ORDERING MULTIATTRIBUTE ESTIMATING METHODS
FROM THE UTILITY POINT OF VIEW*

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Abstract: In the multi-attribute utility theory the utility functions are usually constructed by dots. It concerns both the lottery's method and the value increasing method. In the both cases the utility function is constructed in the absolute scale [0, 1] that causes inconveniences for experts. The comparative assessments look more preferable for decision-makers. The paired comparison matrix (PCM) looks as a natural model representing the preference structure of decision-maker (DM).

The PCM provides the extended possibilities to DMs to form comparative assessments both the qualitative ones (as better-worse) and the quantitative ones reflecting winnings and losses of DMs. In the paper we consider methods for utility function construction having different forms of its presentation. Among them there are utility functions based on attributes measured in nominal scales.

Keywords: utility function, paired comparison matrix, scale points, priority function.

ACM Classification Keywords: H.4.2 [Information Systems Applications]: Types of Systems decision support

Introduction

In English-language papers as the first stage finding the best option on a finite set of alternatives, as a rule, the dominant analysis uses for the Pareto set determination. As theoretically proved that the best option belongs to the Pareto set, the other options are excluded from consideration. Multiobjective optimization methods are used for finding the best option among the remaining options. In order to obtain a linear order on the set of alternatives vector estimates are converted to scalars using aggregate objective functions. The same functions are applied for the best variant finding by multiattribute utility theory methods. Despite the same approach to aggregation of the considered methods of optimization have no common paradigm.

Meanwhile, the decision is not limited to finding the best option. It is often necessary to look for the "bottleneck link" as the opposite of the best option, the removal of which allows us to solve the problem. The weak link may be lagging in the technology section of the production chain, department, impairing the results of the whole organization, the employee, in good faith to perform his duties, etc. In some cases, the need is to identify the properties of secondary options. For example, as the target may be made to find the average ("typical") unit or employee.

The presence of many different choice tasks and methods for solving them difficult to understand the user as to which approach is most suitable for him. To solve the problem systematization of multiattribute estimation methods is need. To systematize those methods is an objective of the present work.

Statement of the problem

To systematize multiattribute estimation methods it is necessary to find the common application. As a task suitable for the use of different multiattribute estimation methods assume the task of objects ordering (ranking) [Mikoni, 2009]. Ordering of objects applicable to search any object – the best, worst, or average. Since all methods of choice are based on the preferences, the result of their application may be to find an order relation on the set of objects. Consequently, the task of ordering allows you to map any of the methods of choice.

As a common property for objects estimation assumes its usefulness. The concept of utility was introduced by von Neumann and Morgenstern [Neumann, Morgenstern, 1953]. Multiattribute utility or value functions are elicited and used to identify the most preferred alternative or to rank order the alternatives. Elaborate interview techniques, which exist for eliciting linear additive utility functions and multiplicative nonlinear utility functions, are used [Keeney and Raiffa, 1976]. At present, the English-language literature, no distinction between the utility and value functions. Indeed, it is hard to imagine that the concept of value and utility in conflict. The value and usefulness functions are measured in the same scale [0, 1]. The difference in the names of functions was due to the author ideas to offer them. Value function based on the assumption for the disparity of attribute scale points, and for the utility functions proposed a method of lotteries (preferences associated with excitement and risk.) Naturally, the value of the divisions obtained by the lottery function is also different.

The concept of attributes usefulness is applied to defined object. In utility theory multiattribute utility function value corresponds to the object. Object *A* is more useful then object *B*, if the multiattribute utility function value of object *A* more than object *B*. This allows you to arrange all objects by multiattribute utility functions. Legitimate and the inverse problem – determining the usefulness of objects based on the preference relation.

Since the value of the utility is used for ordering objects in a single method for comparing the methods with each other will use the complexity of the utility functions. The utility function is more complex if it reflects more diverse preferences on the scale of attribute. According that non-linear function is more complex then linear one and continuous function is more complex then partially-linear one. Thus, the comparison task of the choice methods is reduce to comparison of utility function forms used by the methods.

Utility measurement

To measure the utility, as the normalized values, we use a scale of [–1, +1]. Positive segment of the scale [0, 1] is associated measure of property or the object utility (up to 100% of expected). Negative scale interval [–1, 0] matches the measure of damage (up to 100% of expected). In an economic sense, damage means loss of the subject. In a broader sense, damage – is all that threatens the existence of the subject. Value $u_j(x) = 0$ corresponds to the uselessness of the *j*-th property in terms of expected utility (useful equal 0%).

Positive and negative segments of the scale have all the properties of absolute scale. Above those measured in numbers it allows any arithmetic. In particular cases, the utility is measured in less informative scales. Consider the example of the positive segment of the scale.

Most primitive binary scale is {0, 1}. It is measured either utility ($u(x) = 1$), or the uselessness ($u(x) = 0$) of the object *x*. In the binary scale all the intermediate values of the scale [0, 1] are absent. In other words, the only boundary values are used in the scale.

Usefulness of $u(x)$ of the object *x*, measured in ordinal scale, can be calculated through the assigned normalized rank $\rho(x)$, as follows:

$$u(x)=(\rho_{max}-\rho(x))/(\rho_{max}-1) \tag{1}$$

where ρ_{max} – maximum (worst case) the rank of the object.

Distances between intermediate values of the ordinal scale are the same ones. Its magnitude depend on the maximum rank ρ_{max} and the number of intermediate scale points.

Aggregation measured in different scales, preceded by their normalized relative to the boundaries of the scale. In terms of utility, the normalizing function of the growing rate is interpreted utility u_{max} (y_j):

$$u_{max}(y_j) = \frac{y_j - y_{j,min}}{y_{j,max} - y_{j,min}}, \quad j = \overline{1,n}, \tag{2}$$

and the normalizing function of diminishing utility index is interpreted u_{min} (y_j):

$$u_{min}(y_j) = \frac{y_{j,max} - y_j}{y_{j,max} - y_{j,min}}, \quad j = \overline{1,n}. \tag{3}$$

Both functions u_{max} (y_j) and u_{min} (y_j) are continuous and linear.

The most common utility functions, reflecting the decrease and increase in the scale of preferences that obtained the erection of (2) and (3) to the power k. Negative power of k characterizes no inclination of decision maker to risk, and the positive degree, opposite, – inclination to risk. Depending on the value of k no inclination to risk describes a family of curves, convex upward (Fig. 1a), and inclination to risk – a family of curves, convex downwards (Fig. 1b).
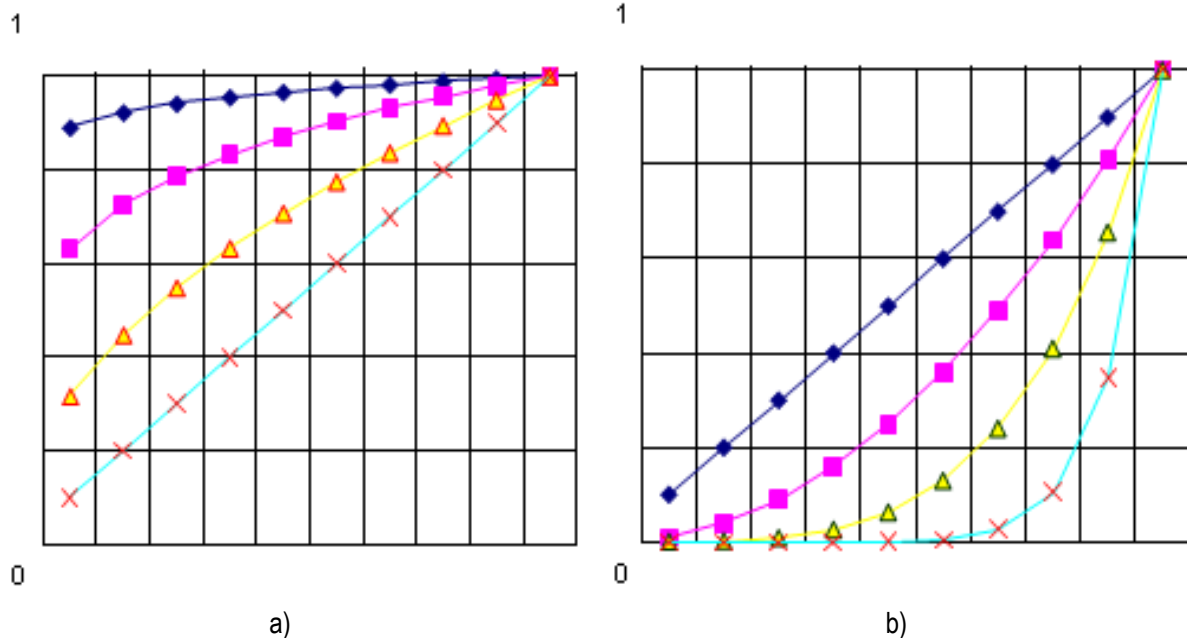

Figure 1.

More complex useful function combines inclination to risk on the initial section of an attribute scale and no inclination to risk after a certain threshold c_j. In Fig. 2 the logistic function is presented, which combined the both properties.
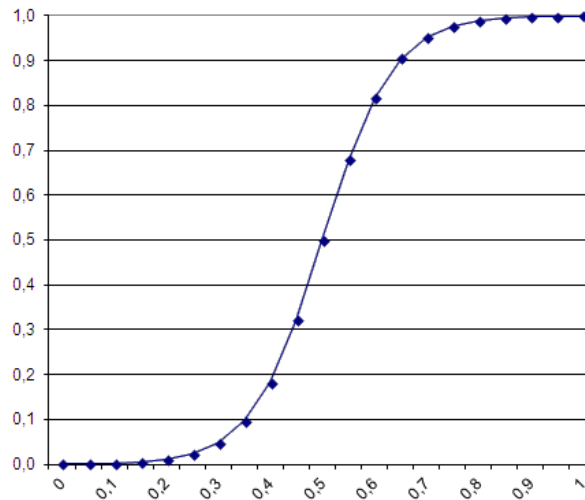

Figure 2.

Assessment of the utility of objects by different methods of decision making

In decision theory the following groups of multiattribute estimation methods were formed:

- o Dominant analysis;
- o Verbal decision analysis (Prioritization by Similarity to Ideal Solution);
- o Multiobjective optimization (Weighted sum or product model);
- o Multiobjective optimization with constraints;
- o Goal programming (minimization of weighted deviations from the target values of attributes);
- o Multiattribute utility theory;
- o Analytic hierarchy process;
- o Selection and Classification.

Selection of non-dominated alternatives gives the simplest estimation of usefulness. Alternatives included in the Pareto set can be regarded as useful, but not included in them – as useless. Here, an interim measure of utility between zero and one is missing. Selection methods from the utility view point are hard ones. Meanwhile, under some conditions an object no belonging to Pareto set may be more preferable according its aggregate objective function then an object belonging to Pareto set.

If we consider all objects Pareto-ordered set, then the measure of their usefulness can be calculated by the formula (1), based on the level they occupy in the ranked graph domination. Thus, the formula (1) is a discrete linear utility function, defined on the levels ranged graph. Due to the consideration of all objects without exception methods of ordering, in contrast to the methods of selection, should be attributed to the soft ones. Methods of dominant and verbal analysis measure alternatives in ordinal scale.

The ability to measure the utility of alternatives in an interval scale provides multiobjective optimization methods. Since the aggregation of heterogeneous attributes requires normalization of the values used formulas (2) and (3) are used for the purpose. They convert the values from the interval scale in the absolute scale [0, 1]. This allows us to calculate the attribute utility at any point of its scale, and the dependence of the utility value of the objective criterion is expressed in a linear continuous function.

Due to the target c_j on the attribute scale the normalization of the constraint criterion produces a piece-wise linear utility function $u(c_j)$, which is calculated on the intervals of the scale $[y_{j,min}, c_j)$ and $(c_j, y_{j,max}]$. A linear dependence is established on these scale parts according criterion and Decision-maker can set the utility at the point c_j. Thus, the utility function corresponded to a constraint criterion is more informative than function corresponded to an objective criterion. This applies to the scalarization of the absolute values of attributes and their deviations from the fixed target c_j, $j = \overline{1, n}$.

In fact, multiobjective optimization methods no apply utility functions but interpretation normalized functions by utility functions permit us to set relationship between multiple-criteria decision analysis and multiattribute utility theory. If multiobjective optimization with constraints no excludes any objects it can suppose that utility functions for all attributes are generated automatically. Decision-maker no participates in its creation. One can refer such method to interim between multiobjective optimization and multiattribute utility theory methods. That method can be named soft one with respect to multiobjective optimization with constraints method.

According to multiattribute utility theory decision-maker participates in utility functions creation by assessing his preferences upon the attribute scale. Thus non-linear utility functions are created. They contain more information about decision-maker preferences then linear and piece-wise linear utility functions. Consequently, non-linear utility functions are more complex regarding preferences. If one evaluates the function complexity by preferences number on the attribute scale, then the most complex functions are non-linear continuous ones.

The greater complexity of nonlinear functions and defines highly informative models created in the utility theory.

In [Mikoni, Gharina, 2012], the utility function of the attribute can be created not only decision-makers but also to calculate the membership functions of fuzzy classes. This property links fuzzy classification with the multiattribute utility theory. Of course, the creation of a utility function based on the membership functions of a class of more labor intensive than creating it directly. But it solves two problems: classification and ordering of objects.

Priorities objects, calculated based on the pair comparisons matrix, it is easy to express in terms of utility by their normalization relative to the maximum value:

$$u(x) = u(x_i) = w_i / w_{i,max} \qquad (4)$$

Thus, discrete utility function, defined on the set of objects, is formed from its priorities calculated on the base the pair comparisons matrix [Mikoni, 2012].

Systematization of methods for multiattribute estimation evaluating by the usefulness

Based on the analysis of information on the utility, made in the previous section, we order estimation methods multiattribute alternatives. Ordering methods towards more complex utility functions is given in Table 1.

Table 1

NN п/п	Group of methods	Method	The utility function (UF)
1	Selection	with constraint	binary
		non dominated alternatives	
2	Ordering an ordinal scale	dominant analysis	linear discrete

		verbal analysis	
3	Multiobjective optimization	objective functions only	linear continuous
4	Multiobjective optimization with constraints	objective functions and constraints in common	linear continuous for selecting objects
5	Multiobjective optimization with constraints without screening	attribute values	piece-wise linear
		deviations from the target	
6	Multiattribute utility optimization	creation UF by points	nonlinear discrete
		by paired comparison matrix	
		by membership functions	nonlinear continuous
		typical UF	

Conclusion

Ordering methods multiattribute evaluation of alternatives for the utility functions to evaluate the properties of these methods. The more a function indicators show the real benefits of their values, the more accurate assessment of alternatives. It should be remembered that the accuracy of the estimates and the influence of factors such as the importance of correct assignment and selection of indicators summarizing functions. But it refers to the quality of any examination and is the subject of a separate study.

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