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A MODEL FOR THE UNIVERSITY COURSE TIMETABLE PROBLEM

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***Abstract**: In this paper a model of the university course timetabling problem is presented. The basic data structures, parameters, vectors and matrices, which are used in the definition of the problem, are presented. The hard constraints, which include checking simultaneous involvement of lecturers, students or auditoriums, checking for exceeded capacity of auditoriums as well as verification of commitment of students and lecturers exceeding the predetermined number of hours, are considered. Objective functions for each of the soft constraints, which will be united in a common objective function, are defined. Thus the problem under investigation, will acquire multicriteria optimization nature. The future trends of work are presented.*

***Keywords:** timetable problem, multicriteria optimization.*

Introduction

In solving the university course timetabling problem, all hard constraints which are defined must necessarily be fulfilled. If in finding a solution, even one of these constraints is violated, then the whole solution is pronounced as unacceptable and needs searching for another solution. The solution of problem is limited to distribution of events in the week schedule. It is necessary to meet all hard constraints, while at the same time, be violated at least soft constraints. When defining the problem discussed in this paper, four hard and three soft constraints are defined. Although there are other examples of definitions of this problem (Burke at al., 2004; Carter and Laporte 1998, Rossi-Doria, at al., 2002, 2003; Socha at al., 2002), we propose a new model where events may be of varying lengths, in addition, each resource and its interactions will participate with a corresponding weight.

In our model a matrix of the preferences of lecturers and a common matrix of the preferences of students are provided. They combine some of the soft constraints of two data structures. If necessary it is also possible to introduce a similar matrix of the auditorium.

The advantages of our model, which has been discussed here are three:

- First, in the problem definition all the resources "lecturers", "students", "auditoriums" and "events" are included. Every element of the resource "events" represents the interaction of one or more elements from the three other resources that occur within a specified timeslot. A variant of the problem which is often considered is that in which some of the resources (eg "lecturers" and/or "auditoriums") are not taken into consideration. This is done in order to compile a model more easily and to create algorithms to solve the problem.
- Second, maintaining blocks of consecutive events that appear inseparable.
- Third, the soft constraints are combined into three groups - a) a minimum length of timetable; b) a minimum number of violations according to the lecturers' preferences and c) a minimum number of violated students' preferences. For each group an objective function with a certain weight is defined.

Basic Definitions

Basic data structures, parameters, vectors and matrices, which will be used to create the model will be presented.

Vector of events (EV) contains information about events (or activities). In our model, the events are marked with unique numbers from 1 to N:

(1) $EV = \{n_i\}_{i=1}^{N} = \{n_1, n_2, \dots, n_N\}$, where n_i is the *i*-th event.

Also a vector of the duration of events (EDV) and a vector of weights of events (WEV) are introduced, which contain additional information about each event:

(2) $EDV = \{n_i^d\}_{i=1}^{N} = \{n_1^d, n_2^d, \dots, n_N^d\}$, where n_i^d is the duration of the *i*-th event and

(3) $WEV = \{n_i^w\}_{i=1}^{N} = \{n_1^w, n_2^w, \dots, n_N^w\}$, where $n_i^w \in (0,1]$ is the weight of the *i*-th event.

The weight of each event is determined based on subjective factors and indicates the relative weight of an event to other events.

Students vector (SV), lecturers vector (LV) and auditoriums vector (AV) are with length respectively S, L and A. In our model, students, lecturers and auditoriums are identified by unique numbers, respectively, from 1 to S from 1 to L and from 1 to A:

(4) $SV = \{s_i\}_{i=1}^{S} = \{s_1, s_2, \dots, s_S\}$, where s_i is the *i*-th student.

(5) $LV = \{l_i\}_{i=1}^{L} = \{l_1, l_2, \dots, l_L\}$, where l_i is the *i*-th lecturer.

(6) $AV = \{a_i\}_{i=1}^{A} = \{a_1, a_2, \dots, a_A\}$, where a_i is the *i*-th auditorium.

It also introduces the weight vector of lecturers (WLV), the weight vector of auditoriums (WAV) and the vector capacity of auditoriums (ACV), which contain additional information of related resources:

(7) $WLV = \{l_i^w\}_{i=1}^{L} = \{l_1^w, l_2^w, \dots, l_L^w\}$, where $l_i^w \in (0,1]$ is the weight of the *i*-th lecturer.

(8) $WAV = \{a_i^w\}_{i=1}^{A} = \{a_1^w, a_2^w, \dots, a_L^w\}$, where $a_i^w \in (0,1]$ is the weight of the *i*-th auditorium.

(9) $ACV = \{a_i^c\}_{i=1}^{A} = \{a_1^c, a_2^c, \dots, a_A^c\}$, where a_i^c is the capacity of the *i*-th auditorium.

To have the matrix of weekly schedule (WPM) defined, it is necessary first to define: the days vector (DV), a vector of hours (HV) and the vector of absolute hours (AHV), which respectively have a length D, H and T:

(10) $DV = \{d_i\}_{i=1}^{D} = \{d_1, d_2, \ldots, d_D\}$, where d_i is the *i*-th day.

(11) $HV = \{h_i\}_{i=1}^{H} = \{h_1, h_2, \ldots, h_H\}$, where h_i is the *i*-th hour in each day.

(12) $AHV = \{t_i\}_{i=1}^{T} = \{t_1, t_2, \ldots, t_T\}$, where $T = D * H$, and t_i is the *i*-th absolute hour, which corresponds to a time interval (timeslot) of the weekly schedule.

Each timeslot is given by ordered pair $\langle d_i, h_j \rangle$, where $i = 1, 2, ...D; j = 1, 2, ...H$. A presentation of the matrix of weekly schedule (WPM) shown in (13):

(13) $WPM = (p_{ij})_{DxH}$, where p_{ij} is the timeslot in the *i*-th day and *j*-th hour.

The matrix of students and events (SEM) has a length of $S \times N$ where each element is given by "1" or "0". Value "1" indicates the *i*-th student takes *j*-th event where $i \in \{1..S\}$, $j \in \{1..N\}$ and "0" otherwise:

(14) $SEM = (p_{ij})_{SxN}$, where $p_{ij} = \begin{cases} 1, & if \ \ student \ \ i \ takes \ event \ \ j \\ 0, & otherwise \end{cases}$

Also the vector of the number of students at events (NSEV), the vector of the number of events by students (NESV) and the vector of students' hours of work (LHSV) are defined:

(15) $NSEV = \{n_i^s\}_{i=1}^{N} = \{n_1^s, n_2^s, \ldots, n_N^s\}$, where n_i^s is the number of students attending the *i*-th event and $n_i^s = \sum_{k=1}^{S} v_{ki}$, where $v_{ki} \in SEM$.

(16) $NESV = \{s_i^n\}_{i=1}^{S} = \{s_1^n, s_2^n, \ldots, s_S^n\}$, where s_i^n is the number of events, which must be attended by the *i*-th student and $s_i^n = \sum_{k=1}^{N} v_{ik}$, where $v_{ik} \in SEM$.

(17) $LHSV = \{s_i^t\}_{i=1}^{S} = \{s_1^t, s_2^t, \ldots, s_S^t\}$, where s_i^t is the total number of hours during which the *i*-th student is engaged and $s_i^t = \sum_{k=1}^{N} u_{ik} v_k$, where $u_{ik} \in SEM$, $v_k \in EDV$.

Because each event is held by only one lecturer, that information can be represented as a vector of events and lecturers (ELV) with length L:

(18) $ELV = \left\{ n_i^l \right\}_{i=1}^{N} = \left\{ n_1^l, n_2^l, \ldots, n_N^l \right\}$, where $n_i^l \in LV$ is the index of the lecturer who held the i-th event.

Also a vector of the number of events by lecturers (NELV), and a vector of lecturers' hours per week (LHLV) are defined:

(19) $NELV = \left\{ l_i^n \right\}_{i=1}^{L} = \left\{ l_1^n, l_2^n, \ldots, l_L^n \right\}$, where l_i^n is the number of events to be held by the *i*-th lecturer and $l_i^n = \sum_{k=1}^{N} \delta(k,i)$, where $\delta(k,i) = \begin{cases} 1, \ if \ v_k = u_i \\ 0 \ otherwise \end{cases}$, where $v_k \in ELV,\ u_i \in LV$, ie the function $\delta(k,i)$ will take value "1" if the i-th lecturer holds the k-th event.

(20) $LHLV = \left\{ l_i^t \right\}_{i=1}^{L} = \left\{ l_1^t, l_2^t, \ldots, l_L^t \right\}$, where l_i^t is the total number of hours during which the *i*-th lecturer is engaged and $l_i^t = \sum_{k=1}^{N} \delta(k,i)\, d_k$, where $d_k \in EDV$ is the length of the *k*-th event.

To be defined the matrix for appropriate auditoriums (SAEM) it is necessary that the vector of the kind of events (EKV) and the vector of the kind of auditoriums (AKV) to be defined before it, which have respectively length N and A:

(21) $EKV = \left\{ n_i^k \right\}_{i=1}^{N} = \left\{ n_1^k, n_2^k, \ldots, n_N^k \right\}$, where n_i^k is the kind of *i*-th event.

(22) $AKV = \left\{ a_i^k \right\}_{i=1}^{A} = \left\{ a_1^k, a_2^k, \ldots, a_A^k \right\}$, where a_i^k is the kind of *i*-th auditorium.

SAEM matrix is defined by the vectors ACV, NSEV, EKV and AKV. The dimensionality of this matrix is $A \times N$. Its elements are represented by "1" and "0" indicating that the auditorium a_i where $i \in \{1, 2, ..., A\}$, respectively, may or may not be used for the event n_j, where $j \in \{1, 2, ..., N\}$:

(23) $SAEM = (p_{ij})_{AxN}$, where $p_{ij} = \begin{cases} 1, \ if \ \left(a_i^c \geq n_j^s \right) \wedge \left(a_i^k = n_j^k \right) \\ 0 \ otherwise \end{cases}$, where $a_i^c \in ACV$ is the capacity of *i*-th auditorium, $n_j^s \in NSEV$ is the number of students who visit the *j*-th event, $a_i^k \in AKV$ is the kind of the *i*-th auditorium and $n_j^k \in EKV$ is the kind of the *j*-th event.

Similar to the lecturers, a vector of events and auditoriums (EAV), which has a length N is defined:

(24) $EAV = \left\{ n_i^a \right\}_{i=1}^{N} = \left\{ n_1^a, n_2^a, \ldots, n_N^a \right\}$, where $n_i^a \in AV$ is the index of auditorium in which the the *i*-th event is held.

Similar to the lecturers, the vector of the number of events by auditoriums (NEAV) and the vector of auditorium hours per week (LHAV) are defined:

(25) $NEAV = \left\{ a_i^n \right\}_{i=1}^{A} = \left\{ a_1^n, a_2^n, \ldots, a_A^n \right\}$, where a_i^n is the number of events to be held in the *i*-th auditorium.

(26) $LHAV = \left\{ a_i^t \right\}_{i=1}^{A} = \left\{ a_1^t, a_2^t, \ldots, a_A^t \right\}$, where a_i^t is the total number of hours during which the *i*-th auditorium will be occupied.

The matrix of conflicts of events arising from students (CESM) has dimension $N \times N$ and is generated by using the vector EV and the matrix SEM. Each element $p_{ij} \in CESM$ shows the number of the students who must attend the event i and event j as well, where $i, j \in \{1, 2, ..., N\}$. CESM is a symmetric matrix to its main diagonal and its definition is shown in (27):

(27) $CESM = (p_{ij})_{NxN}$, where $p_{ij} = \begin{cases} \sum_{k=1}^{S} v_{ki} v_{kj}, & i \neq j \\ 0, & otherwise \end{cases}$, where $v_{ki}, v_{kj} \in SEM$.

Similarly, the matrix of conflicts arising from lecturers and events (CELM) and the matrix of conflicts arising from auditoriums and events (CEAM) are defined and have dimension $N \times N$ and are generated using vectors EV, NSEV, ELV and EAV. If two events i and j are in conflict because of one lecturer or one auditorium, the element value p_{ij} is the number of students who must attend both events i and j, where $i, j \in \{1, 2, ..., N\}$. CELM and CEAM matrices are symmetrical to their main diagonal and are shown in (28) and (29):

(28) $CELM = (p_{ij})_{NxN}$, where $p_{ij} = \begin{cases} v_i + v_j, & if\ u_i = u_j \\ 0, & otherwise \end{cases}$, where $v_i, v_j \in NSEV$, $u_i, u_j \in ELV$.

(29) $CEAM = (p_{ij})_{NxN}$, where $p_{ij} = \begin{cases} v_i + v_j, & if\ u_i = u_j \\ 0, & otherwise \end{cases}$, where $v_i, v_j \in NSEV$, $u_i, u_j \in EAV$.

Also the vector of the number of conflicts arising from students and events (NCESV), the vector of the number of conflicts arising from lecturers and events (NCELV) and the vector of the number of conflicts arising from auditoriums and events (NCEAV) are defined, respectively:

(30) $NCESV = \left\{n_i^{cs}\right\}_{i=1}^{N} = \left\{n_1^{cs}, n_2^{cs}, \ldots, n_N^{cs}\right\}$, where $n_i^{cs} = \sum_{j=1}^{N} 1 \; if \; v_{ij} > 0$, $v_{ij} \in CESM$.

(31) $NCELV = \left\{n_i^{cl}\right\}_{i=1}^{N} = \left\{n_1^{cl}, n_2^{cl}, \ldots, n_N^{cl}\right\}$, where $n_i^{cl} = \sum_{j=1}^{N} 1 \; if \; u_{ij} > 0$, $u_{ij} \in CELM$.

(32) $NCEAV = \left\{n_i^{ca}\right\}_{i=1}^{N} = \left\{n_1^{ca}, n_2^{ca}, \ldots, n_N^{ca}\right\}$, where $n_i^{ca} = \sum_{j=1}^{N} 1 \; if \; x_{ij} > 0$, $x_{ij} \in CEAM$.

A summary matrix of conflicts arising from students, lecturers and auditoriums and events (GCEM) has dimensionality $N \times N$. It is generated using matrices CESM, CELM, CEAM and vectors WEV, WLV and WAV. The value of the element $p_{ij} \in GCEM$ showing the cost of the conflict among the events n_i and n_j, where $i, j \in \{1, 2, ..., N\}$. GCEM matrix is symmetrical and its main diagonal is shown in (33):

(33) $GCEM = (p_{ij})_{NxN}$, where $p_{ij} = \left\lceil \left(n_i^w + n_j^w\right) u_{ij} + f\left(b_i\right) v_{ij} + g\left(c_i\right) x_{ij} \right\rceil$, $u_{ij} \in CESM$, $v_{ij} \in CELM$, $x_{ij} \in CEAM$, $n_i^w \in WEV$, $n_j^w \in WEV$, $f(b_i) = l_k^w$, $l_k^w \in WLV$, $g(c_i) = a_q^w$, $a_q^w \in WAV$.

Also the vector of the number of conflicts arising from students, lecturers, auditorium and the events (NGCEV), the vector of the weights of conflicts arising from students, lecturers, auditorium and the events (GWCEV) and the vector for the total length by conflicting groups (GDCGV), which have a length N are defined:

(34) $NGCEV = \left\{n_i^{cg}\right\}_{i=1}^{N} = \left\{n_1^{cg}, n_2^{cg}, \ldots, n_N^{cg}\right\}$, where $n_i^{cg} = \sum_{j=1}^{N} 1, \; if \; v_{ij} > 0$, $v_{ij} \in GCEM$.

(35) $GWCEV = \left\{n_i^{cw}\right\}_{i=1}^{N} = \left\{n_1^{cw}, n_2^{cw}, \ldots, n_N^{cw}\right\}$, where $n_i^{cw} = \sum_{j=1}^{N} v_{ij}$, $v_{ij} \in GCEM$.

(36) $GDCGV = \left\{n_i^{cdg}\right\}_{i=1}^{N} = \left\{n_1^{cdg}, n_2^{cdg}, \ldots, n_N^{cdg}\right\}$, where $n_i^{cdg} = d_i + \sum_{j=1}^{N} b_j \delta\left(i, j\right)$, where

$\delta(i,j)=\begin{cases}1, \; if \;\; c_{ij}>0 \\ 0, \; otherwise\end{cases}$, where $d_i \in EDV$, $b_j \in EDV$, $c_{ij} \in GCEM$.

The matrix for the weekly lecturers' preferences (LWPM), can be represented by a matrix of dimensionality $L \times D \times H$, where L is the number of lecturers, D is the number of days in the week schedule and H respectively, the number of hours in one day. The elements in the matrix that have value "0", showing that these hours are the most preferred. The elements with value "1" indicate that this is the next favorite hours for the lecturer, etc. LWPM matrix is shown in (37):

(37) $LWPM=\{m_k\}_{k=1}^{L}=\{m_1,m_2,...,m_L\}$, where m_k is the matrix of weekly preference of the k-th lecturer and $m_k=(p_{ij})_{DxH}$, where $p_{ij} \in \left[0, \left\lceil \frac{T}{l_k^t} \right\rceil -1\right]$, $l_k^t \in LHLV$.

The common matrix for the preferences of students (CSWPM), can be represented by a matrix of dimensionality $D \times H$. Similarly, as in the matrix LWPM, the lower the value of the element, the more preferred the timeslot corresponding to that element is in the matrix CSWPM preferences belong to all students. The definition of the matrix CSWPM, is shown in (38):

(38) $CSWPM=(p_{ij})_{DxH}$, where $p_{ij} \in \left[0, \left\lceil \frac{TS}{\sum_{i=1}^{S} s_i^t} \right\rceil -1\right]$, $s_i^t \in LHSV$.

Decision vector (RV) can be represented as a list of length N. Each element of the vector RV is an ordered pair of elements $\langle t_i^s, t_i^f \rangle$, respectively, for the beginning and end of the i-th event. The definition of the vector RV is shown in (39):

(39) $RV=\left\{n_i^r\right\}_{i=1}^{N}=\left\{n_1^r,n_2^r,\ldots,n_N^r\right\}$, where $n_i^r=\langle t_i^s,t_i^f \rangle$, $t_i^s \in AHV$, $t_i^f \in AHV$, $t_i^f=(t_i^s+v_i)-1$, $v_i \in EDV$.

The vector of hours allocated by days (HDRV) and the vector of fixed events (FEV) is defined, respectively:

(40) $HDRV=\left\{n_i^{rd}\right\}_{i=1}^{N}=\left\{n_1^{rd},n_2^{rd},\ldots,n_N^{rd}\right\}$, where $n_i^{rd}=g(i)$ and

$$g(i)=\begin{cases}\dfrac{t_i^s}{H},\ if\ t_i^s \bmod H=0\\ \left\lfloor\dfrac{t_i^s}{H}\right\rfloor+1,\ otherwise\end{cases}$$, where $t_i^s \in AHV$.

(41) $FEV=\left\{n_i^{fx}\right\}_{i=1}^{N}=\left\{n_1^{fx},n_2^{fx},\ldots,n_N^{fx}\right\}$, where $n_i^{fx}=\begin{cases}-1,\ if\ user\ is\ fixed\ event\ i\\ 0,\ otherwise\end{cases}$.

A matrix for coefficients of events distance (CDEM) has dimension $N\times N$ and is generated using an EV vector, GCEM matrix, RV vector and HDRV vector. The value of each element in the matrix CDEM, shows the number of hours which put the events of a distance i and j (if they are in conflict), where $i\neq j$, $i,j\in\{1,2,...,N\}$. CDEM is a symmetric matrix to its main diagonal and its definition is shown in (42):

(42) $CDEM=(p_{ij})_{NxN}$, where $p_{ij}=\delta(i,j)*\lambda(t_i,t_j)$, where $\delta(i,j)=\begin{cases}1,\ c_{ij}>0\\ 0,\ c_{ij}=0\end{cases}$, $c_{ij}\in GCEM$,

$$\lambda(t_i,t_j)=\begin{cases}\left(t_j^s-t_i^f\right)-1,\ if\ \begin{pmatrix}\left(d_i=d_j\right)\wedge\left(t_i^s<t_j^s\right)\wedge\left(t_i^s<t_j^f\right)\wedge\\ \left(t_i^f<t_j^s\right)\wedge\left(t_i^f<t_j^f\right)\end{pmatrix}\\ \left(t_i^s-t_j^f\right)-1,\ if\ \begin{pmatrix}\left(d_i=d_j\right)\wedge\left(t_i^s>t_j^s\right)\wedge\left(t_i^s>t_j^f\right)\wedge\\ \left(t_i^f>t_j^s\right)\wedge\left(t_i^f>t_j^f\right)\end{pmatrix}\\ 0,\ if\ \left(d_i\neq d_j\right)\end{cases}$$

where $d_i,d_j\in HDRV$, $t_i,\ t_j\in RV$, $t_i=\left\langle t_i^s,t_i^f\right\rangle,\ t_j=\left\langle t_j^s,t_j^f\right\rangle$.

The matrix for the students' weekly schedule (SWLM) is a three-dimensional matrix with dimensionality $S\times D\times H$. This matrix is generated using a SEM matrix, RV vector, ELV vector and EAV vector. The definition of the SWLM matrix is shown in (43):

(43) $SWLM=\{m_k\}_{k=1}^{S}=\{m_1,m_2,...,m_S\}$, where $m_k=(p_{ij})_{DxH}$,

$$p_{ij}=\begin{cases}\langle n,l,a\rangle,\ where\ \exists!n,\ which\ \left(s_{kn}=1\right)\wedge\left(t_n^s\leq h\leq t_n^f\right),\ n=1,2,...,N\\ 0,\ otherwise\end{cases},$$

$s_{kn}\in SEM$, $t_n=\left\langle t_n^s,t_n^f\right\rangle\in RV$, $h=\left(i-1\right)H+j$, $l=u_n,\ u_n\in ELV$, $a=v_n,\ v_n\in EAV$.

Similarly, the matrices for lecturers and auditoriums weekly schedule, respectively (LWLM) and (AWLM) are presented. For them each element contains the value "0" or an ordered pair, respectively, $\langle n,a\rangle$ or $\langle n,l\rangle$, where n is the event which will be held by lecturer l in auditorium a.

Conditions for the Existence of a Solution

The following three conditions must be satisfied in order to have a possible solution:

1) For $\forall k,\ k=1,2,...,L$, to be met:

(44) $\left\lceil \frac{l_k^t}{t_m^l} \right\rceil \leq D$, where $l_k^t \in LHLV$ and is the maximum number of hours a day, which may have the k-th lecturer.

2) For $\forall p,\ p=1,2,...,S$, to be met:

(45) $\left\lceil \frac{s_p^t}{t_m^s} \right\rceil \leq D$, where $s_p^t \in LHSV$ and t_m^s is the maximum number of hours a day, in which the p-th student may be involved.

3) The total duration of the timetable must be less than or equal to the absolute number of hours in the weekly schedule.

To formulate this condition, we construct graph $G=(V,A)$ as follows: each event n_i will correspond to the vertex $v_i \in V$ with weight $w(v_i)$, equal to the length of the i-th event i.e. $w(v_i)=n_i^d$, $n_i^d \in EDV$ and the edge between vertex i and j exists if and only if events i and j are involved in a conflict of common resources (students, lecturers or auditoriums).

The graph $G=(V,A)$ is called *r-chromatic* if its set of vertices V can be broken to *r* subsets $V_1,V_2,...,V_r$, such that $\bigcap_{k=1}^{r} V_k=\varnothing$, $\bigcup_{k=1}^{r} V_k=V$ and which have two vertices of the same subset which are not joined with edge. Classes $V_1,V_2,...,V_r$ are called chromatic classes. The *chromatic number* $\gamma_v(G)$ of the graph *G* is called the minimum number *r*, for which the graph *G* is *r*-colorable [6].

To determine the minimum length of the timetable it is necessary for all chromatic classes to find vertices with maximum weights and to form their sum. It must be less than or equal to the absolute number of hours, i.e.:

(46) $T \geq \sum_{k=1}^{r} \max_{i\in\{1,...,|V_k|\}} \left(w(v_i)\right)$, where $r=\gamma_v(G)$ is the chromatic number of graph *G*, $|V_k|$ is the cardinal number of the *k*-th chromatic class and $w(v_i)$ is the weight of the i-th vertex belonging to the *k*-th chromatic class.

The problem of finding the chromatic number of graph is NP-hard and its solution in a large number of vertices (or edges) is impossible. It is possible to use heuristic algorithms, in which it is not always guaranteed that the found chromatic number is minimal.

It is necessary to note that if one of the three inequalities (44), (45) and (46) are not met, then the number of days D in a weekly schedule is not sufficient and should be increased.

Hard Constraints

For a solution to be acceptable the following four soft constraints must be fulfilled:

1) Any element of any resource cannot be in two events at the same time, i.e.:

(47) $\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} p_{ij} * \delta\left(t_i, t_j\right) = 0$, where

$$\delta\left(t_i,t_j\right)=\begin{cases} 0,\ if\left(d_i=d_j\right)\wedge \\ \qquad \left(\left(t_i^s<t_j^s\right)\wedge\left(t_i^s<t_j^f\right)\wedge\left(t_i^f<t_j^s\right)\wedge\left(t_i^f<t_j^f\right)\right)\vee \\ \qquad \left(\left(t_i^s>t_j^s\right)\wedge\left(t_i^s>t_j^f\right)\wedge\left(t_i^f>t_j^s\right)\wedge\left(t_i^f>t_j^f\right)\right) \\ 1,\ otherwise \end{cases},$$

$p_{ij} \in GCEM$, $t_i \in RV$, $t_j \in RV$, $t_i = \left\langle t_i^s, t_i^f \right\rangle$, $t_j = \left\langle t_j^s, t_j^f \right\rangle$, $d_i \in HDRV$.

This condition requires the events that are in conflict to be allocated in different timeslots in the weekly schedule.

The number of students must be less than or equal to the capacity of the auditorium where the event takes place and the kind of auditorium, must match the kind of event, i.e. for $\forall i,\ i=1,2,...,N$ must be met:

(48) $\left(n_i^s \le a_k^c\right) \wedge \left(a_k^k = n_i^k\right) \Leftrightarrow n_i^a = a_k,\ k=1,2,...,A$, where $a_k^c \in ACV$, $a_k \in AV$, $n_i^a \in EAV$, $n_i^s \in NSEV$, $a_k^k \in AKV$, $n_i^k \in EKV$.

A lecturer cannot teach in more than t_m^l number of hours per day, i.e. for $\forall m_k, k=1,2,...,L$, where $m_k \in LWLM$, to be met:

(49) $l_{k_d} \le t_m^l,\ d=1,2,...,D$, where $l_{k_d} = \sum_{j=1}^{H} 1,\ if\ \ p_{ij} \ne 0$, $p_{ij} \in m_k$.

4) A student may not be involved in more than t_m^s number of hours per day, i.e. for $\forall m_k, k=1,2,...,S$ where $m_k \in SWLM$, to be met:

(50) $s_{k_d} \le t_m^s,\ d = 1,2,\ldots,D$, where $s_{k_d} = \sum_{j=1}^{H} 1,\ ако\ p_{ij} \neq 0,\ p_{ij} \in m_k$.

Soft Constraints

The soft constraints may be of different weights relative to each other. The quality of a solution is determined by the number of violations of those constraints. The higher this number is small the solution is better. We define the following three soft constraints:

1) The time span between the events to be the shortest possible.

To take into account the weights of the conflicts themselves, we will introduce the term "average cost of distance". For an event n_i, that cost is the sum of products between weights of conflicts and the coefficients of distance divided by the number of events.

We formulate this constraint as an objective function G_1:

(51) $G_1(i,j) = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} p_{ij} q_{ij}}{N^2} \to \min$, where $p_{ij} \in GCEM$, $q_{ij} \in CDEM$.

Because the matrix GCEM (respectively CDEM) is symmetrical to its main diagonal products of the elements above the main diagonal can be summed up and the resulting amount can be divided in half by the square of the number of events, i.e.:

$$G_1(i,j) = 2\frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} p_{ij} q_{ij}}{N^2} \to \min$$

A function $G_1(i,j)$ reaches a minimum at $\min\left(G_1(i,j)\right) = 0$.

2) Satisfaction of lecturers preferences to be maximized.

We formulate this constraint as an objective function G_2:

(52) $G_2(l,d,h) = \sum_{l=1}^{L} w_l \sum_{d=1}^{D}\sum_{h=1}^{H} p_{ldh}\delta(l,d,h) \to \min$, where $p_{ldh} \in LWPM$, $w_l \in WLV$ and

$\delta(l,d,h) = \begin{cases} 1, & x_{ldh} \neq 0 \\ 0, & otherwise \end{cases}$, where $x_{ldh} \in LWLM$.

A function $G_2(l,d,h)$ reaches a minimum at $\min\left(G_2(l,d,h)\right) = 0$.

3) Satisfaction of students preferences to be maximized.

Assuming that both resources "lecturers" and "students" have the same total weight, the weight of each student receives:

(53) $w_s = \frac{\sum_{i=1}^{L} l_i^w}{S}$, where $l_i^w \in WLV$.

Now we can formulate a third soft constraint as an objective function G_3 :

(54) $G_3\left(s,d,h\right) = \sum_{s=1}^{S} w_s \sum_{d=1}^{D} \sum_{h=1}^{H} p_{dh} \delta\left(s,d,h\right) \to \min$, where $p_{dh} \in CSWPM$ and

$\delta\left(s,d,h\right) = \begin{cases} 1, \ x_{sdh} \neq 0 \\ 0, \ otherwise \end{cases}$, where $x_{sdh} \in SWLM$.

Objective Function

The objective function of the problem can be formulated as a function that combines the three objectives G_1 , G_2 and G_3 , which were defined above.

It is necessary that a problem to be formulated mathematically as a linear multicriteria optimization.

The basic idea of multicriteria optimization is to convert output multicriteria problem in a problem with an objective function. The resulting problem leads to an efficient solution, which may not be optimal for all objectives of the original problem, because some of the objectives may conflict.

One possible approach for solving the multicriteria problems is to bring the relevant problem to problem with one criteria by replacing the set of objective functions $f_1,...,f_p$ with a single function $F(f_1,...,f_p)$. Defined in this way, the multicriteria problem is reduced to minimization (respectively, maximization) of total function $F(f_1,...,f_p)$, which is a function of the given criteria $f_1,...,f_p$.

One of the commonly used methods for the formation of the function $F(f_1,...,f_p)$ is the use of weight sums of functions $f_1,...,f_p$, presenting the criteria (objectives) in a given problem.

Let the multicriteria problem have *p* criteria (objectives) and *i*-th criteria be set as follows:

$\min(\max)\left\{f_{i,} \ i = 1,...,p\right\}$.

Then the total objective function if used the weighing method is:

$\min(\max)F\left(f_i,...,f_p\right) = \sum_{i=1}^{p} w_i * f_i$, where $w_i \geq 0, \ i = 1,...,p$ are weights that are set by the decision maker under the relative importance of each criteria.

The weighing method will be used for the task under investigation in this paper.

Definition. If X is convex set and $x_1,...,x_p \in X$, $w_i \geq 0$, $\sum_{i=1}^{p} w_i = 1$, then $x = \sum_{i=1}^{p} w_i x_i \in X$ is a convex combination of $x_1,...,x_p$ with coefficients $w_1,...,w_p$.

We determine the relative importance of each of the three criteria in such a way that the amount of weight equals to one.

Resources which interest us are lecturers L and students S because we have not defined soft constraints connected with auditoriums. The first objective G_1 is common for students and lecturers. The second objective

G_2 is connected only to the lecturers and the third objective G_3 is connected only to the students. The relative weight of the first objective is proportional to the sum of the number of lecturers and students i.e. $L+S$. The weighing of the second objective is proportional to the number of students S, and the weight of the third objective is proportional to the number of lecturers L. For for the three objectives we get a relative weight equal to $2(S+L)$. Since we have assumed that the sum of the weights of the objectives will be equal to one then for the weight coefficients we get the following values:

(55.1) $$w_1 = \frac{L+S}{2(L+S)} = \frac{1}{2}$$

(55.2) $$w_2 = \frac{S}{2(L+S)} = \frac{1}{2}\frac{S}{L+S}$$

(55.3) $$w_3 = \frac{L}{2(L+S)} = \frac{1}{2}\frac{L}{L+S}$$

For the common objective function G we get:

(56) $$G = w_1G_1 + w_2G_2 + w_3G_3 \to \min.$$

Because $w_1 \geq 0$, $w_2 \geq 0$, $w_3 \geq 0$ and $\sum_{i=1}^{3} w_i = 1$ the function G is a convex combination of G_1, G_2 and G_3, with coefficients w_1, w_2 and w_3.

By replacing the weights with their values and simplify (56), we obtain the objective function given in (57):

(57) $$G(i,j,l,d_1,h_1,s,d_2,h_2) = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} p_{ij}q_{ij}}{N^2} +$$
$$+\frac{1}{2}\frac{S}{L+S}\sum_{l=1}^{L} w_l \sum_{d_1=1}^{D}\sum_{h_1=1}^{H} u_{ld_1h_1}\delta_1(l,d_1,h_1) + \frac{1}{2}\frac{L}{L+S} w_s \sum_{s=1}^{S}\sum_{d_2=1}^{D}\sum_{h_2=1}^{H} v_{d_2h_2}\delta_2(s,d_2,h_2) \to \min$$

where N is the number of events S is the number of students, L is the number of lecturers, D is the number of days in the weekly schedule, H is the number of hours a day of the weekly schedule, $p_{ij} \in GCEM$, $q_{ij} \in CDEM$, $u_{ldh} \in LWPM$, $w_l \in WLV$,

$$\delta_1(l,d_1,h_1) = \begin{cases} 1, & x_{ld_1h_1} \neq 0 \\ 0, & otherwise \end{cases},$$

$x_{ld_1h_1} \in LWLM$, $v_{d_2h_2} \in CSWPM$,

$$w_s = \frac{\sum_{i=1}^{L} l_i^w}{S},\ l_i^w \in WLV,$$

$$\delta_2(s,d_2,h_2) = \begin{cases} 1, & x_{sd_2h_2} \neq 0 \\ 0, & otherwise \end{cases},$$

$x_{sd_2h_2} \in SWLM$,

provided that the conditions for the existence of solution (44), (45), (46) and hard constraints (47), (48), (49) and (50) are satisfied.

The function G reaches a minimum at $\min\left(G\left(i, j, l, d_1, h_1, s, d_2, h_2\right)\right) = 0$.

Conclusion

In this paper a model for the university course timetable problem is presented. Fundamental data structures, parameters, vectors and matrices, which are used in defining the problem are presented. Hard constraints on the verification of concurrent involvement of lecturers, students and auditoriums, checking for excess capacity auditoriums and verification of commitment of students and lecturers for more hours per day than a predefined number are presented. Also, the soft constraints formulated as the objective functions are presented. The total objective function which is a convex combination of the others (with appropriate weighing) is also defined.

As future trends of work we may mention: the creation of algorithms for solving the problem which is based on the proposed model. Their integration into real information systems aims at demonstrating the usefulness of the developed model in solving combinational optimization problems.

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