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GENERALIZED INTERVAL ESTIMATIONS IN DECISION MAKING AND SCENARIO ANALYSIS

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***Abstract**: Main features of a new method of expert knowledge elicitation – method of generalized interval estimations – and peculiarities of its using in decision making are presented. Expert estimation of an analyzed quantitative parameter is defined in the framework of the method by a set of intervals. This set may be shown on a plane (X, Y) by a curvilinear trapezoid. If densities of probability distributions were defined on Y-axis ("weights" of different interval estimations at their set) and on X-axis (for all interval estimations at the set) we received generalized interval estimation (GIE) of the parameter. GIE method may be used at different directions. Firstly, we may reduce GIE to mono-interval estimation averaging distributions on X-axis, taking into account their weights, and use then famous probabilistic methods to analyze problem. The average distribution is in fact a probability mixture of distributions on intervals of their set. Secondly, we may study complete structure of interval estimations that reflects expert knowledge in details. Here we automatically receive for resulting diagrams (X - parameter value, Y - level of probability) such probability distributions that are boundaries of all distributions of GIE (probability tube, or box). Besides we may receive probability distributions for different sections of p-tube both for parameter values and for probability levels and use these curves in the process of decision-making. At last, we may use GIE method as an instrument of scenario analysis in decision-making. Illustrative examples are given in the paper to demonstrate the decision-making, expert knowledge and scenario analysis aspects of the proposed approach.*

***Keywords**: Generalized interval estimations, scenario analysis, generalized probability distributions, probability tubes.*

***ACM Classification Keywords**: H.4.2 Types of Systems – Decision support; G.3 Probability and Statistics – Distribution functions.*

Introduction

During analysis of many practical problems there are rather often situations when decisions are made based on certain quantitative indicators, or criteria, important for decision-makers (e.g. net present value of investment projects in project analysis; present and future exchange rates, interest rates and stock prices in financial markets analysis; volumes of oil/gas reserves and prices in oil/gas property evaluation and others). The values of these indicators are usually the outcomes of various computational schemes (mathematical models). As a rule such models are already well known for many areas of human activity. However the complexity of real-life problems inevitably brings uncertainties into the values of input data (parameters) of models. In many situations, especially for multidisciplinary tasks, expert knowledge, in fact, could be the only way to overcome these difficulties and provide reasonable estimates for parameters. Thus to analyze many problems under uncertainty we should combine methods of expert knowledge elicitation, representation and methods of decision making. This stimulates the development of various approaches to expert judgment formalization, of methods and tools to elicit and represent expert knowledge in an adequate way.

In this paper, we present a survey of main results of a method to elicit and represent expert knowledge on quantitative parameters in problems under uncertainty – the method of Generalized Interval Estimations (GIE) [Shepelyov and Sternin, 2003; Chugunov et al., 2006; Chugunov et al., 2008 A; Chugunov et al., 2008 B; Shepelyov and Sternin, 2008]. We provide several illustrative examples to demonstrate both the decision-making, expert knowledge and scenario analysis aspects of the proposed approach.

Interval and Probabilistic Estimations

Until now, in spite of presence of other tools (e.g., fuzzy sets and possibility theory, rough sets), the main language that experts of real economy branches use to characterize uncertainties in input parameters of models is based on interval analysis and probability theory. Methods of interval analysis are adequate in situations under maximal uncertainty, when the expert is able to provide only the minimum and maximum value for the estimated parameter. But if one applies only interval methods to quantify uncertainty in the model input parameters, the width of the interval for the model outcome is likely to be very wide, and the value of such information often can hardly help one to make decisions. The combination of interval analysis and probability theory could be advantageous when the expert has more precise knowledge on the estimated parameters. In this case, the input data of models is represented by experts as random variables defined on certain intervals by their probability distribution functions that characterize uncertainty in the framework of interval estimations. If interval and probabilistic expert estimations for parameters of used models were received, final results of models could be calculated with methods of the Monte-Carlo family. Results of probabilistic calculations may be represented on a graph displaying the probability “not less than” (probability of guaranteed result) as a function of analyzed criteria.

Figure 1 shows an example of such graph for an oil reservoir evaluation for field that is in the predrilling (initial) stages of exploitation. Monte-Carlo calculations were made on base of the standard volumetric model [Welsh et al., 2004]. Estimated reserves Q are calculated as the product of input parameters of the volumetric model representing reservoir properties and the expert specified the input parameters as random variables.

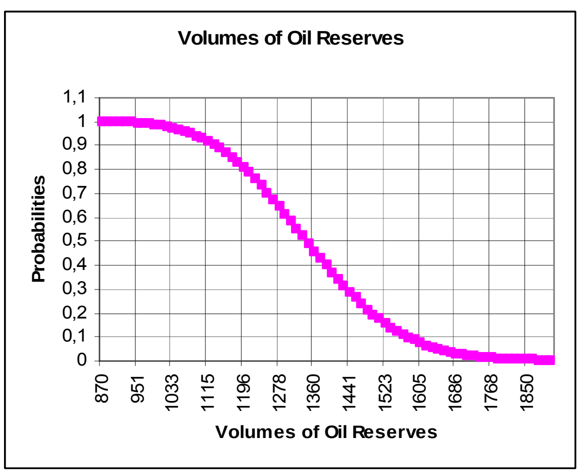


Fig. 1. An example of probabilistic reserves estimate

The decision-maker (DM) may analyze this graph from two points of view: what is the level of probability (chances on the realization of a chosen outcome) that corresponds to a desired value of oil reserves in place, or what is the value of the analyzed indicator for a given level of confidence? DM can then make a reasonable decision based on this interpretation. Based on the international classification of oil and gas reserves, the DM (investor) may obtain the reserves estimates (proved, probable and possible reserves categories) needed for a final decision.

For example, according to the classification, proved reserves would correspond to the probability level of 0.9; proved + probable reserves to the probability level of 0.6; proved + probable + possible ones to the probability level of 0.2. Let us note that these estimations are point ones though initial parameters had interval estimations. Various measures of risk connected with different possible decisions may be introduced and used in the framework of this approach.

Poly-interval and Generalized Interval Estimations

It may seem that the “mono-interval” approach could be enough to quantify uncertainty in decision making problems of mentioned above class. However, our joint work with geologists as experts showed some narrowness of such way. Experts, giving interval estimation of a parameter, may meet some difficulties both on length and on location of estimation. Attempts to eliminate these difficulties defining very wide interval estimations decrease usefulness of expert knowledge. On the other hand, if an expert tries to be too precise (i. e. the interval estimation is too narrow) there is a high probability that the true value of the parameter could be outside of the given range. Psychologists [Kahneman et al., 1981] observed this effect for a more general environment, not only related to oil and gas industries. Thus the problem has sufficiently universal character.

This motivated us to develop a method that could allow expressing an expert knowledge on an estimated parameter in a less restrictive way and thus quantify expert’s knowledge more precisely. Namely, we started with the idea to represent possible variability of interval estimation length and location by letting the expert to specify a set of intervals, rather than just one interval. We named such construction poly-interval estimation (PIE) of a parameter. In many situations experts would feel more comfortable thinking of several possible scenarios represented by several corresponding intervals, rather than assessing only one interval to describe “overall” uncertainty in parameter. Thinking in scenario-oriented framework would also make expert pay more attention to each possible scenario and might reduce effects from heuristics [Morgan and Henrion, 1992].

Expert defines in the framework of the method, as an initial estimation of parameter D, an interval of minimal, in his opinion, length (or “up” interval – “best guess”) $[D_{lu}, D_{ru}]$ – here l, r mean left, right boundaries of the interval - and then, taking into account all possible uncertainties, an interval of maximal, in his opinion, length (“down”, or base, interval – “safe bet”) $[D_{ld}, Drd]$. Now, one may further assume that a subset of intermediate intervals between up and base intervals also belongs to the PIE, although this is not necessary, since the subset of intermediate intervals may be selected from all possible ones based on the expert knowledge. PIE may be represented on a plane (X, Y) by a curvilinear trapezoid. The simplest example of a PIE for nested set of intervals is shown in figure 2.

Here Y-axis (α-axis in our notation) is the axis of marks of intervals and we suppose that $0 \le \alpha \le 1$, and X-axis (D-axis in our notation) is the axis of values of parameter under consideration. Side boundaries of trapezoid could be also specified by the expert when he/she defines the subset of intervals in PIE. Each interval of the trapezoid represents a possible scenario of parameter realization and the expert may also specify chances or probability for each scenario (and thus probability distribution on α–axis). If the expert defines distributions both on α- and D-axes we obtain generalized interval estimation (GIE) corresponding to the PIE. Probability distribution on α-axis represents, in a sense, “weights” of scenario-bands. Probability distributions on D-axis represent chances, according to expert knowledge, on realization of possible values of parameter for a given scenario-interval. Thus GIE of a parameter is characterized by joint distribution function of two random variables α and D with density function $\Psi(\alpha, D) = f_1(\alpha)f_2(D|\alpha)$, defined on PIE of the parameter. Hence, there are several ways to represent uncertainty in the framework of GIE method: specification of initial (characteristic) interval estimations; shape of PIE; specification of distributions on scenario-intervals; specification of “chances” or “weights” for scenario-intervals. This also means that generalized interval estimations are able to represent the following two aspects of uncertainty. On the one hand, the expert may express certain confidence in that some subset of intervals from PIE adequately represents his/her understanding of the estimated parameter’s value (this is described by $f_1(\alpha)$). On the other hand, each possible scenario implies inherent variability of parameter’s value bound by the corresponding interval (this is described by $f_2(D|\alpha)$).

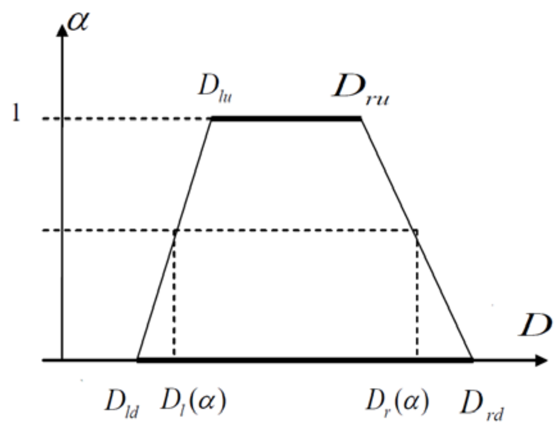


Fig. 2. Poly-interval estimation of an input parameter for nested set of intervals

One may build more general PIE and corresponding GIE, for example if an expert defines initial characteristic interval estimations to represent optimistic, realistic and pessimistic estimations corresponding to the bottom, middle and top intervals shown on Figure 3. Here we have shifted, not embedded as above, set of intervals.

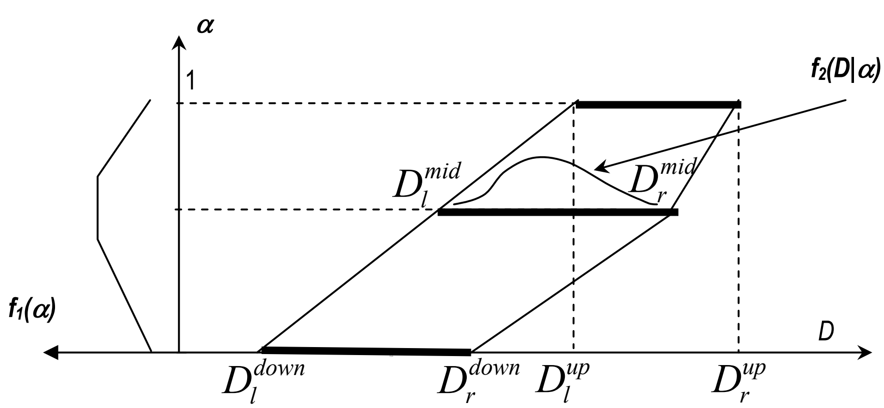


Fig. 3. Generalized interval estimation of an input parameter for shifted set of intervals

Another example of “shifted” PIE may arise in estimating volumes of oil recoverable reserves for ill-investigated fields. Volumes of recoverable reserves would depend not only on geology of the reservoir and technologic conditions but also on the oil prices. As the oil prices goes up, so does the estimation of recoverable reserves volumes. Let us construct the following PIE: Y-axis is axis of the oil prices, X-axis is axis of volumes of recoverable reserves. For each point estimate of the oil price there exists an interval estimation of recoverable reserves. The shape of such PIE would resemble a shape of the tower at Italian town Pisa, i.e. PIE is built on shifted intervals. This example also demonstrates additional possibilities of PIE/GIE method to quantify uncertainty in estimation of dependent parameters. Let also note that Y-axis is not here axis of interval marks but has real meaning.

Now there are three ways to use information provided by the expert in the framework of GIE approach for decision-making and expert analysis. GIE method allows one to obtain an aggregated (averaged) probabilistic distribution, which we also refer to as GIE projection, defined on the base interval of PIE (an interval, containing all possible values for D) and reduces calculation of the model output to a well-known statistical simulation procedure, which can be performed by Monte-Carlo family methods [Sternin and Shepelyov, 2003]. Alternatively, GIE can be treated as a set of intervals with a certain internal structure that represents uncertainty in expert knowledge concerning in estimated parameter [Chugunov et al, 2006]. At last GIE approach may be used as tool of scenario analysis in decision making [Shepelyov and Sternin, 2008]. All cases will be considered in the next sections.

GIE Projection: Mono-interval Approach in GIE Method

For simple case of nested intervals in PIE with straight boundaries (fig. 2) we can obtain the following expression for a probability density function of averaged distribution function $f_{av}(D)$:

$$f_{av}(D) = \begin{cases} \int\limits_0^{\alpha_l(D)} f_1(\alpha) f_2(D|\alpha) d\alpha, & D \in [D_{ld}, D_{lu}) \\ \int\limits_0^1 f_1(\alpha) f_2(D|\alpha) d\alpha, & D \in [D_{lu}, D_{ru}) \\ \int\limits_0^{\alpha_r(D)} f_1(\alpha) f_2(D|\alpha) d\alpha, & D \in [D_{ru}, D_{rd}]. \end{cases} \quad (1A)$$

Here $\alpha_l(D) = \frac{D - D_{ld}}{D_{lu} - D_{ld}}$, $\alpha_r(D) = \frac{D_{rd} - D}{D_{rd} - D_{ru}}$.

Then for averaged probability distribution function $P_{av}(D < D_S)$ we have:

$$P_{av}(D < D_S) = \begin{cases} \int\limits_0^{\alpha_l(D_S)} \int\limits_{D_l(\alpha)}^{D_S} f_1(\alpha) f_2(D|\alpha) d\alpha dD, & D \in [D_{ld}, D_{lu}) \\ \int\limits_0^1 \int\limits_{D_l(\alpha)}^{D_S} f_1(\alpha) f_2(D|\alpha) d\alpha dD, & D \in [D_{lu}, D_{ru}) \\ 1 - \int\limits_0^{\alpha_r(D_S)} \int\limits_{D_S}^{D_r(\alpha)} f_1(\alpha) f_2(D|\alpha) d\alpha dD, & D \in [D_{ru}, D_{rd}]. \end{cases} \quad (1B)$$

Here $D_l(\alpha) = \alpha D_{lu} + (1 - \alpha) D_{ld}$, $D_r(\alpha) = \alpha D_{ru} + (1 - \alpha) D_{rd}$. Similar but more complicated expressions could be derived for GIE with PIE of more general shapes, e.g., as the one in figure 3.

For simple distributions defined by functions f_1 and f_2 (e.g., uniform and triangular distributions) one can obtain direct analytical expression for GIE projection [Shepelyov and Sternin, 2003]. We will consider the case of uniform distributions in the paper some later.

Let us consider an illustrative example for a more complicated case, when GIE has different distributions on different scenario-intervals [Chugunov et al, 2006]. During the evaluation of investment projects in real-estate development business, the final decisions could be made only after detailed analysis of the lot under consideration and business perspectives of the area. Due to uncertainties of various natures, the values of some project's parameters are unknown at the moment of the analysis and are estimated by the expert. In case of business center development, such parameters would include the future price for square meter.

Based on the analysis and data available, the expert is asked to estimate this parameter. GIE method is used to elicit and quantify expert judgments. The results of elicitation process is given in figure 4, where we show density and cumulative probability functions of guaranteed results for the price for sq. m. as random variable. We also provide probability distributions for case where expert use mono-interval approach.

Interval from 2 to 11 thousand money units (m.u.) was specified by the expert as a base interval. Top interval was defined symmetrically – from 5 to 8 thousands m.u. Initially joint distribution function of GIE was built based on uniform distributions on both α and D.

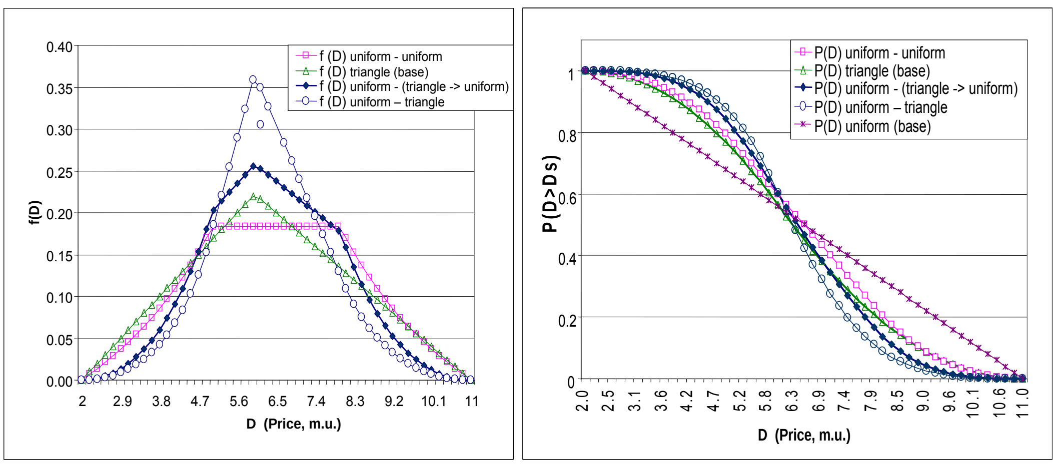


Fig. 4. Density function and cumulative probability function of guaranteed results for future price of sq. m.

Then expert tested a hypothesis with selection of the most preferable value for the price. The expert defined GIE as a combination of uniform distribution on α and triangle one on D with modal value at 6 thousands m.u. (this value is shifted to the left relative to the center of mini interval). This setup led to unreasonably high, in expert's opinion, chances for values in a neighborhood of the modal value. Since, according to the expert, the narrower the interval the more equiprobable the values are, the next hypothesis included uniform distribution on α and triangle distribution on D, with the same as earlier modal value on base interval, but it steadily transformed into uniform distribution on top (mini) interval.

Below we provide expression (2) for density $f(D|\alpha)$ of similar distribution, which could resemble one a shape of country house with roof and walls. On the base interval, the "house" has only "roof" (triangular distribution) and on mini interval it has only "walls" (uniform distribution).

$$f(D\,|\,\alpha)=\frac{1}{(D_r-D_l)}\begin{cases}\dfrac{2D-2D_l+\alpha(D_l+D_p-2D)}{D_p-D_l}, & D\in\left[D_l,D_p\right]\\ \dfrac{2D_r-2D-\alpha(D_r+D_p-2D)}{D_r-D_p}, & D\in\left(D_p,D_r\right]\end{cases} \quad (2)$$

Here D_r, D_l are respectively right and left boundaries of corresponding intervals of PIE (for a straight line case D_r, D_l are given by (1B)), D_p is modal value of distribution.

According to fig. 4, one can see that at level P = 0.85 specified by the expert as his “comfort level”, the difference in estimations of prices for sq. m., corresponding to the results from using mono-interval triangular distribution and expert-defined GIE, is more than 10%: 4340 m.u. and 4880 m.u. respectively.

GIE approach allows one to generate new mathematical objects - probability distributions that are generalizations of traditional distributions. For example, one can construct generalized uniform distribution (GUD) as a projection of GIE when uniform distributions are defined on both axes. Let us consider this simple but important for practice case in more details. In fact a class of GUD may be constructed. Some representatives of GUD will be used later in the process of discussing relationship of GIE method and scenario analysis. Now we consider the simplest case assuming the straight line boundaries of PIE on embedded intervals. Uniform distributions on both α and D axes are defined by $f_1(\alpha) = 1$, $f_2(D|\alpha) = 1/[D_r(\alpha) – D_l(\alpha)]$. The integrals in (1A, B) may integrate by parts to obtain the probability density function for the GUD:

$$f(D)=\frac{1}{B-M}\begin{cases}Ln\dfrac{B(D_{lu}-D_{ld})}{B(D_{lu}-D)+M(D-D_{ld})}, & D\in\left[D_{ld},D_{lu}\right]\\ Ln\dfrac{B}{M}, & D\in\left(D_{lu},D_{ru}\right)\\ Ln\dfrac{B(D_{ru}-D_{rd})}{B(D_{ru}-D)+M(D-D_{rd})}, & D\in\left[D_{ru},D_{rd}\right],\end{cases} \quad (3)$$

where $B = D_{rd} – D_{ld}$ and $M = D_{ru} - D_{lu}$. In the limit $M \to B$ this distribution transforms to traditional uniform one: $f(D) = 1/M$.

Corresponding expressions for cumulative probability of generalized uniform distribution are:

$$P(D<D_S)=\frac{1}{B-M}\begin{cases}\left(D_{lu}-D_{ld}\right)\alpha_l+\dfrac{B(D_{lu}-D_S)+M(D_S-D_{ld})}{B-M}Ln\dfrac{B-\alpha_l(B-M)}{B}, & D\in\left[D_{ld},D_{lu}\right]\\ D_{lu}-D_{ld}+\dfrac{B(D_{lu}-D_S)+M(D_S-D_{ld})}{B-M}Ln\dfrac{M}{B}, & D\in\left(D_{lu},D_{ru}\right)\\ \left(D_{lu}-D_{ld}\right)\alpha_r+\dfrac{B(D_{lu}-D_S)+M(D_S-D_{ld})}{B-M}Ln\dfrac{B-\alpha_r(B-M)}{B}+ & \\ +(B-M)(1-\alpha_r), & D\in\left[D_{ru},D_{rd}\right]\end{cases} \quad (4)$$

Here α_l and α_r are defined by (1A). It should be mentioned, that GUD may be interpreted as a probability mixture of uniform distributions with uniform mixing function. Specific distributions on the intervals in the mixture are defined by the shape of PIE.

It is interesting to compare behavior of uniform distributions defined on the base and mini intervals and GUD on the base interval. All three functions would intersect in one point. This point is defined by the following coordinates: $D_i = (BD_{lu} – MD_{ld})/(B – M)$, $P_i = (D_{lu} – D_{ld})/(B – M)$. Figure 5 shows three distribution functions for the following case: $D_{ld} = 10$, $D_{rd} = 50$, $D_{lu} = 30$, $D_{ru} = 35$.

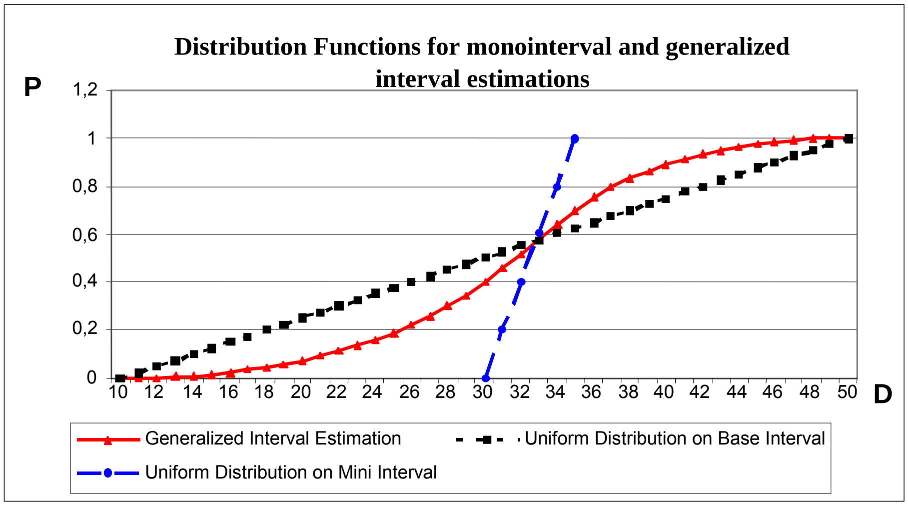


Fig. 5. Cumulative uniform probability functions for mono-interval and generalized cases

This interpretation of the GIE method may be used to analyze the decision making problems that earlier were approached with probabilistic mono-interval methods. In particular, oil reserves evaluation may be performed by GIE method when some or all of the input parameters of volumetric model are described by the expert as GIE estimations. Comparison of two final probabilistic curves for the oil reserves estimations shows that behavior of cumulative curve for GIE case is, in a sense, more balanced and reliable than the outcome for probabilistic mono-interval case. Specifically, GIE increases probabilities of small volumes of reserves and decreases probabilities of large volumes.

Let us now analyze the moments of GUD. Following traditional calculations for expectation <D>, we obtain: $<D> = (D_{ld} + D_{rd} + D_{lu} + D_{ru})/4$. Properties of GDU allow one to derive simple recurrence formula for central moments of n-th $<D_n>_c$ and (n-1)-th $<D_{n-1}>_c$ orders. To derive this expression, let us use integral definition of central moments and formulae (2) for probability density of GUD. After integration by parts, one can see that algebraic sum of logarithm terms equals zero, and residual can be expressed through polynomial term and moment $<D_{n-1}>_c$. Because $BDlu – MD_{ld} = BD_{ru} - MD_{rd}$, we finally obtain:

$$<D_n>_c = [\frac{(D_{lu} - <D>)^{n+1} - (D_{ld} - <D>)^{n+1} + (D_{rd} - <D>)^{n+1} - (D_{ru} - <D>)^{n+1}}{n+1} - \\ -n\frac{(D_{rd} + D_{ld} - D_{ru} - D_{lu})(B+M)}{4} <D_{n-1}>_c]/[(n+1)(B-M)], \quad n \geq 2. \qquad (5)$$

Note that for symmetric PIE $D_{ld} + D_{rd} = D_{lu} + D_{ru}$, and the second term in (5) is equal to zero for this case. Formula (5) allows one to calculate central moment of (n+1)-th order, if one has central moment of n-th order and expectation for GUD.

From the theory of probability we know that the shape of probability distributions can be well characterized by the first four moments. In case of GUD, these moments can be calculated from (5). But for the first three of them we may obtain exact analytical expressions. Let us introduce the following notation $<D_d> = (D_{ld} + D_{rd})/2$, $<D_u> = (D_{lu} + D_{ru})/2$, $L_r = D_{rd} – D_{ru}$, $L_l = D_{lu} – D_{ld}$. Note that $L_r = L_l$ for symmetric PIE.

Then for the variance $<D_2>_c$ of GUD we have:

$$< D_2 >_c = \frac{B^2 + M^2 + MB}{36} + \frac{(< D_d > - < D_u >)^2}{12} = \frac{B^2 + M^2 + MB}{36} + \frac{(L_r - L_l)^2}{48}.$$

The first term of this expression shows dependence of variance on the lengths of intervals in PIE, the second one shows dependence of variance on a possible asymmetry of PIE.

For the coefficient of skewness $<D_3>_c$ we obtain:

$$< D_3 >_c = \frac{(B^2 - M^2)(< D_d > - < D_u >)}{48} = \frac{(< D_{2d} >_c - < D_{2u} >_c)(< D_d > - < D_u >)}{4}.$$

Here we take into account the fact that variances of mono-interval uniform distributions defined on the base and mini intervals are equal to $B^2/12$ and $M^2/12$ respectively.

The sign of $<D_3>_c$ is determined by the sign of a difference $<D_d>$ - $<D_u>$, because $B^2 > M^2$. This means that GUD has negative asymmetry (shifted to the left) if $<D_d>$ is less than $<D_d>$, and it has positive asymmetry (shifted to the right) if $<D_d>$ is larger than $<D_u>$.

Values of kurtosis may be also calculated from (5). If we compare standardized abrupt normal distribution defined on finite interval with GUD, we see that there are allokurtic GUD: specifically, kurtosis for symmetric GUD may have both positive values and negative values and also zero value (Figure 6).

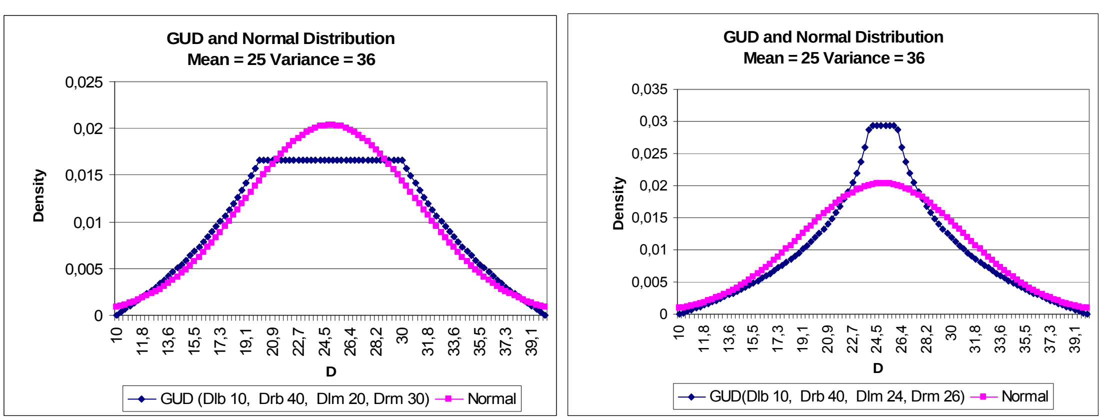


Fig. 6. Densities of generalized uniform and normal distributions for different values of kurtosis

Another set of decision-making problems that can be approached in the framework of the GIE method by GIE projection calculation includes dynamic (time-dependent) problems and problems with dependent parameters.

Figure 7 illustrates situation when the expert describes dynamic behavior of parameter D_2 as a pair of intervals for each time (parameter D_1). This setup could be represented by a “tube” with walls of finite variable thickness. Now we may construct GIE for each value of time if expert could also specify probability distributions on t-sections. The base intervals of the GIE for each time step are defined by outer boundaries of the tube and mini intervals are defined by inner ones. Since time dependence could be considered as just a case of general parameter dependence, GIE for dealing with uncertainty in dependent parameters may be built in a similar way.

When dealing with dynamic tasks in the GIE approach, an expert may define GIE for several characteristic sections (time-steps) and then “propagate” GIE for other time values by interpolation. If an expert thinks that distributions on different time sections belong to the same distribution type, one can derive analytic expressions for the projections of joint distribution functions of GIE. Specifically, this can be done for various combinations of uniform and triangle distributions on α and *D*.

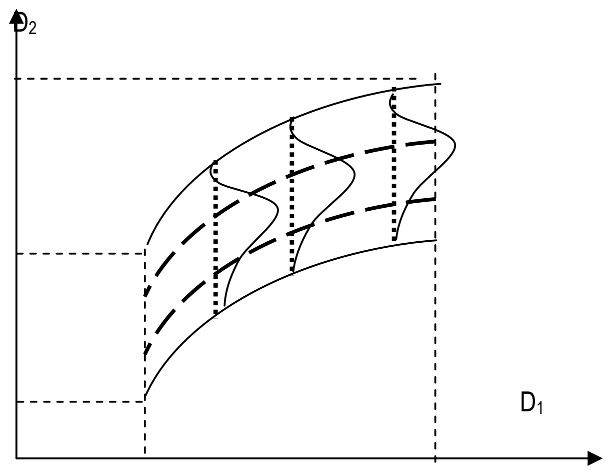


Fig. 7. GIE for parameter D_2 changing with time or parameter D_1

Internal Structure of GIE: Probability Tubes

This way is to reject averaging but analyze inner structure of GIE and use results of such analysis during decision-making (Chugunov et al., 2008). Let us fix some value Ds from the base interval. Since the expert defines probability distribution for each interval (or α-scenario) of GIE, we can try to analyze distribution of cumulative probability functions P(D<Ds|α) with respect to α. Marginal probability distribution function for random variable D is as follows:

$$P(D < Ds) = \int_0^1 \int_{D_{ld}}^{Ds} f_1(\alpha) f_2(D \mid \alpha) dD d\alpha = \int_0^1 f_1(\alpha) P(D < Ds \mid \alpha) d\alpha. \quad (6)$$

We wish to derive up and down bounds for P(D<DS):

$$P(D<Ds)=\int_0^1 f_1(\alpha)P(Ds\,|\,\alpha)d\alpha \le \tilde{P}_{\max}(Ds)\int_{\alpha_d}^{\alpha_u} f_1(\alpha)d\alpha + P_u(Ds)+P_d(Ds)=P_{\max}(Ds) \quad (7)$$

$$P(D<Ds)=\int_0^1 f_1(\alpha)P(Ds\,|\,\alpha)d\alpha \ge \tilde{P}_{\min}(Ds)\int_{\alpha_d}^{\alpha_u} f_1(\alpha)d\alpha + P_u(Ds)+P_d(Ds)=P_{\min}(Ds). \quad (8)$$

Here α_d and α_u correspond respectively to down and up intervals from GIE (figures 2, 3), which contain the fixed value Ds,

$$\tilde{P}_{\max}(Ds)=\max_{\alpha\in[\alpha_d;\alpha_u]} P(D<Ds\,|\,\alpha)\,,\ \tilde{P}_{\min}(Ds)=\min_{\alpha\in[\alpha_d;\alpha_u]} P(D<Ds\,|\,\alpha)\,,$$

$$P_d(Ds)=P(D<Ds\,|\,\alpha_d)\int_0^{\alpha_d} f_1(\alpha)d\alpha,\ \ P_u(Ds)=P(D<Ds\,|\,\alpha_u)\int_{\alpha_u}^{1} f_1(\alpha)d\alpha.$$

Note, that functions P_{min}(Ds), P_{max}(Ds) are non decreasing functions defined on the base interval; they both always equal zero at the left endpoint of this interval, and equal one at its right endpoint. Hence, these functions are some probability distribution functions. Unknown “true” probability distribution function lies between these functions. So we may receive triple of probability distribution functions P_{min}(Ds), P_{max}(Ds) and P(D<Ds) on the base interval. Pair P_{min}(Ds), P_{max}(Ds) constitutes probability tube (p-box) [Dempster, 1967; Williamson and Downs, 1990], which represents uncertainty in assessment of “true” distribution (figure 8).

Let us now go back to distribution of P(D<Ds|α) values for some fixed Ds. Let Ds belong to each interval from the subset, described by the intervals corresponding to α_u and α_d. Then, if all values P(D<Ds|α) are unique, we can define probability distribution for values P(D<Ds|α) with probability density g(P(D<Ds|α)):

$$g(P(D<Ds\,|\,\alpha))=f_1(\alpha)\,/\int_{\alpha_d}^{\alpha_u} f_1(\alpha)d\alpha\ .$$

Probability tubes approach is widely used in real-life problem analysis under uncertainty. Note, that each boundary p-tube is not always correspond to a single possible scenario (e.g. optimistic or pessimistic), but could be specified by a combination of those, with each scenario playing its role on a certain subinterval of the base interval. Therefore, in some complex situations it might be difficult for an expert to specify the bounds for a whole base interval at once. GIE approach allows an expert to construct boundary p-tubes P_{min}(Ds) and P_{max}(Ds) automatically based on naturally driven reasoning about possible scenarios to describe values of estimated parameter and provides comprehensive description of the p-tubes obtained.

It should underline that GIE method permits for each value Ds of some estimated parameter D obtain on the base interval not only boundary tubes [P_{min}(Ds), P_{max}(Ds)], but also a probability distribution on section D = D_S of boundary tubes. In a similar way, for each fixed probability value P_s, one can obtain a distribution for values of estimated parameter D related to this fixed probability level. These two distributions represent in GIE formalism uncertainty in expert judgments on both estimated parameter values and corresponding probability distributions.

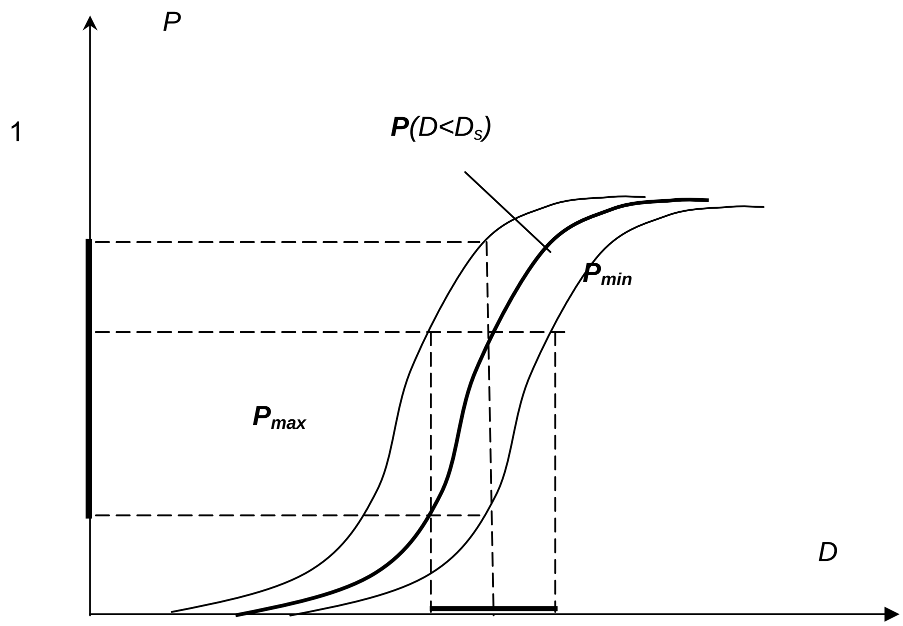


Fig. 8. Probability tubes generated by GIE.

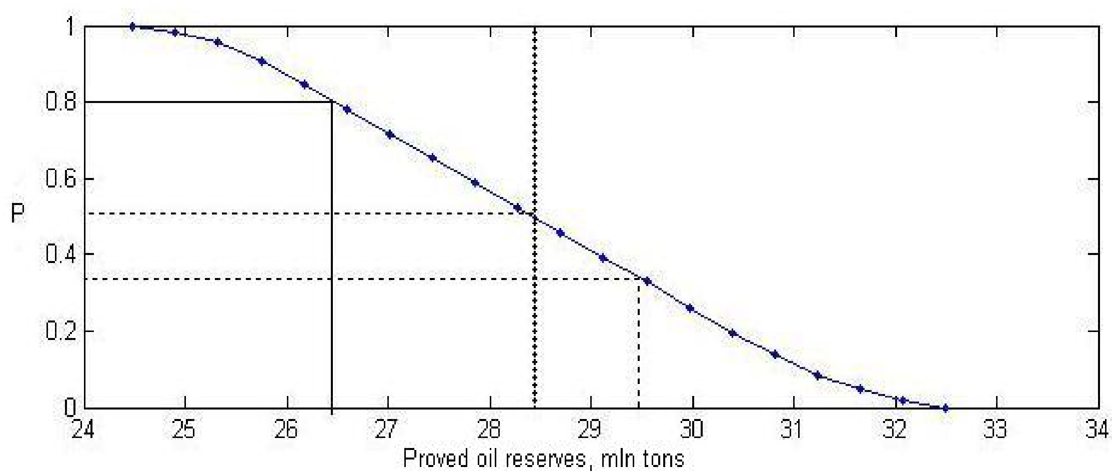


Fig. 9. Probability distribution for proved reserves

Let's again give an illustrative example of oil reserves estimation in the framework of probabilistic approach. Above we do so for mono-interval picture, there point estimations for all categories of oil reserves were received (see fig. 1). Now we present some results of calculations for volumetric model where input parameters have GIE estimations. One can see the results for proved reserves (probability level 0.9) on figure 9. Here values of proved reserves lay in the range from 24.5 to 32.5 ml tons. For the same input parameters mono-interval approach would give us 29.5 ml tons for this reserves category. From figure 9 one can see that this value corresponds to 0.35

confidence level on probabilistic curve, which may be unacceptably low for decision maker. Mean value is 28.4 ml tons. For acceptable confidence level 0.8 the value for proved reserves is 26.4 ml tons, that less than result for mono-interval approach. Similar analysis may be performed for probable and possible reserves categories.

Thus GIE method allow decision maker to obtain not only ranges, but, what is more important for justifiable decision making, cumulative probabilities for each reserves category.

GIE Method in Scenario Analysis

Scenario approach is rather often used in theory and practice of decision-making for the analysis of complex ill-structured problems. The methods of scenario analysis ensure the information-analytical support of the processes of decision making under the conditions of uncertainty. Advantages of scenario approach are accompanied now with a disadvantage due to the need for preliminary labor-consuming preparation of a set of scenarios that should be analyzed. Therefore, at present a finite number of scenarios are usually used for analysis that significantly limits a set of analyzed alternatives and the most rational ones may be exclude in advance. It means that the development of rapid methods for the elaborating alternative scenarios is important. An approach, which gives the set of scenarios by means of indicating its boundaries, may be promising approach. This can be done in the framework of the GIE method. Mentioned situation is a similar to situation of comparison of discrete optimization and a linear programming approach. At the first case choice of the “best” alternative takes place from alternatives prepared earlier.

Above it was noted that the intervals of PIE in the GIE method might be interpreted as the possible scenarios of the realization of the initial parameters or resultant index, and distribution on the Y-axis of PIE defined “the weights” of scenarios. However, for the systematic use of GIE method in the scenario analysis the method should be fitted out for the solution of problems with such dependent parameters where one of the parameters reflects the state of “external environment”, and the second, depending on the first, is the initial parameter or the resultant index of model. Let us note also that in contrast to the original GIE scheme where the Y-axis of PIE was the axis “of the marks” of intervals and had not identifiable “physical” sense, in scenario approach both axes of PIE had real meaning. Thus the expansion of GIE method to the tasks of the scenario analysis requires the modification of mathematical tools developed previously.

Table 1. Configurations of PIE for D ≠ 0, U ≠ 0 (trapezoidal shape), D = 0 или U = 0 (triangular shape)

Shapes of PIE	PIE of trapezoidal shape	PIE of triangular shape
1	$V_{ld} < V_{lu} < V_{rd} < V_{ru}$	$V_{ld} < V_{rd} < V_{u}$
2	$V_{ld} < V_{rd} < V_{lu} < V_{ru}$	$V_{ld} < V_{u} < V_{rd}$
3	$V_{ld} < V_{lu} < V_{ru} < V_{rd}$	$V_{u} < V_{ld} < V_{rd}$
4	$V_{lu} < V_{ld} < V_{rd} < V_{ru}$	$V_{lu} < V_{ru} < V_{d}$
5	$V_{lu} < V_{ld} < V_{ru} < V_{rd}$	$V_{lu} < V_{d} < V_{ru}$
6	$V_{lu} < V_{ru} < V_{ld} < V_{rd}$	$V_{d} < V_{lu} < V_{ru}$

Let us discuss the modifications in PIE shapes that need in scenario analysis. Note by V the values of the test parameters or indicators in problems with dependent variables and α - values of the external factors affecting the occurrence of possible values for V, α ∈ [α_m, α_M], where α_m (α_M) - the minimum (maximum) value of α respectively. Now PIE is set by the quartet V_{ld} (left-lower angel of PIE), V_{rd} (right lower angel), V_{lu} (left upper angel of PIE), V_{ru} (right upper angel), $D = V_{rd} - V_{ld}$, $U = V_{ru} - V_{lu}$. The relationships between members of the quartet determine the shape of PIE.

All possible shapes of PIE for the case $D \neq 0$, $U \neq 0$ (PIE of trapezoidal shape), and D = 0, $V_{ld} = V_{rd} = V_d$, or U = 0, $V_{lu} = V_{ru} = V_u$ (PIE of triangular shape) are presented in Table 1.

Let's pay attention to the fact that if the original pattern of PIE is often a set of nested intervals for the problems with dependent parameters is not the case.

Earlier we studied generalized uniform distribution of probabilities for PIE on nested intervals. In problems with dependent parameters variety of possible shapes of PIE leads to the appearance of a family of generalized uniform distributions. The corresponding relations are presented in Tables 2, 3.

Table 2. Generalized uniform probabilities distributions for trapezoidal PIE

Sectors of PIE	Densities for D≠U	Densities for D = U	Distributions: D≠U	Distributions: D=U
1.1. $V_{ld} \le V < V_{lu}$	$f(V) = I_1$	$f(V) = L_1$	$P(V < V_S) = F_1$	$P(V < V_S) = G_1$
1.2. $V_{lu} \le V \le V_{rd}$	$f(V) = I_2$	$f(V) = L_2$	$P(V < V_S) = F_2$	$P(V < V_S) = G_2$
1.3. $V_{rd} < V \le V_{ru}$	$f(V) = I_3$	$f(V) = L_3$	$P(V < V_S) = F_3$	$P(V < V_S) = G_3$
2.1. $V_{ld} \le V < V_{rd}$	$f(V) = I_1$	$f(V) = L_1$	$P(V < V_S) = F_1$	$P(V < V_S) = G_1$
2.2. $V_{rd} \le V \le V_{lu}$	$f(V) = I_4$	$f(V) = L_4$	$P(V < V_S) = F_4$	$P(V < V_S) = G_4$
2.3. $V_{lu} < V \le V_{ru}$	$f(V) = I_3$	$f(V) = L_3$	$P(V < V_S) = F_3$	$P(V < V_S) = G_3$
3.1. $V_{ld} \le V < V_{lu}$	$f(V) = I_1$	$f(V) = L_1$	$P(V < V_S) = F_1$	$P(V < V_S) = 0$
3.2. $V_{lu} \le V \le V_{ru}$	$f(V) = I_2$	$f(V) = L_2$	$P(V < V_S) = F_2$	$P(V < V_S) = G_5$
3.3. $V_{ru} < V \le V_{rd}$	$f(V) = I_5$	$f(V) = L_5$	$P(V < V_S) = F_5$	$P(V < V_S) = 0$
4.1. $V_{lu} \le V < V_{ld}$	$f(V) = I_6$	$f(V) = L_6$	$P(V < V_S) = F_6$	$P(V < V_S) = 0$
4.2. $V_{ld} \le V \le V_{rd}$	$f(V) = I_2$	$f(V) = L_2$	$P(V < V_S) = F_2$	$P(V < V_S) = G_5$
4.3. $V_{rd} < V \le V_{ru}$	$f(V) = I_3$	$f(V) = L_3$	$P(V < V_S) = F_3$	$P(V < V_S) = 0$
5.1. $V_{lu} \le V < V_{ld}$	$f(V) = I_6$	$f(V) = L_6$	$P(V < V_S) = F_6$	$P(V < V_S) = G_6$
5.2. $V_{ld} \le V \le V_{ru}$	$f(V) = I_2$	$f(V) = L_2$	$P(V < V_S) = F_2$	$P(V < V_S) = G_2$
5.3. $V_{ru} < V \le V_{rd}$	$f(V) = I_5$	$f(V) = L_5$	$P(V < V_S) = F_5$	$P(V < V_S) = G_7$
6.1. $V_{lu} \le V < V_{ru}$	$f(V) = I_6$	$f(V) = L_6$	$P(V < V_S) = F_6$	$P(V < V_S) = G_6$
6.2. $V_{ru} \le V \le V_{ld}$	$f(V) = -I_4$	$f(V) = -L_4$	$P(V < V_S) = F_7$	$P(V < V_S) = G_8$
6.3. $V_{ld} < V \le V_{rd}$	$f(V) = I_5$	$f(V) = L_5$	$P(V < V_S) = F_5$	$P(V < V_S) = G_7$

Here

$$I_1 = \frac{1}{U-D} Ln \frac{D(V_{lu}-V)+U(V-V_{ld})}{D(V_{lu}-V_{ld})}, \quad I_2 = \frac{1}{U-D} Ln \frac{U}{D}, \quad I_3 = \frac{1}{U-D} Ln \frac{U(V_{ru}-V_{rd})}{U(V-V_{rd})+D(V_{ru}-V)},$$

$$I_4 = \frac{1}{U-D} Ln \frac{[D(V_{lu}-V)+U(V-V_{ld})](V_{ru}-V_{rd})}{[D(V_{ru}-V)+U(V-V_{rd})](V_{lu}-V_{ld})}, \quad I_5 = \frac{1}{U-D} Ln \frac{D(V_{ru}-V)+U(V-V_{rd})}{D(V_{ru}-V_{rd})},$$

$$I_6 = \frac{1}{U-D} Ln \frac{U(V_{lu}-V_{ld})}{D(V_{lu}-V)+U(V-V_{ld})}.$$

$$L_1 = \frac{V-V_{ld}}{D(V_{lu}-V_{ld})}, \quad L_2 = \frac{1}{D}, \quad L_3 = \frac{V_{ru}-V}{D(V_{ru}-V_{rd})}, \quad L_4 = \frac{1}{V_{lu}-V_{ld}}, \quad L_5 = \frac{V-V_{rd}}{D(V_{ru}-V_{rd})}, \quad L_6 = \frac{V_{lu}-V}{D(V_{lu}-V_{ld})}.$$

$$F_1 = \frac{1}{D-U}\left[V_S - V_{ld} + \frac{D(V_{lu}-V_S)+U(V_S-V_{ld})}{D-U} Ln \frac{D(V_{lu}-V_S)+U(V_S-V_{ld})}{D(V_{lu}-V_{ld})}\right],$$

$$F_2 = \frac{1}{D-U}\left[V_{lu} - V_{ld} + \frac{D(V_{lu}-V_S)+U(V_S-V_{ld})}{D-U} Ln \frac{U}{D}\right],$$

$$F_3 = \frac{1}{D-U}\left[D + V_{lu} - V_S + \frac{D(V_S-V_{ru})+U(V_{rd}-V_S)}{D-U} Ln \frac{D(V_S-V_{ru})+U(V_{rd}-V_S)}{D(V_{rd}-V_{ru})}\right],$$

$$F_4 = \frac{1}{D-U}\left[D + \frac{D(V_{ru}-V_S)+U(V_S-V_{rd})}{D-U} Ln \frac{V_{ru}-V_{rd}}{V_{lu}-V_{ld}}\right],$$

$$F_5 = \frac{1}{D-U}\left[V_S - V_{ld} - U + \frac{D(V_{ru}-V_S)+U(V_S-V_{rd})}{D-U} Ln \frac{D(V_{ru}-V_S)+U(V_S-V_{rd})}{D(V_{ru}-V_{rd})}\right],$$

$$F_6 = \frac{1}{D-U}\left[V_{lu} - V_S + \frac{D(V_S-V_{lu})+U(V_{ld}-V_S)}{D-U} Ln \frac{D(V_S-V_{lu})+U(V_{ld}-V_S)}{U(V_{ld}-V_{lu})}\right],$$

$$F_7 = \frac{1}{D-U}\left[-U + \frac{D(V_S-V_{ru})+U(V_{rd}-V_S)}{D-U} Ln \frac{V_{rd}-V_{ru}}{V_{ld}-V_{lu}}\right],$$

$$G_1 = (V_S - V_{ld})^2 / [2D(V_{lu}-V_{ld})],$$
$$G_2 = (2V_S - V_{ld} - V_{lu})/(2D),$$
$$G_3 = 1 - (V_{ru} - V_S)^2 / [2D(V_{ru}-V_{rd})],$$
$$G_4 = (2V_S - V_{rd} - V_{ld})/[2(V_{lu}-V_{ld})],$$
$$G_5 = (V_S - V_{ld})/D,$$
$$G_6 = (V_S - V_{lu})^2 / [2D(V_{ld}-V_{lu})],$$
$$G_7 = 1 - (V_S - V_{rd})^2 / [2D(V_{rd}-V_{ru})],$$
$$G_8 = (2V_S - V_{ru} - V_{lu})/[2(V_{ld}-V_{lu})]$$

Table 3. Generalized uniform probabilities distributions for triangular PIE

Sectors of PIE	Densities	Distributions functions
1.1: $V_{ld} \le V < V_{rd}$	$f(V) = K_1$	$P(V < V_S) = H_1$
1.2: $V_{rd} \le V \le V_u$	$f(V) = K_2$	$P(V < V_S) = H_2$
2.1: $V_{ld} \le V < V_u$	$f(V) = K_1$	$P(V < V_S) = H_1$
2.2: $V_u < V \le V_{rd}$	$f(V) = K_3$	$P(V < V_S) = H_3$
3.1: $V_u \le V < V_{ld}$	$f(V) = K_2$	$P(V < V_S) = H_4$
3.2: $V_{ld} \le V \le V_{rd}$	$f(V) = K_3$	$P(V < V_S) = H_3$
4.1: $V_{lu} \le V < V_{ru}$	$f(V) = K_4$	$P(V < V_S) = H_5$
4.2: $V_{ru} \le V \le V_d$	$f(V) = K_5$	$P(V < V_S) = H_6$
5.1: $V_{lu} \le V < V_d$	$f(V) = K_4$	$P(V < V_S) = H_5$
5.2: $V_d < V \le V_{ru}$	$f(V) = K_6$	$P(V < V_S) = H_7$
6.1: $V_d \le V < V_{lu}$	$f(V) = K_5$	$P(V < V_S) = H_8$
6.2: $V_{lu} \le V \le V_{ru}$	$f(V) = K_6$	$P(V < V_S) = H_5$

Here

$$K_1 = \frac{1}{D} Ln \frac{V_u - V_{ld}}{V_u - V},\ K_2 = \frac{1}{D} Ln \frac{V_u - V_{rd}}{V_u - V_{ld}},\ K_3 = \frac{1}{D} Ln \frac{V_u - V_{rd}}{V_u - V},\ K_4 = \frac{1}{U} Ln \frac{V_{lu} - V_d}{V - V_d},$$

$$K_5 = \frac{1}{U} Ln \frac{V_{lu} - V_d}{V_{ru} - V_d},\ K_6 = \frac{1}{U} Ln \frac{V_{ru} - V_d}{V - V_d},\ K_7 = \frac{1}{U} Ln \frac{V_{lu} - V_d}{V_{ru} - V_d},$$

$$H_1 = \frac{1}{D}[V - V_{ld} + (V - V_u) Ln \frac{V_u - V_{ld}}{V_u - V}],\ H_2 = 1 - \frac{V_u - V}{D} Ln \frac{V_u - V_{ld}}{V_u - V_{rd}},\ H_4 = \frac{V - V_u}{D} Ln \frac{V_{rd} - V_u}{V_{ld} - V_u},$$

$$H_3 = \frac{1}{D}[V - V_{ld} + (V_u - V) Ln \frac{V - V_u}{V_{rd} - V_u}],\ H_5 = \frac{1}{U}[V - V_{lu} + (V_d - V) Ln \frac{V_d - V}{V_d - V_{lu}}],$$

$$H_6 = \frac{1}{U}[U + (V_d - V) Ln \frac{V_d - V_{ru}}{V_d - V_{lu}}],\ H_7 = \frac{1}{U}[V - V_{lu} + (V_d - V) Ln \frac{V_d - V}{V_d - V_{ru}}],$$

$$H_8 = \frac{V - V_d}{U} Ln \frac{V_{ru} - V_d}{V_{lu} - V_d},$$

New possibilities that appear in the tasks with the dependent variables due to “the equality of rights” of PIE axes may use in two directions. To more completely consider possible uncertainty of the estimation “of external factors” (e. g. forecast of prices in the above mentioned task about recoverable reserves of oil) one may build on the Y-axis (axis of prices in the task about the reserves) additional GIE besides initial one. Furthermore, in some tasks of the type of the task concerning in the dependence of the volumes of recoverable reserves on the price of the hydrocarbons, may be useful constructing for each level of the prices not mono-interval, but GIE that leads to the appearance “multidimensional” GIE.

Probabilistic Forecast for World Oil Production: Illustrative Example

Here we will give an illustrative example to demonstrate application of GIE method, and, specifically, generalized uniform distributions, in scenario approach to analyze probabilistic forecast for world oil production. There is a wide spectrum of estimations for world oil production and oil reserves base ranging from pessimistic ones that predicts the end of oil era in several years, up to those optimistic that argues there are sufficient oil reserves, at least, up to the end of this century.

The pessimistic scenario arises from the theory of “peak of oil” which is quite popular among many oil analysts. According to this theory consumption of oil will overtake, and then will outstrip rates of discovering new reserves whereas already known reserves will start to be exhausted.

In contrast to a widely discussed theory that world oil production will soon reach a peak and go into sharp decline, a new analysis of the subject by Cambridge Energy Research Associates (CERA) finds that the remaining global oil resource base is actually 3.74 trillion barrels -- three times as large as the 1.2 trillion barrels estimated by the theory’s proponents -- and that the “peak oil” argument is based on faulty analysis [CERA, 2006]. According to the study, the global production profile will not be a simple logistic or bell curve postulated by geologist M. King Hubbert, but it will be asymmetrical – with the slope of decline more gradual and not mirroring the rapid rate of increase - and strongly skewed past the geometric peak. It will be an undulating plateau that may well last for decades [CERA, 2006].

There are two important categories of oil reserves used in these studies: conventional and unconventional oil. Conventional oil refers to a reserves category that has supplied most to date and will likely to dominate oil supply far into the future. These reserves are relatively easy, cheap and fast to produce. Unconventional oil includes heavy oil category, dense and extremely viscous oils, as well as those produced from coal and immature source rocks. This category is characterized by a high resource base but a low extraction rate and net energy yield.

Figure 10 shows CERA’s outlook for conventional and unconventional oil including several possible scenarios and peak oil theory.

Here we are not interested in the accuracy and reliability of these forecasts; we are more interested in providing consistent mathematical tools for an independent expert to develop an individual prediction based on those from the study. These tools should allow the expert to express his/her preferences that would be reflected in the final scenario for the future dynamics of world oil reserves. We will use GIE method as such tools. The GIE framework will allow the expert to include the full spectrum of possible scenarios into the final estimation besides initial ones. The existence of those “intermediate” scenarios could be attributed to the uncertainties of the starting predictions as well as to limited number of models or experts involved in generation of those initial scenarios.

To use GIE method an expert should construct PIEs for each time points of forecasts. According to figure 10, there are three initial forecasts – “Peak Oil”, “Conventional Oil”, “Unconventional Oil”, the forecasts sharply differ after year 2010. Therefore we quantified data from the figure to use them in a subsequent scenario analysis based on GIE, starting from 2010. At the preliminary stage of constructing GIE the expert can define at an every time point (year) the base interval being determined by the corresponding values of the most optimistic scenario (CERA, unconventional oil) - right point of the base interval, and the pessimistic (peak oil) scenario – left point of the base interval. The mini interval could be defined by introducing a 10% spread (± 5%) for the most realistic, from expert’s point of view, forecast (CERA, conventional oil) (figure 11). Naturally we might use PIE of triangle shape but the first option gives more diversity of initial scenarios.

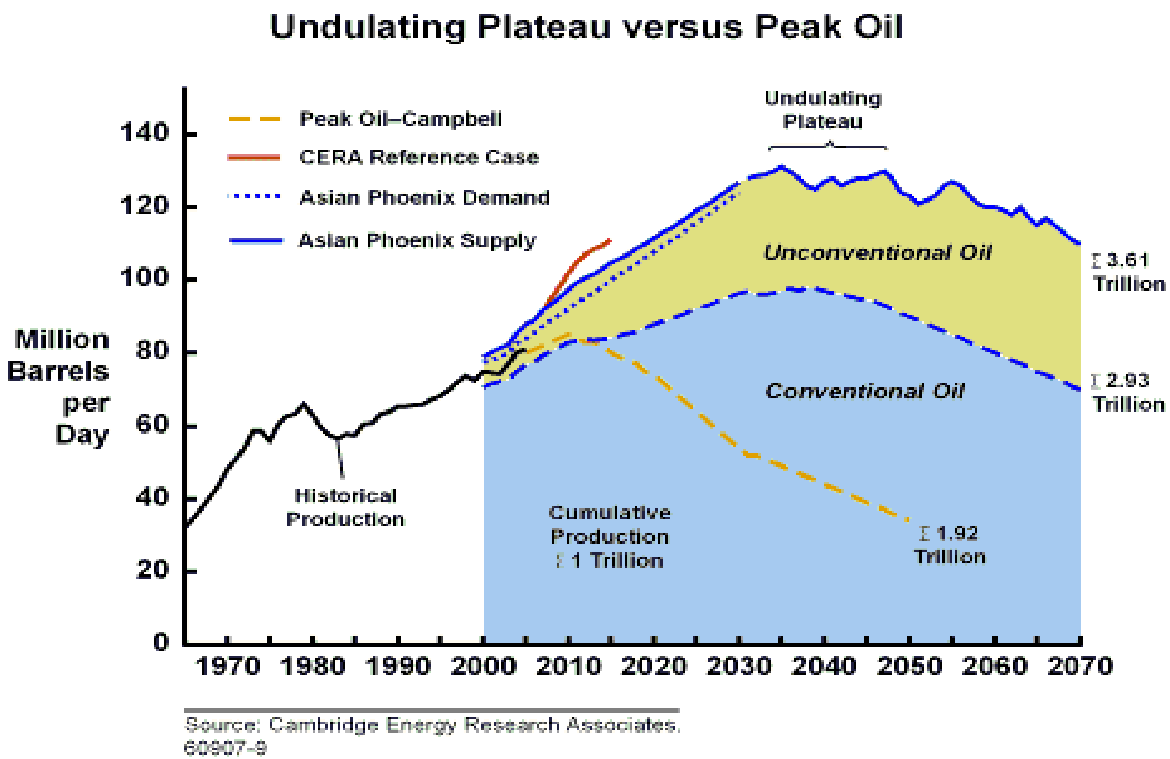


Fig. 10. Wave plateau versus peak theory

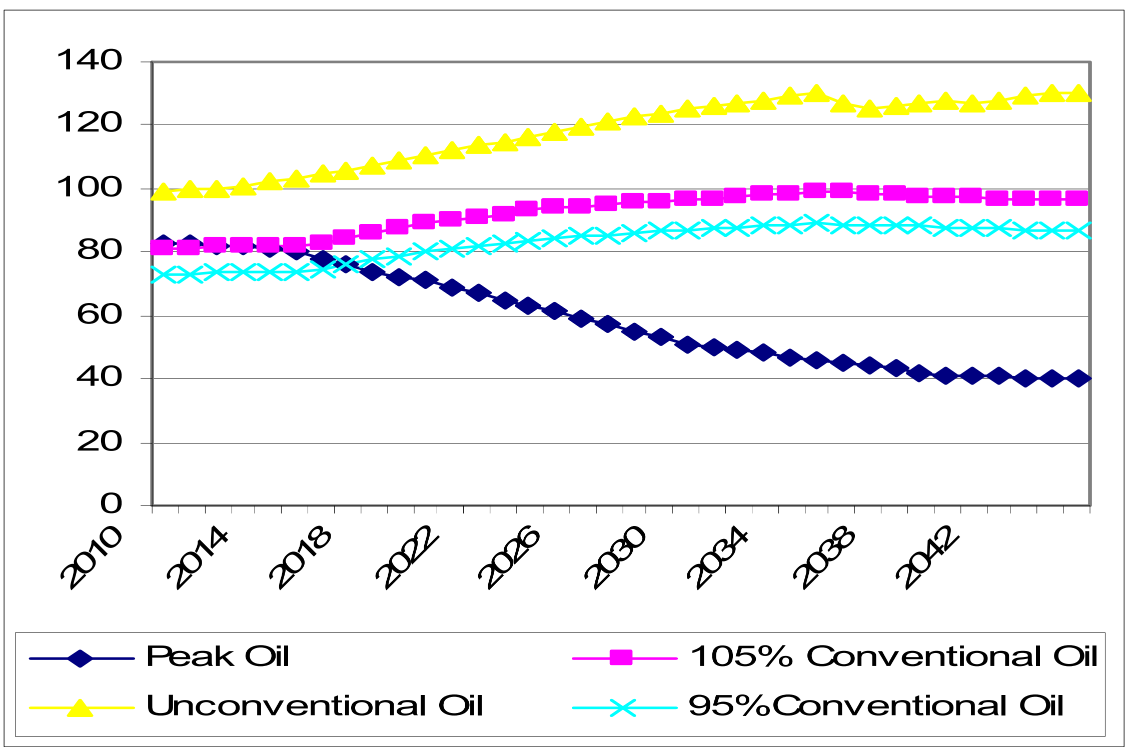
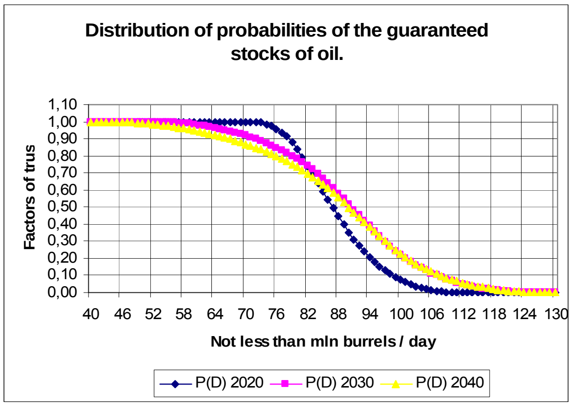


Figure 11. Expert’s perspective on Peak or Plato

So for example at the point t = 2020 we have that the base interval is [71; 110.5] while the mini interval is [84.55; 93.45]. Once the PIE structure is defined, the expert can express his/her beliefs by specifying probability density for distributions on the base and mini intervals for a current point t. Let us consider the case of rapid analysis when expert uses generalized uniform distribution as projection distribution.



Confidence Level	Production not less than (Mln barrels per day)		
	2020	2030	2040
0.9	78	72	67
0.8	81	79	76
0.7	83	83	82
0.6	85	87.5	87
0.5	87	90.5	90
0.4	89	93.5	93.5
0.3	91.5	97	97
0.2	94	101	102
0.1	99	107	108

Figure 12. Distributions of probabilities for the guaranteed levels of oil production

Figure 12 shows curves of probabilities of the guaranteed values for world oil production for three time points - the years 2020, 2030, and 2040. Analysis of these curves shows that for a confidence level of 0.9 the estimation of oil production is at least 80 million barrels per day for the year 2020, at least 73 million barrels per day for 2030, and 67 million barrels per day for the year 2040. Estimation of the world oil production at the confidence level of 0.5 gives at least 87 million barrels per day for the year 2020, at least 90.5 million barrels per day for 2030, and 90 million barrels per day for the year 2040. It is interesting to note that for the confidence level around 0.7 the guaranteed value of the estimation for the world oil production is almost constant for years 2020-2040 and does not go bellow 82 - 83 million barrels of oil per day.

Conclusion

Adequate solutions of many complex real-life problems under uncertainty significantly depend on effective elicitation and formalization of expert knowledge on input parameters of used models. This requires specific methods for expert knowledge elicitation, interpretation and processing.

Poly-interval representation and, specifically, Generalized Interval Estimations method seems to be promising approach to elicit and formalize expert knowledge on quantitative parameters in problems under uncertainty. This survey summarizes and further extends theoretical and practical aspects of GIE approach.

Generalized Interval Estimation structure, being scenario-based expert knowledge representation, is analysis-oriented, since it provides flexibility to add/remove/modify scenarios from expert description of estimated quantity. The practical example on projecting the dynamics of the world oil production illustrates possibilities that an independent expert could use to develop and analyze new scenarios based on initial set of forecasts available from relevant studies and research.

Generalized Probability Tubes structure is more decision-oriented interpretation of expert knowledge, since for given decision makers' confidence level it is able to represent the probability for estimated quantity to have a certain value. Oil reservoir evaluation example demonstrates this property of GIE approach, which, we believe, could be important for a wide variety of interdisciplinary problems including investment project evaluation, technology assessment, development of policies and regulations.

We hope that GIE method may be applied also for analysis of other problems. Among them aggregation information concerning in input data of used models, comparison of PIEs and GIEs and others.

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Bibliography

[CERA, 2006] URL: http://www.cera.com/aspx/cda/public1/news/pressReleases/pressRelease Details.aspx? CID=8444 «Peak Oil Theory – "World Running Out of Oil Soon" – Is Faulty; Could Distort Policy & Energy Debate». 14/11/ 2006.

[Chugunov et al., 2006] N. Chugunov, G. Shepelyov and M. Sternin. A method for uncertainty quantification in expert decision support systems, in F. Adam, P. Brezillon, S. Carlsson and P. Humphreys (Eds.) Creativity and Innovation in Decision Making and Decision Support, London: Decision Support Press, 2006, Vol. 2, pp.851–865.

[Chugunov et al., 2008 A] N. Chugunov, G. Shepelyov and M. Sternin. Probabilistic Methods for Uncertainty Quantification. In: Encyclopedia of Decision Making and Decision Support Technologies. Edited By: Frederic Adam, University College Cork, Ireland; Patrick Humphreys, London School of Economics, UK. Publisher: Information Science Reference, - April 2008, Vol. 1, pp. 732 – 742.

[Chugunov et al., 2008 B] N. Chugunov, G. Shepelyov and M. Sternin. The generalized interval estimations in decision making under uncertainty. International Journal of Technology, Policy and Management, 2008, Vol.8, pp.298–321.

[Dempster, 1967] A.P. Dempster. Upper and Lower Probabilities Induced by a Multivalued Mapping. // The Annals of Statistics, 1967, Vol. 38, pp.325-339.

[Kahneman et al., 1982] D. Kahneman, P. Slovic, A. Tversky. Judgment under uncertainty: heuristics and biases. Cambridge: Cambridge university press. 1982.

[Morgan and Henrion, 1992] G.M. Morgan, M. Henrion. Uncertainty – A guide to Dealing with Uncertainty in Quantitative Risk and Policy Analysis. Cambridge University Press, New York. 1992.

[Shepelyov and Sternin, 2003] G. Shepelyov and M. Sternin. Method of generalized interval estimations for intelligent DSS // "DSS in the uncertainty of the internet age", Proceedings of the Conference, Katowice, University of Economics, 2003, pp.367–377.

[Shepelyov and Sternin, 2008] G. Shepelyov and M. Sternin. Обобщенные интервальные оценки в сценарном анализе. // Supplement to International Journal Information Technologies and Knowledge. – 2008 – Vol 2 – pp. 87-94. (in Russian).

[Welsh et al., 2004] M.B. Welsh, S.H. Begg, R.B. Bratvold and M.D. Lee. Problems with the Elicitation of Uncertainty. / SPE 90338 paper, SPE Annual Technical Conference, Houston, Texas: Society of Petroleum Engineers. 2004.

[Williamson and Downs, 1990] R.C. Williamson, T. Downs. Probabilistic arithmetic I: numerical methods for calculating convolutions and dependency bounds. // International Journal of Approximate Reasoning, 1990, Vol. 4, pp.89–158.

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