BIVIRTUAL ORGANIZATION AS A QUEUING SYSTEM

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Abstract: The main features of virtual organizations are outlined. The mathematical models of functioning of virtual organization are offered on the basis of theory of queuing systems. Characteristics of efficiency are examined.

Keywords: virtual enterprise, virtual laboratory, queuing system.

ACM Classification Keywords: I.6.4 Model Validation and Analysis; K.6.1 Project and People Management - Systems analysis and design.

Introduction

Saturation of goods markets and development of information technologies made possible such kind of organization of productive activity as virtual organization.

In the present work we shall consider such subspecies of virtual organization as a virtual laboratory.

There have been suggested mathematical models of virtual organization functioning on the basis of the queuing theory. The efficiency characteristics are considered.

Problem Statement

Let us consider virtual organization as the combination of three interconnected components: local agents (LA), upper agents (UA) and service [Dawidow, 1992]. For virtual laboratory these components could be interpreted in the following way: local agents – students, upper agents – the educational institution and a server.

In case of a server work we assume, that the assembly of agents represents the source of queries of unlimited capacity, and a server is a queuing system (QS), assigning time for the queries processing [Minzberg, 2001].

The average time for one request handling t_{serv} is composed of the average time of the start of connection t_{start}, time of connection $\bar{t}_{connect}$ and the time of the end of connection $\bar{t}_{end}$: $\bar{t}_{serv} = \bar{t}_{start} + \bar{t}_{connect} + \bar{t}_{end}$, at that the meanings of $\bar{t}_{start}$ and $\bar{t}_{end}$ are small comparing with the value of $\bar{t}_{connect}$. The intensity of handling is a value, inverse to the average time of handling, so we receive:

$$\mu = \frac{1}{\bar{t}_{serv}} = \frac{1}{\bar{t}_{start} + \bar{t}_{connect} + \bar{t}_{end}}.$$

The intensity of arrival of queries from agents we shall indicate as λ.

For the sake of simplification we assume, that the server is the one-channel QS with the refusals. Let's assume that there exist two standard situations in the server operation: 1) the "hot season", when the server can not handle with the flow of requests (such faults could, for example, arise before the session, when the large quantity of lower agents apply with the queries); 2) the "dead season" or the vacations, when the intensity of arrival of service requests sharply drops.

The state of the QS being considered is determined after the number of queries in it:

1) for the virtual organization:
   - S_0 - in the system there is no queries;
   - S_1 - in the system there is one query and the UA is handling it.
2) for the virtual laboratory:
   - S_0 – in the QS there is no queries (the server stands idle);
   - S_1 – in the QS there is one query (the server is busy with the handling of the given query).

The transition from S_0 to S_1 takes place depending of the intensity of requests and the probability of the connection confirmation $p_{connect}$.

From the given description of the QS functioning we receive, that the density $d_{01}(t)$ of the system transition from the state S_0 into the state S_1 is equal to the product of $p_{connect}$ by the intensity λ of the incoming flow of queries, and the density $d_{10}(t)$ of the transition from the state S_1 into the state S_0 – to the intensity μ of the flow of queries handling. That is why the transition graph will be of the form, represented at Fig. 1

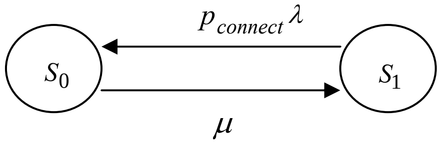

Fig. 1. Transition graph of the interaction model of agents and the server (for virtual laboratory)

Let us assume that all the flows of events in the QS are the simplest. Than in the QS the Markovian process takes place. After the transition graph we receive, that the functioning of the QS is depicted by the Chapman-Kolmogorov differential equation system [Kremer, 2001]:

$$\frac{dp_0(t)}{dt} = -p_{connect}\lambda p_0(t) + \mu p_1(t)\,,$$

$$\frac{dp_1(t)}{dt} = -\mu p_1(t) + p_{connect}\lambda p_0(t)\,,$$

together with the normalization requirement

$$p_0(t) + p_1(t) = 1\,.$$

We consider, that at the initial moment of time in the system there were no queries, i.e. the system was in the state S_0:

$$p_0(0) = 1,\; p_1(0) = 0\,.$$

Excluding the second equation of the system and using the normalization requirement we receive the ordinary differential equation:

$$\frac{dp_0(t)}{dt} = -(p_{connect}\lambda + \mu)p_0(t) + \mu\,.$$

Analytical solution of the mentioned equation taking into account the initial conditions has the looks in the following way:

$$p_0(t) = \frac{\mu}{p_{connect}\lambda + \mu} + \frac{p_{connect}\lambda}{p_{connect}\lambda + \mu} e^{-(p_{connect}\lambda + \mu)t},\; p_1(t) = 1 - p_0(t).$$

Similarly after the transition graph we can obtain the balance equation (for the sake of establishing the working regime of the system):

$$p_{connect}\lambda p_0 = \mu p_1,\; \mu p_1 = p_{connect}\lambda p_0,\; p_0 + p_1 = 1\,,$$

from where the values of probabilities of the system states are calculated:

$$p_0(t) = \frac{\mu}{p_{connect}\,\lambda + \mu},\; p_1(t) = \frac{\mu}{p_{connect}\,\lambda + \mu}\,.$$

Let us consider the simplest model of interaction of the LA and the service. If ν is the intensity of offers from service to the LA, and η is the intensity of theirs accepting by the LA, then there are two stable states of the system: S_0^1 - the proposal is accepted, S_1^1 - the proposal is not accepted.

Model of interaction of the agent and the server is considered similarly. Let ν be the billing intensity by the server to the agent; and η – the intensity of theirs payment by the agent. As earlier, let us assume, that all the flows of the events are the simplest flows and consequently in the system the Markovian process takes place. Two stable states if the system considered could be distinguished: S_0^1 – the bill is paid, S_1^1 – the bill is unpaid.

Then the transition graph will be of the form, represented at Fig. 2.

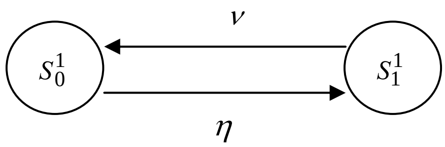

Fig. 2. Transition graph of the interaction model of the local agents and the service

Solving the given problem similarly to the previous one, we shall receive the probability values of the QS state for the steady-state working regime

$$P_0 = \frac{\eta}{\nu}, P_1 = \frac{\eta}{\nu+\eta}.$$

Integrating the received simplest modes we shall obtain the QS with four states:

1) for virtual organization:
   - S_{00} - the UA is free, the proposal has been accepted;
   - S_{01} – the UA is free, the proposal has not been accepted;
   - S_{10} - the UA is occupied, the proposal has been accepted;
   - S_{11} – the UA is occupied, the proposal has not been accepted.
2) for virtual laboratory:
   - S_{00} - the server is free, the bill is paid;
   - S_{01} - the server is free, the bill is unpaid;
   - S_{10} - the server is occupied, the bill is paid;
   - S_{11} - the server is occupied, the bill is unpaid.

Describing functioning of the QS as changes of its states we can obtain the transition graph, represented at Fig. 3.

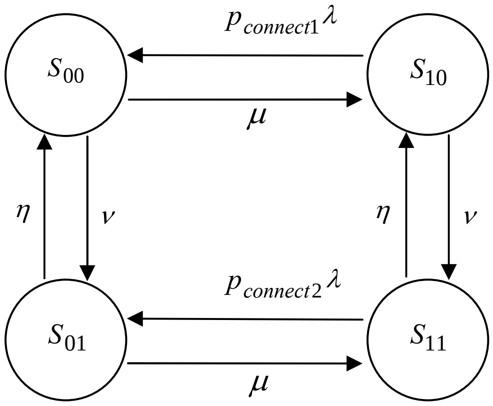

Fig. 3. Transition graph of the integrated model (of the fragment of virtual organization)

Balance equation

After the transition graph we obtain the mathematical model of the system functioning - the balance equation:

$$(p_{connect1}\lambda + \nu)p_{00} = \mu p_{10} + \eta p_{01},$$
$$(\mu + \nu)p_{10} = p_{connect1}\lambda p_{00} + \eta p_{11},$$
$$(\eta + p_{connect1}\lambda)p_{01} = \nu p_{00} + \mu p_{11},$$
$$(\mu + \eta)p_{11} = p_{connect2}\lambda p_{01} + \nu p_{10},$$
$$p_{00} + p_{10} + p_{01} + p_{11} = 1.$$

Solution of the given system of linear algebraic equations looks in the following way:

$$p_{00} = \frac{1-\alpha_2 p_{11}}{\alpha_1},$$

$$p_{01} = \frac{1-\beta_2 p_{11}}{\beta_1},$$

$$p_{10} = \frac{1-\gamma_2 p_{11}}{\gamma_1},$$

$$p_{11} = \frac{2}{(\frac{\alpha_2}{\alpha_1}+\frac{\beta_2}{\beta_1}+\frac{\gamma_2}{\gamma_1})},$$

where

$$\alpha_1 = 1+\frac{\nu}{\eta + p_{connect2}\lambda}+\frac{p_{connect1}\lambda}{\mu+\nu},$$

$$\alpha_2 = 1+\frac{\mu}{\eta + p_{connect2}\lambda}+\frac{\eta}{\mu+\nu},$$

$$\beta_1 = 1+\frac{\eta + p_{connect2}\lambda}{\nu}+\frac{p_{connect2}\lambda}{\nu},$$

$$\beta_2 = 1-\frac{\mu}{\nu}+\frac{\mu+\eta}{\nu}=1+\frac{\eta}{\nu},$$

$$\gamma_1 = 1+\frac{\mu+\nu}{p_{connect1}\lambda}-\frac{\nu}{p_{connect1}\lambda}=1+\frac{\mu}{P_{connect1}\lambda},$$

$$\gamma_2 = 1-\frac{\eta}{p_{connect1}\lambda}-\frac{\mu+\eta}{p_{connect2}\lambda}=1-\frac{\mu}{P_{connect2}\lambda}.$$

Characteristics of functioning of virtual laboratory (virtual organization)

After the proposed model the following system characteristics are calculated:

- probability of the refusal because of the bill, unpaid by the agent $P_{ref,unb} = p_{00}(1-p_{connect1}) + p_{01}(1-p_{connect2})$
- probability of the refusal because of the occupancy of the server (consequently, the UA) $P_{ref,s} = p_{10} + p_{11}$
- probability of the refusal of the servicing $P_{ref} = P_{ref,unb+}P_{ref,s}$
- relative throughput $q = 1 - P_{ref} = p_{00}p_{connect1} + p_{01}p_{connect2}$
- absolute throughput $A = \lambda q$
- average time of staying of the query in the system $\overline{Z} = \frac{A}{\mu} = \frac{\lambda}{\mu} q = \rho(1-P_{ref}) = \rho p_{00} p_{connect1} + \rho p_{01} p_{connect2}$

Conclusion

The mathematical models of functioning of virtual organizations, namely of virtual laboratories in the capacity of one-channel queuing system with the refusals were considered. The characteristics of functioning of the considered objects were analysed.

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