at each stage are demonstrated.

Key words: Fresnel diffraction, fractional Fourier-transform

ACM Classification Keywords: G.1.2 Fast Fourier transforms (FFT)

Introduction

The analysis of a great number of optical systems and devices requires diffraction computing under various conditions. This task can be solved through the implementation of modern methods of optical and digital image processing. Precise computing of the diffraction pattern obtained by illuminating complex transmitting objects or reflecting surfaces is a problem requiring huge computing resources. Thus, the necessity becomes obvious of introducing fast computing algorithms and of reducing the computational volume by simplifying the solution of the wave equation [1]

$$\nabla^2 \upsilon - \frac{1}{c^2}\frac{\partial^2 \upsilon}{\partial t^2} = -s \,, \qquad (1)$$

where c – speed of light, υ - a scalar quantity, describing the wave in an arbitrary point in space, $s(x,y,z,t)$ – a known function of the irradiating surface.

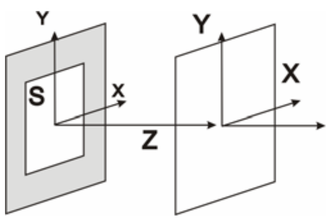

Fig.1. Irradiating and recording surface

In some cases the irradiating and the recording surface (Fig.1) can be presented as parallel planes. Most of the used approximations are based on the solution of Eq.(1) using Kirchhoff's integral [2]:

$$A(X) \approx \int_s \frac{1}{r} P(x) e^{ik(x-X)} \cos\left(\frac{\sqrt{x-X}}{r}\right) dx \qquad (2)$$

$$k = \frac{2\pi}{\lambda}, r = \sqrt{(x-X)^2 + \Delta Z^2} ,$$

where *P(x)* – surface irradiation function *S*; *r* - radius-vector; λ - wavelength. At that, the solution is simplified by using different assumptions. For the sake of simplicity, we consider the solution in the one-dimensional case. The most inaccurate approximation of the diffraction integral, used in optics, is replacing it with its Fourier-transform (FT) [2].

$$A(X) \approx \frac{1}{r} \int_s P(x) e^{ik(x-X)} dx \qquad (3)$$

The necessary condition for applying this approach is that the irradiating object aperture and the size of the diffraction pattern be much smaller than the distance between them. What is normally considered in optical systems is light diffraction in vacuum or light transmission through spherical optical elements (lenses) whose implementation introduces a quadratic phase multiplier in Kirchhoff's integral. In order to solve that form of the diffraction integral, FrFT introduced by Victor Namias in 1980 is successfully used [3]. It has different definitions that are proven to be equivalent and are used depending on the field of application. One of the definitions is:

$$f_a(X) \equiv \int_{-\infty}^{+\infty} \sqrt{1 - i\cot(\alpha)} e^{i\pi\left(\cot\alpha . X^2 - 2\csc\alpha . x . X + \cot\alpha . x^2\right)} f(x) dx , \qquad (4)$$

where a is the fractional Fourier-transform order and $\alpha = \frac{a\pi}{2}$.

The strict (accurate) proof and the conditions for its use in various optical diffraction problems is elaborated by Ozaktas, Zalevsky and Kutay [4]. The diffraction integral for the optical lens system and the propagation of the wave throgh vacuum is described in the following way:

$$\hat{h}_{lens}(x, X) = \delta(X - x) e^{\frac{-i\pi X^2}{\lambda f}}$$
$$\hat{h}_{space}(x, X) = e^{\frac{i2\pi d}{\lambda}} e^{\frac{-i\pi}{4}} \sqrt{\frac{1}{\lambda d}} e^{\frac{i\pi}{\lambda d}(X-x)^2} , \qquad (5)$$
$$\hat{g}(X) = \int_s P(x) \hat{h}\ (x, X) dx$$

where $\hat{g}(X)$ is the complex amplitude of the wave field in the plane of diffraction at a distance *d*, $\hat{h}_{lens}(x, X)$ is the nucleus ised in the case of using thin lenses with a focal distance *f* and $\hat{h}_{space}(x, X)$ is the wave propagation in vacuum.

Like FT, FrFT can be presented in the form of a sum instead of in the form of an integral. The main reason for that transition to discreet functions instead of continuous functions is the possibility of implementing computer processing of the digital images. FrTT can be represented by several sequential operations one of which is FT [4,6]. The natural elaboration of the approach is to seek fast computational algorithms analogous to fast Fourier-transform (FFT). An algorithm and software for fast transform of the authors referred to earlier are used for the computation of FrFT.

Problem

In a number of cases the condition for the implementation of FrFT cannot be fulfilled because of the large aperture of the object compared to the distance at which the diffracted wave is recorded. There is no analytical solution of the integral in that case. It is known that in the far field the diffracted wave function can be described by FT. The function behaviour in its intermediate states in the transition of the function to its Fourier form is represented by FrFT. Under such conditions, the solution of Kirchhoff's integral can be carried out by FrFT and find the FrFT order at which the best approximation is achieved.

Computing algorithm

In the image a line is selected that passes through some complex area, i.e. the line consists of elements of different and, if possible, large amplitude. Thus, the approximation will have a more distinct maximum with the change in the approximation parameter. For this value of the approximation parameter, Kirchhoff's integral is calculated, taking into account Eq.(2) and fast FrFT (FFrFT) (6). The aim is to find the best coincidence of the two solutions by changing the FrFT order. Using the parameter of the selected line found in this way, the FrFT for the whole image is calculated.

The control of the approximation can be carried out by selecting several lines and columns for which the parameter is calculated and its average value is obtained.

Another version of the algorithm is based on calculation of the Kirchhoff's integral followd by reverse transform of the obtained data by means of FrFT. In that case, the reconstructed and the original image can be compared more easily because, normally, there is no complex component, and in the reconstructed image the complex component is present only as a result of inaccurate approximation and calculation.

Results

In order to verify the algorithm, a simple object was chosen – a ring, (Fig. 2) with a constant value of the illuminated areas and a zero background. For the sake of simplicity, only the behaviour of the horizontal component of the diffraction was analysed. In this way, the changes in the image at the periphery of the illuminated zones are seen more clearly. A square aperture sized 102.4 μm was chosen and the discretisation step was 100 nm.

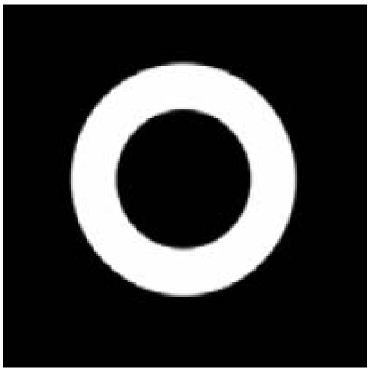

Fig. 2. Original test image

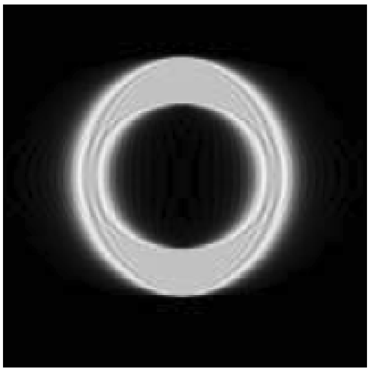

Fig. 3. Diffraction pattern in the Fresnel zone

The calculation of Kirchhoff's integral in the Fresnel zone was carried out at a distance of 10 nm. A wavelength of 533 nm was chosen (Fig.3).

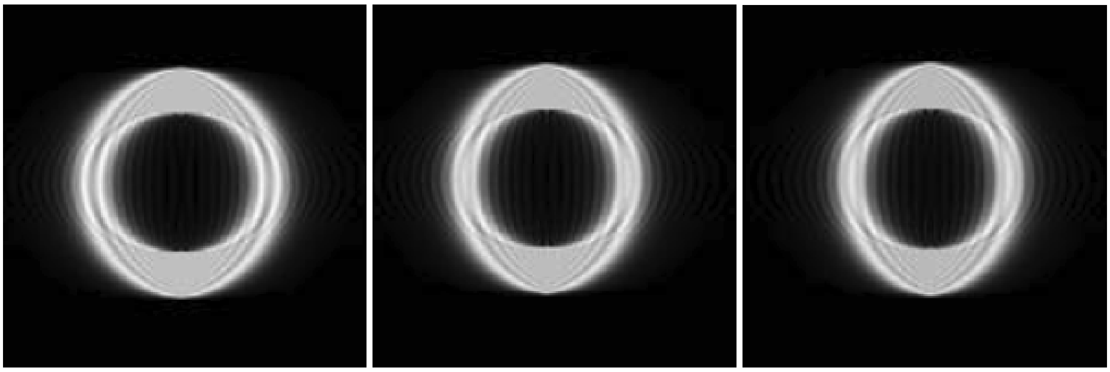

Fig. 4. Reverse transform of the lines -0.2, -0.3 and -0.34 FrFT

The reverse calculation is accomplished by seeking a solution by FrFT. The results at different values of the FrFT order are shown in Fig.4.

When the function is known, the optimisation criterion can be sought as the smallest mean square error of the difference between the original and reconstructed image. If the original is unknown and only the amplitude response is varied and the phase is kept constant, the approach of minimisation of the imaginary component of the reconstructed image can be applied. This is the case of a parallel laser beam passing through an amplitude mask.

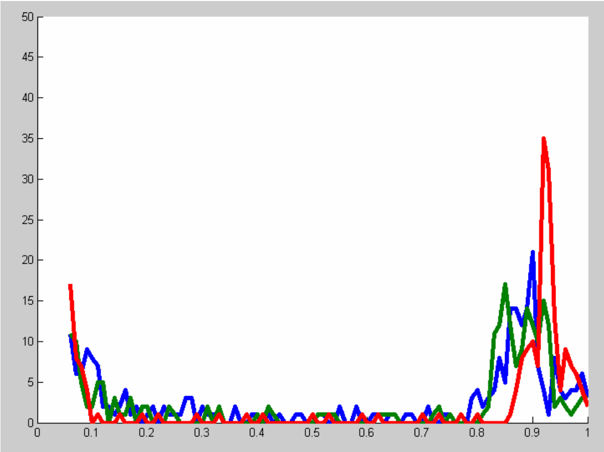

Fig. 5. Histogram of the image values for various FrFT orders

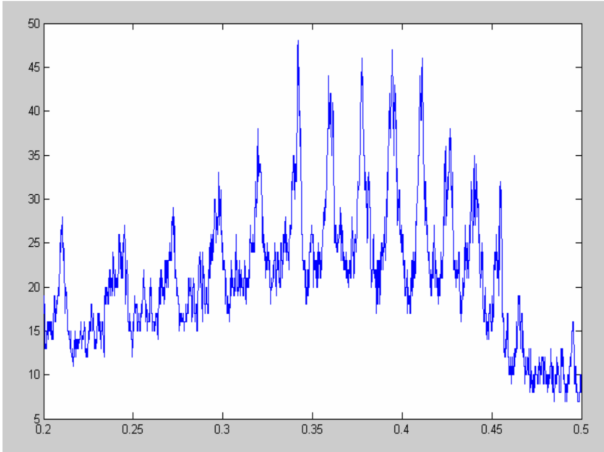

Fig. 6. Maximum in the histogram depending on the FrFT order

In the object that we selected, the amplitude has just two values: 1 and 0. Thus, the clarity of the white part of the image can be used as a criterion for successful reconstruction. Fig.5 displays a histogram of the values for some FrFT orders. In real measurements, the calculation of FrFT and the normalization of the data suppress the real value of the amplitude. If there is a solution close to the target, the values will be grouped in two areas: around 0 and around the amplitude. When normalization is accomplished after FrFT, the amplitude value is slightly below 1. The values close to zero are not shown since we are looking for a maximum close to 1.

The search for a solution of the problem begins by successive changes in the FrFT order in the range from -1 to 0. The solution is an order at which the highest maximum in the histograms in Fig.5 is achieved. Because of the periodicity of the function for searching the global maximum, scanning is possible only in the given range. The calculation of the FrFT order with an accuracy of 0.01 makes possible the implementation of fast converging algorithms, for example, the method of division of the range in two.

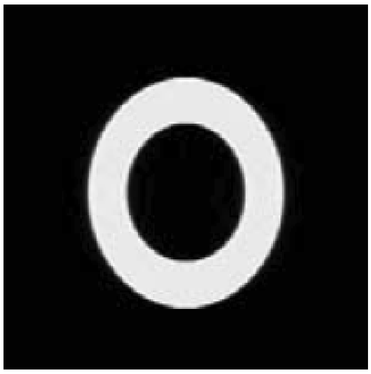

Fig. 7. Reconstructed image

It is seen that the maximum appears at a FrFT order of around 0.34 (exact value 0.3419). The reconstructed image at this value is shown in Fig.7. Scaling of the image is not taken into consideration in the calculation process. When the parameter takes different values, the size of the image varies. The effect exhibits itself as an image deformation in horizontal direction, since the calculations are made in that direction.

Software

A software program is developed in two parts. The programming language is Microsoft Visual C for MS WINDOWS. The first part of the program computes Kirchhoff's integral by the rectangular method. Due to the oscillating nature of the curves, the integration error is slightly higher than in the case of trapezoidal formula or other approximations of higher order. The computational process is based on a two-kernel processor Atlon 64 4400+ and operating system Windows XP. Processing of an array of 256x256 pixels takes 16 minutes, processing of an array 512x512 – 8 hours, and 1024x1024 – 120 hours.

The second module of the program takes care of the FrFT and of its order optimization. Computation of FrFT of an image of 1024x1024 pixels takes 1 minute. In the optimization process, FFrFT is used just for one line containing 1024 pixels. In this case, scanning for optimization purposes in the range from -1 to 0 with a step of 0.01 takes three seconds.

Conclusions

An algorithm is proposed for calculation of light diffraction in the Fresnel zone by finding the most suitable value of the FrFT order in one cross-section and its subsequent use for computing the whole image. Results are shown from test image processing for each stage of the algorithm. For the sake of obtaining the best visualization processing is carried out only along one axis.

References

1. Papoulis A., Systems and Transforms with Applications in Optics, McGRAW-HILL Book Company, 1968.
2. John M.Cowley, Diffraction physics, Amsterdam: North Holland, 1975.
3. V. Namias, The fractional order Fourier transform and its applications to quantum mechanics, J. Inst. Math Appl., 25, 241–265 (1980).
4. H.M. Ozaktas, M.A. Kutay, and G. Bozdagi. , Digital computation of the fractional Fourier transform. IEEE Trans. Sig. Proc., 44:2141{2150, 1996.
5. Ozaktas H., Zalevsky Z., Kutay M., The Fractional Fourier Transform with Application in Optics and Signal Processing, John Wiley & Sons, Ltd, 2001.
6. Bultheel A., K.U.Leuven H. Martnez-Sulbarany , Recent developments in the theory of the fractional Fourier and linear canonical transforms, Bulletin of the Belgian mathematical Society- Simon Stevin, 2006.

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DEVICE FOR COUNTING OF THE GLASS BOTTLES ON THE CONVEYOR BELT

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Abstract: In the present paper the results from designing of device, which is a part of the automated information system for counting, reporting and documenting the quantity of produced bottles in a factory for glass processing are presented. The block diagram of the device is given. The introduced system can be applied in other discrete productions for counting of the quantity of bottled production.

Keywords: device for counting, automated information system

ACM Classification Keywords: J.2 Physical Sciences and Engineering

Introduction

In all discrete productions it is needed the ready production to be counted as well as reporting and documenting of the received data. In the present paper a device for counting the quantity of the produced glass bottles, moving on conveyor belt and which is designed by the authors is presented. It is a part of the automated information system for reporting and documenting of the ready production in a factory for glass processing [Draganov, 2006]. The information system has to meet following requirements: collecting data for the ready production, moving in one direction on the conveyor belts; archiving the data for each shift; reporting the quantity of the production for a shift (eight hours).