SPECTRAL COEFFICIENT OF CONSISTENCY OF FUZZY EXPERT INFORMATION AND ESTIMATION OF ITS SENSITIVITY TO FUZZY SCALES WHEN SOLVING FORESIGHT PROBLEMS

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Abstract: *An estimation of sensitivity of a new measure of consistency of interval pairwise comparison matrix, i.e. interval spectral coefficient of consistency to fuzzy fundamental scales which are the most often used in fuzzy AHP methods is carried out. This coefficient of consistency may be more valid for solving some foresight problems than other known indexes and methods. The coefficient of consistency is theoretical, but not empiric attribute of consistency of pairwise comparison matrix in the sense that on determination of consistency are not used randomly filled pairwise comparison matrices. For determination of admissible level of inconsistency of interval pairwise comparison matrices application and detection thresholds are developed. To estimate the sensitivity of interval spectral coefficient of consistency a computer simulation study was performed. Spectral coefficient of consistency of FPCM is proposed as combination of ISCC in all* α *-levels using linear, multiplicative and min combination rules of the AHP. For determination of admissible level of inconsistency of FPCM necessary and sufficient conditions are proposed. The AHP method and spectral coefficient of consistency were used to evaluate critical technologies of energy conservation and power efficiency in Ukraine.*

Keywords: *interval pairwise comparison matrix, fuzzy pairwise comparison matrix, spectral coefficient of consistency, admissible level of inconsistency, interval weights, fuzzy fundamental scales, foresight problems.*

ACM Classification Keywords: *H.4.2. INFORMATION SYSTEM APPLICATION: type of* system strategy

Introduction

Technological foresight is a decision making relative to complex systems with human factor concerning their potential behaviour in future [Zgurovskiy & Pankratova, 2007]. Reliability of expert estimation of information in problems of technological foresight is of importance under modern conditions of high dynamism of world globalization. Foresight problems have innovation character. Mainly information of qualitative character in a form of expert estimates, which is often fuzzy, contradictory and incomplete, serves as input data for these problems, therefore, technique of decision making support must include methods for processing information of the mentioned character and, moreover, means of estimation of validity and degree of consistency for the obtained results.

One of the methods, which are applied in the technique of scenario analysis [Zgurovskiy & Pankratova, 2007] for solving problems of technological foresight, is the Analytic Hierarchy Process (AHP). Elaborated by T.Saaty AHP and its generalization Analytic Network Process are popular decision tools used to weight items based on pairwise comparisons in terms of multiple criteria [Saaty, 1980]. Nowadays AHP and its extensions are used to determine relative weights (priorities) of items and probabilities of scenarios for solving foresight problems [Pankratova & Nedashkovskaya, 2007 a, b].

In general, estimates, which are given by experts, will not be consistent. T.Saaty [Saaty, 1980] defines the consistency index CI and ratio CR for a crisp pairwise comparison matrix. The CR value is calculated using the average of the CI values for random matrices. Other consistency coefficients have been developed: the harmonic consistency index [Stein & Mizzi, 2007], which can be viewed as an approximation to the CI, the determinant to measure consistency [Pelaez & Lamata, 2003]. Another type of consistency measure is the distance from a specific consistent matrix. The sum of squared deviations of the log of the elements of a matrix from the log of the matrix elements generated by the row geometric mean solution is used as a measure of consistency [Aguaron & Moreno-Jimenez, 2003].

Problems of technological foresight are characterized by the presence of conceptual uncertainty and multiple-factor risks. Inaccuracy in expert estimates and connected with it risks can be expressed in two ways: 1) by means of crisp estimates and probability distribution function; 2) by means of interval estimates without distribution of probabilities. Probabilistic representation of crisp estimates and distribution functions provides creation of several modifications of AHP, which are called stochastic AHP, while the second way of representation of inaccuracy of expert estimates results in necessity of application of interval and fuzzy methods of weights calculation. Methods for obtaining weights from interval pairwise comparison matrixes (IPCM) may be classified as follows: methods, which make it possible to obtain weights from both consistent and inconsistent IPCM [Saaty&Vargas, 1987; Wang & Elhag, 2007], as well as methods, which work only with consistent IPCM [Arbel, 1989] or do not guarantee solution obtaining in the case of inconsistent IPCM [Sugihara et al, 2004].

New measure of consistency of IPCM the interval spectral coefficient of consistency (ISCC) k_y^{interv} was introduced [Pankratova & Nedashkovskaya, 2007 b]. This spectral coefficient is theoretical, but not empiric attribute of consistency of IPCM in the sense that on determination of consistency we do not use random matrices. An application and detection thresholds of the ISCC are developed [Pankratova & Nedashkovskaya, 2007 b] for determination of admissible level of inconsistency of IPCM in the AHP method.

One of the problems when formalizing expert judgment in a form of fuzzy number and linguistic variable is to choose the type and parameters of a membership function. The lognormal [Laininen & Hämäläinen, 2003], logit and probit multinomial [Hahn, 2003] and other [Lipovetsky & Conklin, 2002] distributions of expert judgments are used in modified AHP methods. But justification of the distribution law of expert judgments needs further investigations. In fuzzy AHP methods [Wang & Chen, 2008; Amy Lee et al, 2008; Wu et al, 2008; Amy Lee, 2009; Kreng & Wu, 2007; Kulak& Kahraman, 2005] different fuzzy fundamental scales are used to formalize expert judgment.

In present paper we perform an estimation of sensitivity of ISCC to fuzzy fundamental scales which are the most often used in fuzzy AHP methods.

Problem statement

Let us consider definition of fuzzy pairwise comparison matrix, which is to be used further.

Fuzzy pairwise comparison matrix (FPCM) is a pairwise comparison matrix $A^{fuzzy} = \{(a_{ij}^{fuzzy}) \mid i = \overline{1,n}, j = \overline{1,n}\}$, for which $a_{ij}^{fuzzy} = (x, \mu_{ij}(x))$ is a normal convex fuzzy set (fuzzy number) reflecting the result of paired comparison of objects O_i and O_j, $x \in \Re$, where $\Re$ is the set of real numbers. The value of the membership function $\mu_{ij}(x)$ of the fuzzy set a_{ij}^{fuzzy} is the degree of realization of preference $O_i \succeq O_j$ [Pankratova & Nedashkovskaya, 2007 a].

Let it be given: FPCM A^{fuzzy}; the vector of fuzzy weights $w^{fuzzy} = \{(w_i^{fuzzy}) \mid i = \overline{1,n}\}$, which reflects preferences written in FPCM A^{fuzzy}; coordinate w_i^{fuzzy} of this vector is a fuzzy set.

It is necessary to determine a degree of consistency of FPCM and admissible level of inconsistency of FPCM.

Problem solving. Interval spectral coefficient of consistency (ISCC)

For estimating consistency of expert information we decompose FPCM A^{fuzzy} by sets of level $A(\alpha)$, where $A(\alpha) = \{(a_{ij}(\alpha)) \mid i = \overline{1,n},\ j = \overline{1,n}\}$ is a matrix of sets of level $\alpha \in [0,1]$, $a_{ij}(\alpha) = \{x : \mu_{ij}(x) \geq \alpha\}$, $\mu_{ij}(x)$ is a membership function of fuzzy set a_{ij}^{fuzzy}, $x \in \Re$.

Since elements a_{ij}^{fuzzy} of FPCM serve as estimates for some parameters (in this case as estimates of paired comparisons), then it is convenient to use triangular fuzzy values $a_{ij}^{fuzzy} = (a_{ij}^l, a_{ij}^m, a_{ij}^u)$, $a_{ij}^l \leq a_{ij}^m \leq a_{ij}^u$ for their representation. Then we pass from the initial FPCM A^{fuzzy} to consideration of the set of IPCM $\{A(\alpha) \mid \alpha \in [0,1]\}$, where $A(\alpha) = \{(a_{ij}(\alpha)) \mid i, j = \overline{1,n}\}$, $a_{ij}(\alpha) = [a_{ij}^m - x1_{ij}(\alpha), a_{ij}^m + x2_{ij}(\alpha)]$, a_{ij}^m is the value of interval with the greatest degree of realization of preference, $x1_{ij}(\alpha) = (1-\alpha)(a_{ij}^m - a_{ij}^l)$, $x2_{ij}(\alpha) = (1-\alpha)(a_{ij}^u - a_{ij}^m)$, $x1_{ij}(\alpha) \geq 0$, $x2_{ij}(\alpha) \geq 0$ are values of deviations from the value a_{ij}^m, $i = \overline{1,n}$, $j = \overline{1,n}$.

We state that necessary condition for FPCM A^{fuzzy} to be consistent is as follows: all IPCM $A(\alpha)$ of sets of levels $\alpha \in [0,1]$ are consistent. Thus the problem is reduced to determination of consistency of IPCM. Let us consider IPCM

$$A = \{(a_{ij}) \mid a_{ij} = [l_{ij}, u_{ij}], i = \overline{1,n}, j = \overline{1,n}\},$$

where $l_{ij} = m_{ij} - x1_{ij}$, $u_{ij} = m_{ij} + x2_{ij}$, m_{ij} is the value of the interval with the greatest degree of realization of preference. Values $x1_{ij} \geq 0$ and $x2_{ij} \geq 0$ show the degree of uncertainty associated with approximate equality $m_{ij} \approx w_i / w_j$.

Let $\{A^h \mid h = \overline{1,n}\}$ be the set of matrices generated by rows [Pankratova & Nedashkovskaya, 2007 b] of the matrix A. Suppose every object O_k, $k = \overline{1,n}$ is characterized by n estimates of interval weights $W^k = \{(w^{kh}) \mid h = \overline{1,n}\}$, where $w^{kh} = [w^{khl}, w^{khu}]$ is interval weight of the object O_k, obtained from the matrix A^h using two-staged method, described in [Pankratova & Nedashkovskaya, 2007 a].

Further we assume that estimates of weights are numbers of marks of certain scale $S = \{s_j \mid j = \overline{0,m}\}$ with $(m+1)$ marks. The number of scale marks can be determined, if we set admissible error of attribution of weight estimate to either this or another mark. The zero, first and the last scale marks are equal correspondingly to $s_0 = 0$, $s_1 = 1/m$ and $s_m = 1$. Scale mark s_j is equal to j/m.

We construct mapping $F^k: W^k \underset{F^k}{\rightarrow} S$, $F^k(w^{kh}) = s_j$. The mapping F^k is composition of mappings G and D^k, where $G(d^{kh})$ defines scale mark, which is the least remote from d^{kh}. $D^k = \{d^{kh} \mid d^{kh} \in \Re, h = \overline{1,n}\}$ is the set of distances from interval weights w^{kh} to scale mark s_0. In the case of interval weights we realize attribution of weight w^{kh} to either one or another scale mark according to distance $d^{kh} = D(w^{kh}, O) = \sqrt{\left(\frac{w^{khu} + w^{khl}}{2}\right)^2 + \frac{1}{3}\left(\frac{w^{khu} - w^{khl}}{2}\right)^2}$. Advantage of this method of determination of distance between intervals consists in the fact that we take into account all points in both intervals in contrast to the majority of existing techniques, which are often based only on the left or right boundaries of intervals.

Next, the set W^k is represented by spectrum, which is a vector $R^k = \{(r_j^k) \mid j = \overline{1,m}\}$, where r_j^k is the number of interval weights belonging to scale mark s_j.

To construct ISCC of IPCM we initially determine the ISCC $k_y^{interv}(R^k)$ of spectrum for the set W^k of object O_k interval weights [Pankratova & Nedashkovskaya, 2007 b]:

$$k_y^{interv}(R^k) = \left(1 - \frac{\frac{1}{n}\sum_{j=1}^{m} r_j^k \mid j - a^k \mid - \sum_{j=1}^{m} \frac{r_j^k}{n} \ln\left(\frac{r_j^k}{n}\right)}{G\sum_{j=1}^{m}\left|j - \frac{m+1}{2}\right| + \ln(m)}\right) z,$$

where a^k is mean value of the set of interval weights W^k, $G = n / \ln(n) m \ln(m)$ is scale coefficient, z is Boolean function, which sets necessary and sufficient conditions of equality to zero for ISCC.

Coefficient $k_y^{interv} = \inf_{k \in [1;n]} k_y^{interv}(R^k)$ is called the ***interval spectral coefficient of consistency (ISCC) of IPCM*** *A* [Pankratova & Nedashkovskaya, 2007 b].

For determination of admissible level of inconsistency of IPCM the criterion is proposed.

Criterion of admissible level of inconsistency of IPCM:

- If $k_y^{interv} < T_0^{interv}$, where T_0^{interv} is detection threshold, then IPCM does not contain information and it is necessary to make paired comparisons again;
- If $T_0^{interv} \le k_y^{interv} < T_u^{interv}$, where T_u^{interv} is application threshold, then IPCM contains useful information, but this IPCM is strongly inconsistent and it is necessary to use methods for increase of its consistency;
- If $k_y^{interv} \ge T_u^{interv}$, then the degree of inconsistency of IPCM is supposed as admissible.

Detection threshold T_0^{interv} on estimate of consistency of interval judgements is the ISCC $k_y^{interv}(R^0)$ of the spectrum R^0, which is constructed from spectrum, in which every scale mark was selected exactly by one expert. In spectrum R^0 estimate being at first scale mark is excluded and placed supplementary on scale mark

$[\varepsilon m+1]$, where $[.]$ is operation of taking integer part, $\varepsilon = 0.5$ is minimally registered quantity in terms of the scale.

Application threshold T_u^{interv} on estimate of consistency of interval judgements is the ISCC $k_y^{interv}(R^u)$ of consistency of the spectrum R^u, which contains only two estimates distant for b scale marks, $b=1$.

Spectral coefficient of consistency of fuzzy pairwise comparison matrix

Decomposing FPCM A^{fuzzy} by sets of levels $\alpha_1,...,\alpha_s,...\alpha_S \in [0,1]$ we obtain a set of IPCM $A=\{A(\alpha_s) \mid s=\overline{1,S}\}$ and a set of ISCC:

$$k_y^{interv} = \{k_y^{interv}(\alpha_s) \mid s=\overline{1,S}\},$$

where $k_y^{interv}(\alpha_s)$ is ISCC of IPCM $A(\alpha_s)$.

Using triangular fuzzy values $a_{ij}^{fuzzy}=(a_{ij}^l, a_{ij}^m, a_{ij}^u)$ we have $A(\alpha_s)=\{(a_{ij}(\alpha_s)= =[l_{ij}(\alpha_s), u_{ij}(\alpha_s)]) \mid i,j=\overline{1,n}\}$, $l_{ij}(\alpha_s)=a_{ij}^m-(1-\alpha_s)(a_{ij}^u-a_{ij}^m)$, $u_{ij}(\alpha_s)=a_{ij}^m+(1-\alpha_s)(a_{ij}^u-a_{ij}^m)$.

A low value of α_s corresponds to IPCM with wide intervals, whose bounds have low memberships in the fuzzy set a_{ij}^{fuzzy}. Therefore, a lower value of α_s corresponds to less reliable IPCM with a high level of uncertainty. A greater value of α_s corresponds to IPCM with narrow intervals, whose bounds have higher memberships in the fuzzy set a_{ij}^{fuzzy}. Therefore, a higher value of α_s corresponds to more reliable IPCM. Level $\alpha_s=1$ results in the crisp pairwise comparison matrix, whose elements are the values a_{ij}^m of initial fuzzy judgments a_{ij}^{fuzzy} with the greatest degree of realization of dominance.

In this paper spectral coefficient of consistency of FPCM A^{fuzzy} is defined as combination of $k_y^{interv}(\alpha_s)$ in all levels α_s, $s=\overline{1,S}$ using combination rules of the AHP method. Linear, multiplicative and min combination rules of the AHP method are known in literature of the subject [Pankratova & Nedashkovskaya, 2011]. Let us consider calculation of spectral coefficient of consistency k_y^{fuzzy} of FPCM using these combination rules. Suppose α_s^* are normalized values, $\alpha_s^* = \alpha_s / \sum_{s=1}^{S}\alpha_s$.

Spectral coefficient of consistency k_y^{fuzzy} ***of FPCM*** A^{fuzzy} ***using linear combination rule*** is as follows:

$$k_y^{fuzzy(1)} = \sum_{s=1}^{S} \alpha_s^* k_y^{interv}(\alpha_s).$$

Spectral coefficient of consistency k_y^{fuzzy} ***of FPCM*** A^{fuzzy} ***using multiplicative combination rule*** is as follows:

$$k_y^{fuzzy(2)} = \prod_{s=1}^{S}\left(k_y^{interv}(\alpha_s)\right)^{\alpha_s^*}.$$

Spectral coefficient of consistency k_y^{fuzzy} ***of FPCM*** A^{fuzzy} ***using min combination rule*** is as follows:

$$k_y^{fuzzy(3)} = \min_{s=1,..,S} \alpha_s^* k_y^{interv}(\alpha_s).$$

For determination of admissible level of inconsistency of FPCM necessary and sufficient conditions are proposed. The necessary condition requires admissible level of IPCM at each level $\alpha_1,...,\alpha_s,...\alpha_S \in [0,1]$. The sufficient condition deals with combined spectral coefficient of consistency k_y^{fuzzy} obtained by one of above combination rules.

Necessary condition of admissible level of inconsistency of FPCM:

- If $k_y^{interv}(\alpha_s) < T_0^{interv}$ at each level $\alpha_s \in [0,1]$, $s = \overline{1,S}$, then FPCM does not contain information and it is necessary to make paired comparisons again.

- If $T_0^{interv} \le k_y^{interv}(\alpha_s) < T_u^{interv}$ at each level $\alpha_s \in [0,1]$, $s = \overline{1,S}$, then FPCM contains useful information, but this FPCM is strongly inconsistent and it is necessary to use methods for increase of its consistency.

- If $k_y^{interv}(\alpha_s) \ge T_u^{interv}$ at each level $\alpha_s \in [0,1]$, $s = \overline{1,S}$, then the degree of inconsistency of FPCM is supposed as admissible.

Sufficient condition of admissible level of inconsistency of FPCM has the same formulation, but combined value k_y^{fuzzy} obtained by one of above combination rules is used instead of $k_y^{interv}(\alpha_s)$.

Estimation of sensitivity of ISCC to fuzzy fundamental scales

In AHP the fundamental scale of relative importance [Saaty, 1980] is the most commonly used numerical scale to quantify the pairwise comparisons. Marks of this scale (linguistic terms) are as follows: “equal importance” (corresponding scalar value is equal to 1), “moderate preference” (3), “strong preference” (5), “very strong preference” (7), “absolute preference” (9) and intermediate marks. This scale is one of the advantages of AHP over other expert methods since it optimally takes into account psychophysiological features of a human being [Arbel, 1989].

We consider fuzzy fundamental scales [Wang & Chen, 2008; Amy Lee et al, 2008; Wu et al, 2008; Amy Lee, 2009; Kreng & Wu, 2007; Kulak& Kahraman, 2005], which are the most often used in fuzzy AHP methods. Linguistic terms of the fuzzy scales are the same as the terms of traditional (crisp) fundamental scale and their numerical values are represented as triangular fuzzy numbers (see Table 1). The fuzzy numbers of different fuzzy scales have different parameters. Let us perform an estimation of sensitivity of ISCC to the fuzzy fundamental scales shown in the Table 1.

Table 1. Fuzzy fundamental scales (FFSs)

	Triangular fuzzy numbers					
Linguistic terms	FFS 1 [Wang & Chen, 2008]	FFS 2 [Amy Lee et al, 2008]	FFS 3 [Wu et al, 2008]	FFS 4 [Amy Lee, 2009]	FFS 5 [Kreng & Wu, 2007]	FFS 6 [Kulak& Kahraman, 2005]
Equal $\widetilde{1}$	$\widetilde{1} = (1,1,3)$	$\widetilde{1} = (1,1,1)$	$\widetilde{1} = (1,1,1)$	$\widetilde{1} = (1,1,2)$	$\widetilde{1} = (1,1,1)$	$\widetilde{1} = (1/2,1,3/2)$

Moderate preference $\widetilde{3}$	$\widetilde{3}=(1,3,5)$	$\widetilde{3}=(2,3,4)$	$\widetilde{3}=(2,3,4)$	$\widetilde{3}=(2,3,4)$	$\widetilde{3}=(1,3,5)$	$\widetilde{3}=(1,3/2,2)$
Strong preference $\widetilde{5}$	$\widetilde{5}=(3,5,7)$	$\widetilde{5}=(4,5,6)$	$\widetilde{5}=(4,5,6)$	$\widetilde{5}=(4,5,6)$	$\widetilde{5}=(3,5,7)$	$\widetilde{5}=(3/2,2,5/2)$
Very strong preference $\widetilde{7}$	$\widetilde{7}=(5,7,9)$	$\widetilde{7}=(6,7,8)$	$\widetilde{7}=(6,7,8)$	$\widetilde{7}=(6,7,8)$	$\widetilde{7}=(5,7,9)$	$\widetilde{7}=(2,5/2,3)$
Absolute preference $\widetilde{9}$	$\widetilde{9}=(7,9,9)$	$\widetilde{9}=(9,9,9)$	$\widetilde{9}=(8,9,10)$	$\widetilde{9}=(8,9,9)$	$\widetilde{9}=(7,9,11)$	$\widetilde{9}=(5/2,3,7/2)$
Intermediate terms: $\widetilde{2}$, $\widetilde{4}$, $\widetilde{6}$, $\widetilde{8}$	$\widetilde{2}=(1,2,4)$ $\widetilde{4}=(2,4,6)$ $\widetilde{6}=(4,6,8)$ $\widetilde{8}=(6,8,9)$	$\widetilde{2}=(1,2,3)$ $\widetilde{4}=(3,4,5)$ $\widetilde{6}=(5,6,7)$ $\widetilde{8}=(7,8,9)$	$\widetilde{2}=(1,2,3)$ $\widetilde{4}=(3,4,5)$ $\widetilde{6}=(5,6,7)$ $\widetilde{8}=(7,8,9)$	$\widetilde{2}=(1,2,3)$ $\widetilde{4}=(3,4,5)$ $\widetilde{6}=(5,6,7)$ $\widetilde{8}=(7,8,9)$	$\widetilde{2}=(1,2,4)$ $\widetilde{4}=(2,4,6)$ $\widetilde{6}=(4,6,8)$ $\widetilde{8}=(6,8,10)$	$\widetilde{2}=(3/4,5/4,7/4)$ $\widetilde{4}=(5/4,7/4,9/4)$ $\widetilde{6}=(7/4,9/4,11/4)$ $\widetilde{8}=(9/4,11/4,13/4)$

The ***ISCC*** k_y^{interv} ***of IPCM is sensitive to fuzzy scale*** if there are different results about admissible level of inconsistency of IPCM according to above Criterion when different fuzzy scales are used. Inadmissible level of inconsistency means that IPCM do not contain information and it is necessary to make paired comparisons again ($k_y^{interv} < T_0^{interv}$) or IPCM is strongly inconsistent and it is necessary to use methods for increase of its consistency ($T_0^{interv} \le k_y^{interv} < T_u^{interv}$).

To estimate a sensitivity of ISCC to fuzzy scale a computer simulation study was performed. Random test FPCM were examined while dimentions of the FPCM were varied. Lenguistic terms which represented elements of one triangular part of these test FPCM were chosen randomly from the set $\{\widetilde{1}, \widetilde{2},..., \widetilde{9}\}$. Then FPCM were formed in six different FFS shown in the Table 1. Elements of another triangular part of the FPCM were calculated according to the property of reciprocity as follows: $1/\widetilde{x} = (1/c, 1/b, 1/a)$. $1/\widetilde{x}$ is an element of lower triangular part of the FPCM, where $\widetilde{x} = (a, b, c)$ is symmetric element of upper triangular part of the FPCM. Elements on a diagonal of the FPCM are equal to a fuzzy number $\widetilde{1} = (1, 1, 1)$. Each FPCM was decomposed by sets of level $\alpha \in [0, 1]$ and IPCM were built. Level α was possessed the values $\alpha_1 = 0$, $\alpha_2 = 0.1$, ..., $\alpha_{11} = 1$. Interval weights were calculated for each of the matrices generated by rows of the IPCM using two-staged method [Pankratova & Nedashkovskaya, 2007 a]. ISCC k_y^{interv} of IPCM was determined. Random test FPCM's were formed with 10000 replications per each case in order to derive statistically significant results. Cases of sensitivity of the ISCC of IPCM to fuzzy scale were recorded.

The results reveal that in general the ISCC is not sensitive to the considered fuzzy scales.

To better understand method of estimation of sensitivity of ISCC to fuzzy fundamental scales, the following test decision problem with four alternatives is considered. Pairwise comparisons of these alternatives in linguistic terms are shown in Fig.1. Let two different FFS from the Table 1, for instance FFS 1 and FFS 2 are chosen to form FPCM. These FPCM A^{FFS_1} and A^{FFS_2} are shown in Fig. 2. Let us consider sets of the level $\alpha_2 = 0.1$ of the FPCM that result in two IPCM, i.e. $IPCM^{FFS_1}(\alpha_2)$ and $IPCM^{FFS_2}(\alpha_2)$ shown in Fig.3.

$$A = \begin{pmatrix} \tilde{1} & \tilde{3} & \tilde{2} & \tilde{4} \\ 1/\tilde{3} & \tilde{1} & \tilde{4} & \tilde{7} \\ 1/\tilde{2} & 1/\tilde{4} & \tilde{1} & \tilde{9} \\ 1/\tilde{4} & 1/\tilde{7} & 1/\tilde{9} & \tilde{1} \end{pmatrix}$$

Figure 1. Pairwise comparison matrix in linguistic terms

$$A^{FFS_1} = \begin{pmatrix} (1,1,1) & (1,3,5) & (1,2,4) & (2,4,6) \\ (1/5,1/3,1) & (1,1,1) & (2,4,6) & (5,7,9) \\ (1/4,1/2,1) & (1/6,1/4,1/2) & (1,1,1) & (7,9,9) \\ (1/6,1/4,1/2) & (1/9,1/7,1/5) & (1/9,1/9,1/7) & (1,1,1) \end{pmatrix}$$

(a)

$$A^{FFS_2} = \begin{pmatrix} (1,1,1) & (2,3,4) & (1,2,3) & (3,4,5) \\ (1/4,1/3,1/2) & (1,1,1) & (3,4,5) & (6,7,8) \\ (1/3,1/2,1) & (1/5,1/4,1/3) & (1,1,1) & (9,9,9) \\ (1/5,1/4,1/3) & (1/8,1/7,1/6) & (1/9,1/9,1/9) & (1,1,1) \end{pmatrix}$$

(b)

Figure 2. FPCMs on the FFS 1 (a) and on the FFS 2 (b)

$$IPCM^{FFS_1}(\alpha_2) = \begin{pmatrix} [1, 1] & [1.2, 4.8] & [1.1, 3.8] & [2.2, 5.8] \\ [0.21, 0.93] & [1, 1] & [2.2, 5.8] & [5.2, 8.8] \\ [0.28, 0.95] & [0.18, 0.48] & [1, 1] & [7.2, 9] \\ [0.18, 0.48] & [0.11, 0.19] & [0.11, 0.14] & [1, 1] \end{pmatrix}$$

(a)

$$IPCM^{FFS_2}(\alpha_2) = \begin{pmatrix} [1, 1] & [2.1, 3.9] & [1.1, 2.9] & [3.1, 4.9] \\ [0.26, 0.49] & [1, 1] & [3.1, 4.9] & [6.1, 7.9] \\ [0.35, 0.95] & [0.21, 0.33] & [1, 1] & [9, 9] \\ [0.21, 0.33] & [0.13, 0.16] & [0.11, 0.11] & [1, 1] \end{pmatrix}$$

(b)

Figure 3. IPCMs for FPCM on the FFS 1 (a) and on the FFS 2 (b) when $\alpha_2 = 0.1$

Then matrices generated by rows of these IPCM are formed. For instance, IPCM generated by *h*-th row of the $IPCM^{FFS_1}(\alpha_2)$, *h*=1 is shown in Fig 4. Weights for two matrices generated by *h*-th rows of the $IPCM^{FFS_1}(\alpha_2)$ and $IPCM^{FFS_2}(\alpha_2)$, *h*=1 are calculated using two-staged method [Pankratova & Nedashkovskaya, 2007 a] and are as follows: $w^{h\,FFS_1} = (0.40, 0.36, 0.16, 0.07)$, $w^{h\,FFS_2} = (0.35, 0.44, 0.14, 0.07)$.

Also weights for matrices generated by other rows of the IPCM are calculated.

$$IPCM^{h\ FFS_1} = \begin{pmatrix} [1,1] & [1.2, 4.8] & [1.1, 3.8] & [2.2, 5.8] \\ [0.21, 0.93] & [1,1] & [0.8, 0.9] & [1.2, 1.8] \\ [0.28, 0.95] & [1.11, 1.25] & [1,1] & [1.5, 2.0] \\ [0.18, 0.48] & [0.56, 0.83] & [0.50, 0.67] & [1,1] \end{pmatrix}$$

Figure 4. IPCM generated by *h*-th row of the $IPCM^{FFS_1}(\alpha_2)$ when *h*=1

Values of ISCC of the $IPCM^{FFS_1}$ and $IPCM^{FFS_2}$, detection and application thresholds are equal correspondingly to $k_y^{interv}(IPCM^{FFS_1}) = 0.851$, $k_y^{interv}(IPCM^{FFS_2}) = 0.784$, $T_0^{interv} = 0.398$ and $T_u^{interv} = 0.878$. Since $T_0^{interv} \le k_y^{interv} < T_u^{interv}$ for both IPCMs, then both IPCMs contain useful information, but are strongly inconsistent and it is necessary to use methods for increase of their consistency. Hence ISCC of IPCM when using different scales, namely FFS 1 and FFS 2, leads to the same conclusion about strong inconsistency of IPCM. Therefore ISCC is not sensitive to the considered fuzzy scales. This result holds for majority of the random FPCM.

Evaluation of critical technologies (CTs) of energy conservation and power efficiency in Ukraine

The AHP method and ISCC were used to calculate consistent relative priority values for critical technologies (CTs) of energy conservation and power efficiency in Ukraine. Quantitative information of passports of CTs and qualitative information in a form of expert estimates serves as input data for this problem. A list of 14 CTs and their technical passports were presented by leading organizations in energy sector of Ukraine on a first stage of foresight process. Then the CTs were clustered as follows: energy conservation CTs, renewable energy CTs and eco-house CT. Energy conservation CTs include energy conservation while producing energy (cogeneration technologies and power machine building) and in energy networks (electrical power engineering and technologies of burning). Renewable energy CTs include geothermal, wind, solar and bioenergetics technologies. Problem of power efficiency is included in a notion of eco-house only as a part along with building materials production, construction of eco-house and waste utilization. Therefore the technology of effective eco-house was considered separately.

Using information of passports of CTs and estimates of 12 experts about different factors of risk for CTs and importance of decision criteria (Fig.5), consistent relative priority values for CTs were calculated. On basis of these priority values rating of CTs of energy conservation and power efficiency in Ukraine was determined.

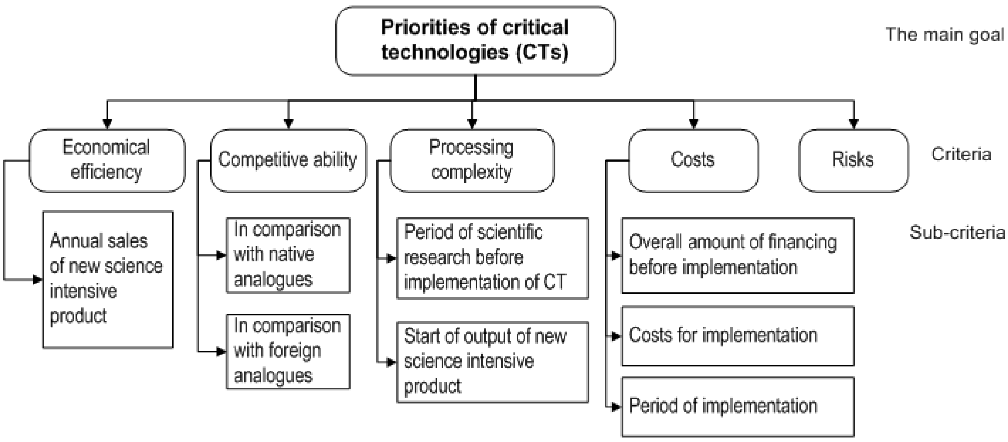

Figure 5. Hierarchy of criteria for choice of priority CTs

Economical efficiency includes annual sales of new science intensive product in value indicator (millions of dollars). Competitive ability of CTs is evaluated in comparison with native and foreign analogues. Processing complexity of CTs is measured by two parameters: period of scientific research before implementation of CT (years) and start of output of new science intensive product (in x years). Costs include costs in terms of money and time, namely: overall amount of financing before implementation (thousands of grivnas), costs for implementation (thousands of grivnas), period of implementation (years).

Table 2. Relative priority values (weights) of decision criteria

Criterion	Sub-criterion	Weight
Economical efficiency(0.486)	Annual sales of new science intensive product (1)	0.486
Competitive ability (0.137)	Competitive ability in comparison with native analogues (0.125)	0.017
	Competitive ability in comparison with foreign analogues (0.875)	0.120
Processing complexity (0.066)	Period of scientific research before implementation of CT (0.833)	0.054
	Start of output of new science intensive product (0.167)	0.011
Costs (0.149)	Overall amount of financing before implementation (0.481)	0.072
	Costs for implementation (0.405)	0.060
	Period of implementation (0.114)	0.017
Risks (0.162)	Risks (1)	0.162

Global Priorities of clusters of CTs, calculated on basis of judgements of 12 experts are shown in Table 3 (values without sign *). Spectral coefficients of consistency of individual global priorities spectrums for clusters “Energy conservation CTs”, “Renewable energy CTs” and “Eco-house” (Fig. 6) which are equal to 0.682, 0.737 and 0.658 respectively, indicate that expert judgements for these clusters are strongly inconsistent ($k_y<T_u$), T_0=0.398, T_u=0.790 and it is necessary to use methods for increase of its consistency. Revised values are marked in Table3 with sign *. Spectrums of revised values are more consistent, spectral coefficients take values 0.758, 0.771 and 0.770 respectively (Fig.7).

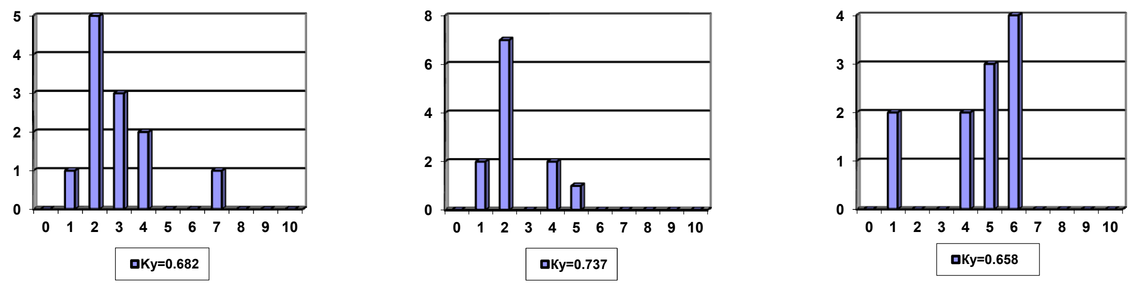

Figure 6. Spectrums of individual global priorities for clusters “Energy conservation CTs” (a), “Renewable energy CTs” (b) and “Eco-house” (c)

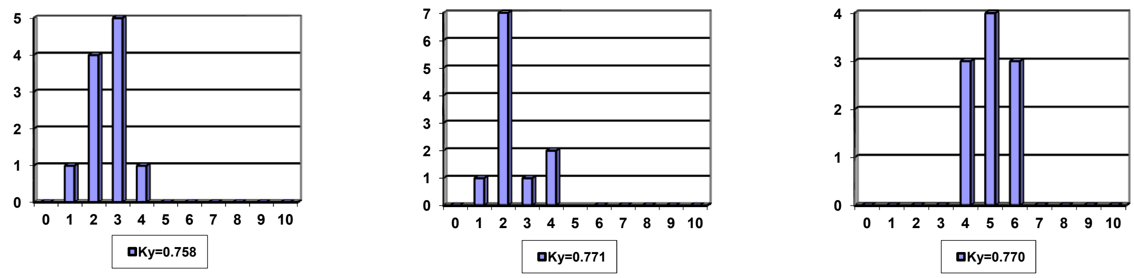

Figure 7. Spectrums of revised individual global priorities for clusters “Energy conservation CTs” (a), “Renewable energy CTs” (b) and “Eco-house” (c)

Table 3. Individual global priorities of clusters of CTs

Expert’s No	Individual global priorities			Expert’s No	Individual global priorities		
	Energy conservation CTs	Renewable energy CTs	Eco-house		Energy conservation CTs	Renewable energy CTs	Eco-house
1	0.247/0.257*	0.115/0.225*	0.638/0.519*	7	0.251	0.165	0.584
2	0.186/0.254*	0.210/0.269*	0.604/0.477*	8	0.358	0.176	0.466
3	0.677/-	0.230	0.093/ -	9	0.229	0.391	0.380
4	0.223	0.416	0.361	10	0.160	0.240	0.600
5	0.143	0.248	0.609	11	0.319	0.179	0.502
6	0.322	0.535/-	0.143/-	12	0.441/0.216*	0.102	0.457/0.379*

Group global priorities (Table 4) result in cluster “Eco-house” as the most priority cluster of CTs of energy conservation and power efficiency in Ukraine (corresponding priority value equals 0.426). Clusters “Energy conservation CTs” and “Renewable energy CTs” receive lower priority values. Cluster “Eco-house” contains only one technology “Technology of power efficient eco-house with renewable energy”. This CT receives the highest priority value, i.e. the first rank, thus, cluster “Eco-house” and CT “Technology of power efficient eco-house with renewable energy” are not considered in further analysis. Then, CTs with second, third and other ranks, which form clusters “Energy conservation CTs” and “Renewable energy CTs” are founded during further analysis.

Consistent normalized relative priority values of CTs, obtained as aggregated values in terms of clusters with group global priorities of clusters as weighted coefficients are shown in Table 5.

Table 4. Group global priorities of clusters of CTs

Clusters of CTs	Group global priorities
Energy conservation CTs	0.284
Renewable energy CTs	0.290
Eco-house	0.426

Table 5. Normalized consistent group global relative priority values of CTs

No	CTs	Relative priority values, *10
1	Technology of power efficient eco-house with renewable energy	4.260
2	Technology of improvement and structural optimization of energy networks in accordance with a purpose of harmonization with energy system of countries of the European Union	0.764
3	Technology of effective usage of soil and groundwater heat in complex thermal pump systems	0.571
4	Technology of diverse renewable energy sources usage in integrated thermal pump systems	0.568
5	Technology of steam compressor thermal pumps	0.557
6	Technology of usage of high-temperature conductivity in electrical machines and devices	0.545
7	Technology of magneto-liquid sealing for considerable increasing energy equipment's service life	0.499
8	Technology of production of generative power capacities on basis of integrated co-generation and thermal pump plants	0.443
9	Technology of energy loss saving in transit power networks	0.437
10	Technology of production of thermostable and corrosion-proof heat-insulating materials for thermal networks	0.420
11	Technology of synthetic fuel (gas) production	0.415
12	Technology of heating and housing and domestic hot-water supply on basis of usage of solar energy	0.380
13	Technology of production of engine oil and methanol on basis of Ukrainian deposit(s) of brown coal, peat, shales, coal and other carbon raw materials	0.377
14	Technology of usage of modular systems in low wind power engineering	0.299

Thus, first rank, i.e. the highest priority has cluster “Eco-house” and technology “Technology of power efficient eco-house with renewable energy”. Second rank has technology “Technology of improvement and structural optimization of energy networks in accordance with a purpose of harmonization with energy system of countries of the European Union”. Three thermal pump technologies “Technology of effective usage of soil and groundwater heat in complex thermal pump systems”, “Technology of diverse renewable energy sources usage in integrated thermal pump systems” and “Technology of steam compressor thermal pumps” receive third rank, differences between their priority values are rather small. Other technologies have lower priorities (Table 5).

Conclusion

In the present paper we provide a research of a new measure of consistency of interval pairwise comparison matrix, i.e. interval spectral coefficient of consistency. This coefficient is theoretical, but not empiric attribute of consistency of pairwise comparison matrix in the sense that on determination of consistency we do not use randomly filled pairwise comparison matrices. For determination of admissible level of inconsistency of IPCM application and detection thresholds are developed.

A computer simulation study was performed to estimate a sensitivity of interval spectral coefficient of consistency to fuzzy fundamental scales, which are the most often used in fuzzy AHP methods. The results reveal that in general the ISCC of interval pairwise comparison matrices is not sensitive to the considered fuzzy scales. This means that this coefficient of consistency leads to the same conclusion about inconsistency of interval pairwise comparison matrix when using different fuzzy scales to represent linguistic terms.

Spectral coefficient of consistency of FPCM is proposed as combination of ISCC in all α -levels using linear, multiplicative and min combination rules of the AHP. For determination of admissible level of inconsistency of FPCM strong and weak criteria are proposed.

The AHP method and spectral coefficient of consistency were used to evaluate critical technologies of energy conservation and power efficiency in Ukraine.

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