DIRECT AND DUAL PROBLEM OF INVESTMENT PORTFOLIO OPTIMIZATION UNDER UNCERTAINTY

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***Abstract**: The problem of forming an optimal portfolio of securities under uncertainty was considered. The main objective of portfolio investment is to improve the investment environment, giving securities such investment characteristics that are only possible in their combination. Careful processing and accounting of investment risks have become an integral and important part of the success of each company. However, the global market crisis of recent years has shown that the existing theory of optimization of investment portfolios and forecasting stock indices themselves exhausted and needed overhaul of the basic theory of portfolio management. Therefore the fuzzy sets theory was used for getting an optimal portfolio.*

The direct problem with the use of triangular, bell-shaped and Gaussian membership functions and dual problem by using the fuzzy sets theory were considered in this work. In direct task we define structure of a portfolio which will provide the maximum profitableness at the set risk level. In dual task we define structure of a portfolio which will provide the minimum risk level at the set level of critical profitableness. The input data for the optimization system were predicted by using the Fuzzy Group Method of Data Handling (FGMDH). The optimal portfolios for asset were determined. The comparative analysis of optimal portfolio obtained by using of different membership functions was fulfilled. Investment portfolio optimization system is an effective tool for the operational management of portfolio investments. This is an opportunity to carry out scientific and reasonable management of their investment portfolio with the ability to reject the planned loss of possession or overvalued risky assets, which increases business efficiency and maximize gain on the stock market.

***Keywords**: membership function, fuzzy sets theory, optimal portfolio, investments, stock securities, fuzzy number, FGMDH.*

***ACM Classification Keywords**: G.1.0 Mathematics of Computing– General – Error analysis; G.1.6 Mathematics of Computing – Numerical Analysis – Optimization - Gradient methods, Least squares methods; I.2.3 Computing Methodologies - Artificial Intelligence - Uncertainty, “fuzzy”, and probabilistic reasoning.*

Introduction

The investment process is the adoption of a decision by the investor in securities, in which investments are made, and the amounts and timing of investment.

The main objective of portfolio investment is to improve the investment environment, giving securities such investment characteristics that are only possible in their combination.

Careful processing and accounting of investment risks have become an integral part and important part of the success of each company. However, more and more companies have to make decisions under uncertainty, which may lead to unintended consequences and, therefore, undesirable results statement. Particularly serious consequences may have the wrong decisions on long-term investments. Therefore, early detection and adequate and the most accurate assessment of risk is one of the biggest problems of modern investment analysis.

Historically, the first and the most common way to take account of uncertainties is the use of probability. The beginning of modern investment theory was in article H. Markowitz, "Portfolio Selection", which was released in 1952. Mathematical model of optimal portfolio of securities was first proposed. Methods of constructing such portfolios under certain conditions based on theoretical and probabilistic formalization of the concept profitability and risk. Thus, the theory is the classical theory of Markowitz portfolio construction of stocks, after which most of the remaining theories are only modifications of the base.

However, the global market crisis of recent years has shown that the existing theory of optimization of investment portfolios and forecasting stock indices themselves exhausted and needed overhaul of the basic theory of portfolio management.

Important investment fuzzy sets theory was created about half a century ago in the fundamental work of Lotfi Zadeh [Zadeh, 1999]. This theory was coming into use in the economy in the late 70's. By using fuzzy numbers in the forecast parameters decision making person not required to form a point probability estimates.

Investment portfolio optimization system - is an effective tool for the operational management of portfolio investments. This is an opportunity to carry out scientific and reasonable management of their investment portfolio with the ability to reject the planned loss of possession or overvalued risky assets, which increases business efficiency and maximize gain on the stock market. And thanks to the results obtained using FGMDH system devoid of subjective risk experts. For now calculate input values yield component of the portfolio is automated, and therefore denied the need to involve experts.

Problem statement of portfolio optimization

The purpose of the analysis and optimization of an investment portfolio is research in area of portfolio optimization, and also the comparative analysis of structure of the effective portfolios received at use of model Markovitz and fuzzy-set model of a share portfolio optimization.

Let us consider a share portfolio from N components and its expected behavior at time interval $[0, T]$. Each of a portfolio component $i = \overline{1, \mathrm{N}}$ at the moment T is characterized by its financial profitableness $\mathrm{r_i}$ (evaluated at a point T as a relative increase in the price of the asset for the period) [Zaychenko, 2008].

The holder of a share portfolio – the private investor, the investment company, mutual fund – operates the investments, being guided by certain reasons. On the one hand, the investor tries to maximize the profitableness. On the other hand, it fixes maximum permissible risk of an inefficiency of the investments. We will assume the capital of the investor be equal 1. The problem of optimization of a share portfolio consists in a finding of a vector of share price distribution of papers in a portfolio $x = \{\mathrm{x_i}\}, i = \overline{1, \mathrm{N}}$ of the investor maximizing the income at the set risk level (obviously, that $\sum_{i=1}^{N} x_i = 1$).

In process of practical application of model Markovitz its lacks were found out:

1. The hypothesis about normality profitableness distributions in practice does not prove to be true.
2. Stationary of price processes also not always is in practice.
3. At last, the risk of actives is considered as a dispersion standard deviation of the prices of securities from expected value i.e. as decrease in profitableness of securities in relation to expected value, and profitableness increase in relation to an average are estimated absolutely the same.

Though for the proprietor of securities these events are absolutely not the same.

These weaknesses of Markovitz theory define necessity of use of essentially new approach of definition of an optimum investment portfolio.

Let review the main principles and idea of a method.

The risk of a portfolio is not its volatility, but possibility that expected profitableness of a portfolio will appear below some pre established planned value.

- Correlation of assets in a portfolio is not considered and not accounted;
- Profitableness of each asset is not random fuzzy number. Similarly, restriction on extremely low level of profitableness can be both usual scalar and fuzzy number of any kind. Therefore optimize a portfolio in such statement may mean, in that specific case, the requirement to maximize expected profitableness of a portfolio in a point of time T at the fixed risk level of a portfolio;
- Profitableness of a security on termination of ownership term is expected to be equal r and is in a settlement range. For i-th security:

  $\bar{r}_i$ – Expected profitableness of i-th security;

  r_{1i} – The lower border of profitableness of i-th security;

  r_{2i} – The upper border of profitableness of i-th security.

  $r_i = \left(r_{1i}, \bar{r}_i, r_{2i}\right)$ – Profitableness of i-th security is triangular fuzzy number.

Then profitableness of a portfolio:

$$r = (r_{\min} = \sum_{i=1}^{N} x_i r_{1i}; \bar{r} = \sum_{i=1}^{N} x_i \bar{r}_i; r_{\max} = \sum_{i=1}^{N} x_i r_{2i}) \tag{1}$$

where x_i - weight of i-th asset in portfolio, and

$$\sum_{i=1}^{N} x_i = 1, \quad 0 \le x_i \le 1 \tag{2}$$

Critical level of profitableness of a portfolio at the moment of T may be fuzzy triangular type number $r^* = \left(r_1^*; \bar{r}^*; r_2^*\right)$.

Fuzzy-set approach with triangular membership functions

To define structure of a portfolio which will provide the maximum profitableness at the set risk level, it is required to solve the following problem (3):

$$\{x_{opt}\} = \{x\} \mid r \to \max, \ \beta = const \tag{3}$$

where r is profitableness, β is a desired risk, vector's components x satisfy (2).

The most expected value risk degree of a portfolio is defined:

$$\beta = \begin{cases} 0, \ if \ r^* < r_{\min} \\ R\left(1 + \dfrac{1-\alpha_1}{\alpha_1}\ln(1-a_1)\right), \ if \ r_{\min} \le r^* \le \tilde{r} \\ 1-(1-R)\left(1 + \dfrac{1-\alpha_1}{\alpha_1}\ln(1-a_1)\right), \ if \ \tilde{r} \le r^* < r_{\max} \\ 1, \ if \ r^* \ge r_{\max} \end{cases} \tag{4}$$

where

$$R = \begin{cases} \dfrac{r^* - r_{\min}}{r_{\max} - r_{\min}}, & if \quad r^* < r_{\max} \\ 1, \quad if \quad r^* \geq \mathrm{r}_{\max} \end{cases}$$

$$\alpha_1 = \begin{cases} 0, & if \quad r^* < r_{\min} \\ \dfrac{r^* - r_{\min}}{\tilde{r} - r_{\min}}, & if \quad r_{\min} \leq r^* < \tilde{r} \\ 1, & if \quad r^* = \tilde{r} \\ \dfrac{r_{\max} - r^*}{r_{\max} - \tilde{r}}, & if \quad \tilde{r} < r^* < r_{\max} \\ 0, & if \quad r^* \geq r_{\max} \end{cases} \tag{5}$$

Having recollected also, that profitableness of a portfolio is:

$$r = (r_{\min} = \sum_{i=1}^{N} x_i r_{1i};\ \tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i;\ r_{\min} = \sum x_i r_{2i}),$$

where $(r_{1i}, \overline{r_i}, r_{2i})$ – profitableness of i-th security, we receive the following problem of optimization (6) - (8):

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \rightarrow \max \tag{6}$$

$$\beta = const \tag{7}$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1, N} \tag{8}$$

At a risk level variation β 3 cases are possible. We will consider in detail each of them.

1. $\beta = 0$

From (4) it is evident, that this case is possible when $r^* < \sum_{i=1}^{N} x_i r_{i1}$.

We receive the following problem of linear programming:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \rightarrow \max \tag{9}$$

$$\sum_{i=1}^{N} x_i r_{i1} > r^* \tag{10}$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1, N} \tag{11}$$

Found result of the problem decision (9)-(11) vector $x = \{x_i\},\ i = \overline{1, N}$ is a required structure of an optimum portfolio for the given risk level.

2. $\beta = 1$

From (4) follows, that this case is possible when $r^* \geq \sum_{i=1}^{N} x_i r_{i2}$.

We receive the following problem

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max, \ \sum_{i=1}^{N} x_i r_{i2} \le r^*, \ \sum_{i=1}^{N} x_i = 1, \ x_i \ge 0, \ i = \overline{1,N}.$$

Found result of the problem decision (9)-(11) vector $x = \{x_i\}$, $i = \overline{1,N}$ is a required structure of an optimum portfolio for the given risk level.

3. $0 < \beta < 1$

From (4) it is evident, that this case is possible when $\sum_{i=1}^{N} x_i r_{i1} \le r^* \le \sum_{i=1}^{N} x_i \tilde{r}_i$, or when $\sum_{i=1}^{N} x_i \tilde{r}_i \le r^* \le \sum_{i=1}^{N} x_i r_{i2}$.

a) Let $\sum_{i=1}^{N} x_i r_{i1} \le r^* \le \sum_{i=1}^{N} x_i \tilde{r}_i$. Then using (4) - (5) problem (6) - (8) is reduced to the following problem of nonlinear programming:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max,$$

$$\left(\left(r^* - \sum_{i=1}^{N} x_i r_{i1} \right) + \left(\sum_{i=1}^{N} x_i \tilde{r}_i - r^* \right) \cdot \ln \left(\frac{\sum_{i=1}^{N} x_i \tilde{r}_i - r^*}{\sum_{i=1}^{N} x_i \tilde{r}_i - \sum_{i=1}^{N} x_i r_{i1}} \right) \right) \cdot \tag{12}$$

$$\cdot \frac{1}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} = \beta, \tag{13}$$

$$\sum_{i=1}^{N} x_i r_{i1} \le r^* \tag{14}$$

$$\sum_{i=1}^{N} x_i \tilde{r}_i > r^* \tag{15}$$

$$\sum_{i=1}^{N} x_i = 1, \ x_i \ge 0, \ i = \overline{1,N}. \tag{16}$$

б) Let $\sum_{i=1}^{N} x_i \tilde{r}_i \le r^* \le \sum_{i=1}^{N} x_i r_{i2}$. Then the problem (6) - (8) is reduced to the following problem of nonlinear programming:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max \tag{17}$$

$$\left(\left(r^* - \sum_{i=1}^{N} x_i r_{i1} \right) - \left(r^* - \sum_{i=1}^{N} x_i \tilde{r}_i \right) \cdot \ln \left(\frac{r^* - \sum_{i=1}^{N} x_i \tilde{r}_i}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} \right) \right) \cdot$$

$$\cdot \frac{1}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} = \beta \tag{18}$$

$$\sum_{i=1}^{N} x_i r_{i2} > r^* \tag{19}$$

$$\sum_{i=1}^{N} x_i \widetilde{r}_i \le r^* \tag{20}$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \ge 0,\ i = \overline{1,N} \tag{21}$$

The R-algorithm of minimization of not differentiated functions is applied to the decision of problems (12) - (16) and (17) - (21). Let both problems: (12) - (16) and (17) - (21) solvable. Then to the structure of a required optimum portfolio will correspond a vector – $x = \{x_i\}\ i = \overline{1,N}$ the decision of that problem (12) - (16), (17) - (21) the criterion function value of which will be greater.

Fuzzy-set approach with bell-shaped membership functions

In case of using bell-shaped MF we should solve the problem of portfolio optimization where parameter $r_i = \left(r_{1i}, \bar{r}_i, r_{2i}\right)$, the profitability of the *i*-th asset, is fuzzy number with bell-shaped form $\mu(x) = \dfrac{1}{1+\left(\dfrac{x-a}{c}\right)^2}$, as shown in Figure 1:

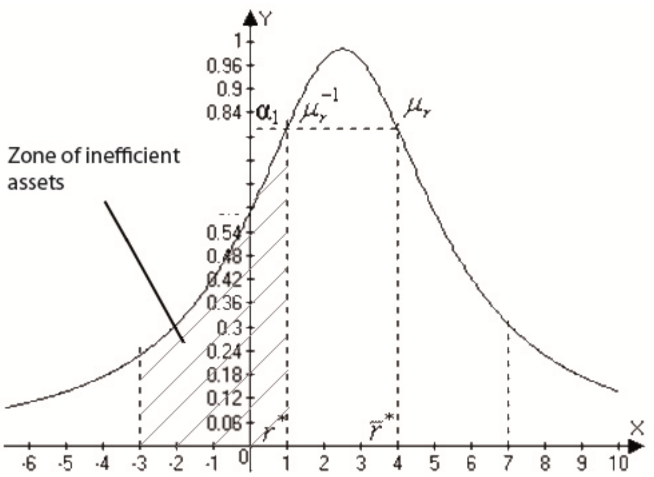

Figure 1. Clear efficiency criterion for the bell-shaped MF

In this case:

$$\alpha_1 = \begin{cases} \dfrac{1}{1+\left(\dfrac{r^*-\widetilde{r}}{r_{\max}-r_{\min}}\right)^2}, \text{ïðè}\quad r_{\min} < r^* < r_{\max} \\ 0,\ \text{ïðè}\quad r_{\min} > r^* \text{ ³ } r^* > r_{\max} \\ 1,\ \text{ïðè}\quad r^* = \widetilde{r} \end{cases} \tag{22}$$

$$\beta = \begin{cases} 0, \ \ddot{\imath}\check{\partial}\grave{e} \quad r^* < r_{\min} \\ \frac{1}{2}\alpha_1 + \frac{1}{2}\frac{r^* - \bar{r}}{r_{\max} - r_{\min}} * L, \ \ddot{\imath}\check{\partial}\grave{e} \quad r_{\min} < r^* < \bar{r} \\ 1 - \left(\frac{1}{2}\alpha_1 + \frac{1}{2}\frac{r^* - \bar{r}}{r_{\max} - r_{\min}} * L\right), \ \ddot{\imath}\check{\partial}\grave{e} \quad \bar{r} < r^* < r_{\max} \\ 1, \ \ddot{\imath}\check{\partial}\grave{e} \quad r^* > r_{\max} \end{cases} \tag{23}$$

Where α1 we can find from (22), L is $L = \arcsin\left(\sqrt{a_1}\right) - \sqrt{a_1 - a_1^2}$

In order to determine the structure of the portfolio, which will provide maximum return with the given level of risk, we need to solve the problem (3) where r and β are determined from formulas (22), (23).

$$\beta = \begin{cases} 0, \ddot{\imath}\check{\partial}\grave{e} \ \ r^* \le r_{\min} \\ \frac{1}{2}(\alpha_1 - \alpha_0) + \frac{1}{2}\frac{(r^* - \tilde{r})}{(r_{\max} - r_{\min})} * L + \frac{(r^* - r_{\min})}{(r_{\max} - r_{\min})} * \alpha_0, \ddot{\imath}\check{\partial}\grave{e} \ \ r_{\min} \le r^* \le \tilde{r} \\ 1 - \left(\frac{1}{2}(\alpha_1 - \alpha_0) + \frac{1}{2}\frac{(r^* - \tilde{r})}{(r_{\max} - r_{\min})} * L + \frac{\left(r_{\max} - r^*\right)}{\left(r_{\max} - r_{\min}\right)} * \alpha_0\right), \ddot{\imath}\check{\partial}\grave{e} \ \ \tilde{r} < r^* \le r_{\max} \\ 1, \ddot{\imath}\check{\partial}\grave{e} \ \ r^* > r_{\max} \end{cases} \tag{24}$$

where

$$\alpha_0 = \begin{cases} \dfrac{1}{1 + \left(\dfrac{\tilde{r} - r_{\min}}{r_{\max} - r_{\min}}\right)^2}, \ddot{\imath}\check{\partial}\grave{e} \ \ r_1 \le r^* \le \tilde{r} \\ \dfrac{1}{1 + \left(\dfrac{r_{\max} - \tilde{r}}{r_{\max} - r_{\min}}\right)^2}, \ddot{\imath}\check{\partial}\grave{e} \ \ \tilde{r} < r^* \le r_2 \end{cases} \tag{25}$$

$$L = \arcsin\left(\sqrt{\alpha_1}\right) - \arcsin\left(\sqrt{\alpha_0}\right) - \sin\left(\arcsin\left(\sqrt{\alpha_1}\right) - \arcsin\left(\sqrt{\alpha_0}\right)\right) * \cos\left(\arcsin\left(\sqrt{\alpha_1}\right) + \arcsin\left(\sqrt{\alpha_0}\right)\right) \tag{26}$$

Optimization problem consists of tasks (3), (23) - (26).

Fuzzy-set approach with Gaussian membership functions

In case of using Gaussian MF we should solve the problem of portfolio optimization where parameter $r_i = \left(r_{1i}, \bar{r}_i, r_{2i}\right)$, the profitability of the *i*-th asset, is fuzzy number with Gaussian form $\mu(x) = e^{-\frac{1}{2}\frac{(x-a)^2}{c^2}}$, as shown in Figure 2:

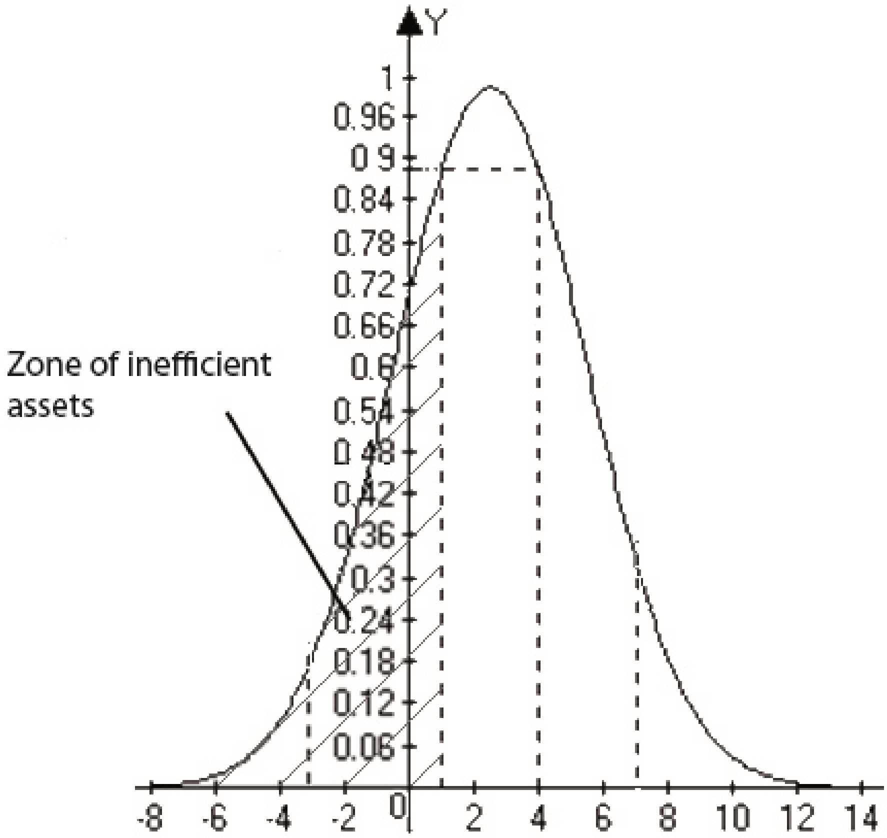

Figure 2. Clear efficiency criterion for the Gaussian MF

In this case:

$$\alpha_1 = \begin{cases} e^{-\frac{1}{2}\frac{\left(r^* - \tilde{r}\right)^2}{\left(r_{\max} - r_{\min}\right)^2}} \\ 0,\ \text{ïðè}\quad r^* < r_{\min}\ {}^{\text{³}}\ r^* > r_{\max} \\ 1,\ \text{ïðè}\quad r^* = \tilde{r} \end{cases} \tag{27}$$

$$\beta = \begin{cases} 0, \text{ïðè}\quad r^* \le r_{\min} \\ \frac{1}{2}(\alpha_1 - \alpha_0) + \frac{\sqrt{\pi}}{2\sqrt{2}} R\left(\hat{O}(R) + \hat{O}\left(\sqrt{\ln\frac{1}{\alpha_0}}\right)\right), \text{ïðè}\quad r_{\min} \le r^* \le \tilde{r} \\ 1 - \left(\frac{1}{2}(\alpha_1 - \alpha_0) - \frac{\sqrt{\pi}}{2\sqrt{2}} R\left(\hat{O}(R) - \hat{O}\left(\sqrt{\ln\frac{1}{\alpha_0}}\right)\right)\right), \text{ïðè}\quad \tilde{r} < r^* \le r_{\max} \\ 1, \text{ïðè}\quad r^* > r_{\max} \end{cases} \tag{28}$$

where α_0 is very small, α1 we can find from (27), R is:

$$R = \frac{\left(r^* - \widetilde{r}\right)}{\left(r_{\max} - r_{\min}\right)} \quad (29)$$

Φ(x) is the Laplace function at x: $\hat{O}(\mathrm{x}) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

In order to determine the structure of the portfolio, which will provide maximum return with the given level of risk, we need to solve the problem (3) where r and β are determined from formulas (27)—(29).

$$\beta = \begin{cases} 0, \text{ïðè} \;\; r^* < r_{\min} \\ \frac{1}{2}(\alpha_1 - \alpha_0) + \frac{\sqrt{\pi}}{2\sqrt{2}} R\left(\hat{O}(\mathrm{R}) + \hat{O}\left(\sqrt{\ln\frac{1}{\alpha_0}}\right)\right) + \frac{\left(r^* - r_{\min}\right)}{\left(r_{\max} - r_{\min}\right)} * \alpha_0, \; \text{ïðè} \;\; r_{\min} \le r^* \le \widetilde{r} \\ 1 - \left(\frac{1}{2}(\alpha_1 - \alpha_0) - \frac{\sqrt{\pi}}{2\sqrt{2}} R\left(\hat{O}(\mathrm{R}) - \hat{O}\left(\sqrt{\ln\frac{1}{\alpha_0}}\right)\right) + \frac{\left(r_{\max} - r^*\right)}{\left(r_{\max} - r_{\min}\right)} * \alpha_0\right), \text{ïðè} \;\; \widetilde{r} < r^* \le r_{\max} \\ 1, \; \text{ïðè} \;\; r^* > r_{\max} \end{cases} \quad (30)$$

where
$$\alpha_0 = \begin{cases} e^{-\frac{1}{2}\frac{(\widetilde{r} - r_{\min})^2}{(r_{\max} - r_{\min})^2}}, \; \text{ïðè} \;\; r_{\min} \le r^* \le \widetilde{r} \\ e^{-\frac{1}{2}\frac{(r_{\max} - \widetilde{r})^2}{(r_{\max} - r_{\min})^2}}, \; \text{ïðè} \;\; \widetilde{r} < r^* \le r_{\max} \end{cases} \quad (31)$$

Optimization problem consists of tasks (3), (27), (29) — (31).

The dual optimization problem

It is necessary to determine the structure of the portfolio, which will provide a minimum level of risk for a given level of profitability of the portfolio.

Obtain the following optimization problem:

$$\min \beta(x), \quad \tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \ge r_{зад} = \mathrm{r}^*, \; \sum_{i=1}^{N} x_i = 1, \; 0 \le x_i \le 1, \; i = \overline{1, N}, \quad (32)$$

where r and β is determined by the used membership function.

Consider building of optimization problem with using the triangular MF.

It is necessary to solve optimization problem (32) where $\beta(x)$ is determined from the formula (4), (5).

Description of the algorithm of FGMDH

Let’s give a brief description of the algorithm.

1. Selection of the overall model view, which will describe the required dependence.
2. Selection of external optimality criteria and freedom of choice.
3. Selection of general form of the support function.
4. Set the counter to zero for the number of models k and the number of series r.
5. Generate a new partial model. Determine the values of the main criteria on it. Assign k = k +1.
6. If $k \geq C_F^2$, than k = 0, r = r + 1. Construct an average criterion of models $N(r)$. If r = 1, then go to step 5, otherwise - to step 7.
7. If $|N(r)-N(r)| \leq \varepsilon$, than go to step 8, otherwise we select the best F models according to the external criteria and go to step 5.
8. Choose the best model from F models by using the regularization criteria. Restoring the analytical form of the best model by using Gödel numbering.

Analysis of the results

The profitableness of leading companies in the period from 03.09.2013 to 17.01.2014 was used as the input data. The companies: Canon Inc. (CAJ), McDonald's Corporation (MCD), PepsiCo, Inc (PEP), The Procter & Gamble Company (PG) , SAP AG (SAP). The corresponding data is presented in the Table 1:

Table 1. The profitableness, %

Companies / Dates	CAJ	MCD	PEP	PG	SAP
06.09.13	0,510	-1,841	1,172	0,772	2,692
13.09.13	-0,880	-0,933	-1,184	-1,139	-0,945
20.09.13	0,990	0,829	-0,889	0,961	-1,509
27.09.13	1,110	0,164	1,012	2,611	-0,582
04.10.13	0,060	1,569	-0,151	-0,569	0,649
11.10.13	-0,410	-0,403	-2,239	-3,741	-3,190
18.10.13	-1,230	-0,507	-2,368	-0,851	-0,477
25.10.13	2,300	-0,201	-1,190	-1,304	-3,979
01.11.13	2,620	-1,961	0,059	0,185	2,494
08.11.13	0,190	0,308	-1,754	-1,451	-0,879
15.11.13	-3,600	0,175	-0,679	-3,136	-1,675
22.11.13	-0,180	-0,635	0,140	-0,449	-0,329
29.11.13	-1,830	1,567	1,066	1,393	-0,510

06.12.13	1,950	-0,300	0,657	-1,416	-0,636
13.12.13	1,650	1,337	2,128	2,843	1,566
20.12.13	-0,060	-1,111	-1,000	-0,184	-2,247
27.12.13	1,560	-0,633	-1,038	-0,861	-1,132
03.01.14	0,880	0,484	0,808	1,890	2,655
10.01.14	1,770	0,052	-1,483	0,422	1,042
17.01.14	**-1,270**	**-0,105**	**0,206**	**0,162**	**0,843**

For forecasting we used the Fuzzy GMDH method with triangular membership functions, linear partial descriptions, training sample of 70%, forecasting for 1 step. The next profitableness to date 17.01.2014 values was gotten (Table 2):

Table 2. The profitableness to date 17.01.2014, %

Companies	Profitableness				MAPE test sample	MSE test sample
	Real value	Low bound	Forecasted value	Upper bound		
CAJ	-1,270	-1,484	-1,246	-1,008	2,2068	0,0295
MCD	-0,105	-0,347	-0,118	0,111	2,5943	0,0091
PEP	0,206	0,001	0,242	0,483	3,0179	0,0177
PG	0,162	0,041	0,170	0,299	1,6251	0,0197
SAP	0,843	0,675	0,867	1,059	2,3065	0,0164

Thus, as the result of application of FGMDH the shares profitableness values were forecasted to the end of 20-th week (17.01.2014):

- Profitableness of CAJ shares lies in the calculated corridor [-1,484; -1,008], the expected value is 1,246%;
- Profitableness of MCD shares lies in the calculated corridor [-0,347; 0,111], the expected value is 0,1179%;
- Profitableness of PEP shares lies in the calculated corridor [0,001; 0,483], the expected value is 0,242%;
- Profitableness of PG shares lies in the calculated corridor [0,041; 0,299], the expected value is 0,17%;
- Profitableness of SAP shares lies in the calculated corridor [0,675; 1,059], the expected value is 0,867%.

In this way the portfolio optimization system stops to be dependent on factor of expert subjectivity. Besides, we can get data for this method automatically, without expert's estimates.

Let the critical profitableness level set by 0,7 %. Varying the risk level we obtain the following results at the end of 2-th week (17.01.2014) for triangular MF. The results are presented in the Tables 3, 4 and the Figure 4:

Table 3. Distribution of components of the optimal portfolio for triangular MF with critical level r *=0,7 %

CAJ	MCD	PEP	PG	SAP
0,05482	**0,00196**	**0,0027**	**0,00234**	**0,93818**
0,06145	0,00113	0,00606	0,0039	0,92746
0,0698	0,00577	0,00235	0,00219	0,91989
0,06871	0,00228	0,0057	0,00244	0,92087
0,07567	0,00569	0,00106	0,00094	0,91664
0,07553	0,00002	0,0029	0,00208	0,91947
0,06774	0,00121	0,006	0,00234	0,92271
0,0764	0,001	0,00612	0,00464	0,91184
0,09072	0,00849	0,00655	0,0039	0,89034

Table 4. Parameters of the optimal portfolio for triangular MF with critical level r *=0,7 %

Low bound	Expected profitableness	Upper bound	Risk level
0,55133	**0,74591**	0,94049	**0,2**
0,53462	0,72954	0,92446	0,25
0,51544	0,71084	0,90624	0,3
0,51894	0,71431	0,90968	0,35
0,5045	0,70018	0,89587	0,4
0,50877	0,70425	0,89973	0,45
0,522	0,71731	0,91262	0,5
0,50197	0,69752	0,89308	0,55
0,46358	0,66014	0,8567	0,6

As we can see on Figure 4 the dependence profitableness - risk has descending type, the greater risk the lesser is profitableness opposite from classical probabilistic methods. It may be explained so that at fuzzy approach by risk is meant the situation when the expected profitableness happens to be less than the given criteria level. When the expected profitableness decreases, the risk grows.

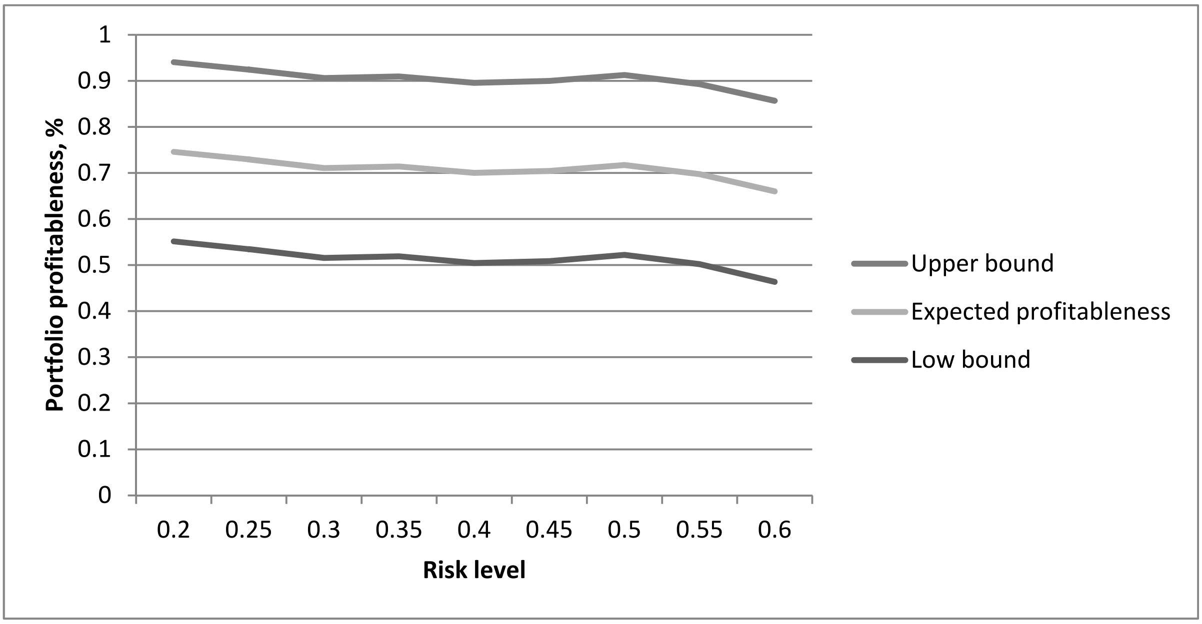

Figure 4. Dependence of expected portfolio profitableness on risk level for triangular MF

The profitableness of the real portfolio is 0,7056 %.This value falls in results calculated corridor of profitableness [0,5346, 0,7295, 0,9245], indicating the highs quality of the forecast.

Now consider the same portfolio using bell-shaped MF (Tables 5, 6, Figure 5).

Table 5. Distribution of components of the optimal portfolio for bell-shaped MF with critical level r *=0,7 %

CAJ	MCD	PEP	PG	SAP
0,00129	**0,0026**	**0,00279**	**0,00316**	**0,99016**
0,00264	0,00249	0,0028	0,0037	0,98837
0,00249	0,00268	0,00246	0,00238	0,98999
0,00209	0,00148	0,00039	0,00024	0,9958
0,00102	0,00105	0,0021	0,00222	0,99361
0,00425	0,00325	0,00343	0,0032	0,98587
0,00128	0,00125	0,00207	0,002	0,9934
0,00084	0,0022	0,0018	0,00163	0,99353
0,00165	0,00105	0,00198	0,00137	0,99395

Table 6. Parameters of the optimal portfolio for bell-shaped MF with critical level r *=0,7 %

Low bound	Expected profitableness	Upper bound	Risk level
0,66568	**0,85777**	1,04986	**0,2**
0,66252	0,85464	1,04676	0,25
0,66372	0,8559	1,04808	0,3
0,6612	0,85335	1,04551	0,35
0,6594	0,85145	1,0435	0,4
0,65816	0,85044	1,04272	0,45
0,63574	0,82782	1,0199	0,5
0,62921	0,82131	1,01341	0,55
0,59079	0,78291	0,97504	0,6

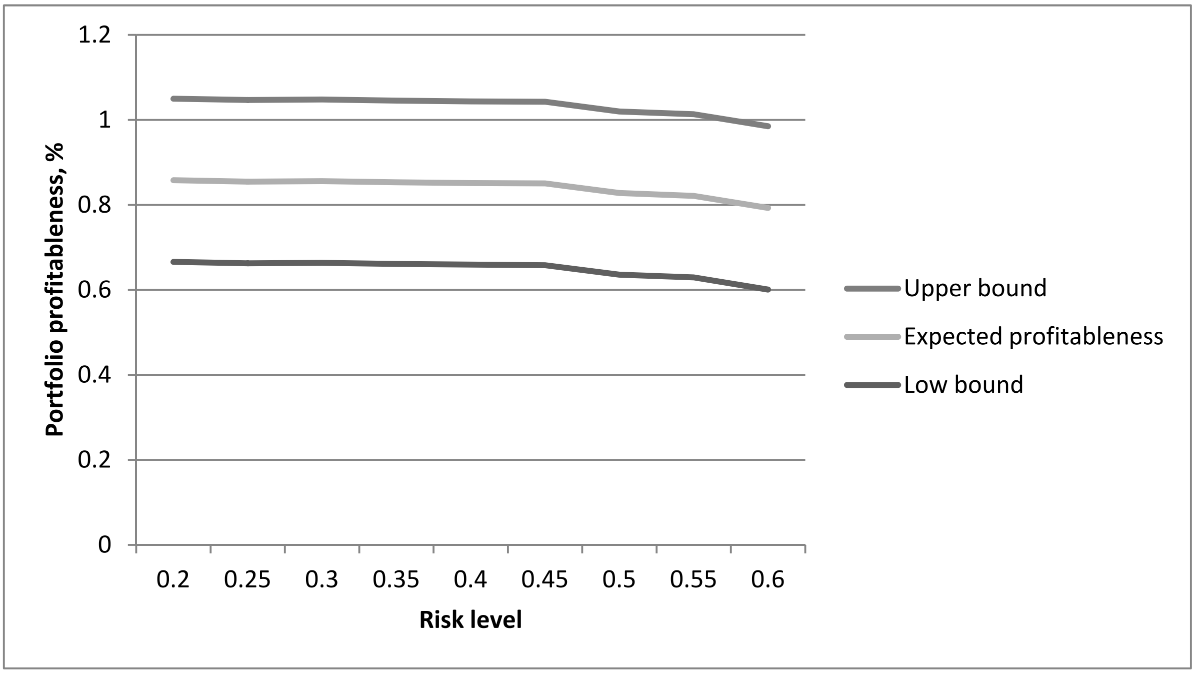

Figure 5. Dependence of expected portfolio profitableness on risk level for bell-shaped MF

The profitableness of the real portfolio is 0,8339 %. This value falls in results calculated corridor of profitableness [0,6657; 0,8578; 1,0499].

Now consider the same portfolio using Gaussian MF (Tables 7, 8, Figure 6).

Table 7. Distribution of components of the optimal portfolio for Gaussian MF with critical level r *= 0,7 %

CAJ	MCD	PEP	PG	SAP
0,0028	**0,00277**	**0,00221**	**0,0021**	**0,99012**
0,0009	0,00126	0,00153	0,00162	0,99469
0,00028	0,00189	0,00232	0,00213	0,99338
0,00193	0,00243	0,00284	0,00278	0,99002
0,00144	0,00096	0,00088	0,00138	0,99534
0,00083	0,001	0,00225	0,00144	0,99448
0,00223	0,0024	0,003	0,00209	0,99028
0,0013	0,00124	0,00129	0,0019	0,99427
0,00261	0,00191	0,00204	0,00239	0,99105

Table 8. Parameters of the optimal portfolio for Gaussian MF with critical level r *= 0,7 %

Low bound	Expected profitableness	Upper bound	Risk level
0,6833	**0,87551**	1,06772	**0,2**
0,66972	0,86178	1,05384	0,25
0,66955	0,86161	1,05368	0,3
0,66468	0,85682	1,04896	0,35
0,64944	0,8415	1,03356	0,4
0,65975	0,85185	1,04394	0,45
0,63439	0,8266	1,0188	0,5
0,63184	0,82389	1,01594	0,55
0,62452	0,81666	1,0088	0,6

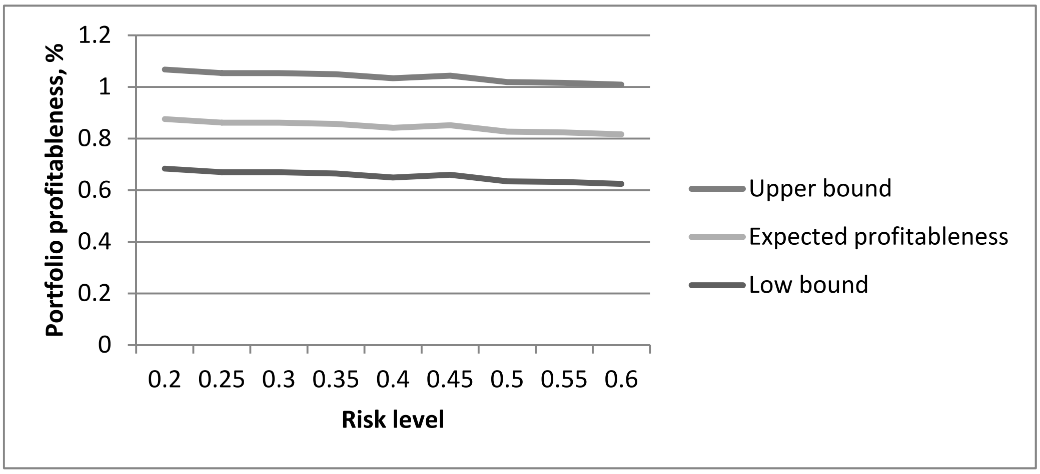

Figure 6. Dependence of expected portfolio profitableness on risk level for Gaussian MF

The profitableness of the real portfolio is 0,8316%. This value falls in results calculated corridor of profitableness [0,6833; 0,8756; 1,0677].

In the above results the optimal portfolio corresponds to the first row of tables. As can be seen from these graphs, the profitableness obtained by using Gaussian and bell-shaped MF is higher than the profitableness obtained using triangular MF. The reason is the looks of used curves. The bell-shaped and Gaussian function is more convex, so an area of inefficient assets is bigger, and the risk of getting into this area is higher.

The optimal portfolio obtained with different MF actually have the same structure, the main part falls on the company SAP, due to high rates of return compared to other companies.

Let's consider the results obtained by solving the dual problem using triangular MF. In this case, the investor sets the rate of return, and the problem is to minimize the risk.

The optimal portfolio is listed in Tables 9, 10, Figure 7:

Table 9. Distribution of components of the optimal portfolio (dual task)

CAJ	MCD	PEP	PG	SAP
0,01627	0,02083	0,02226	0,02231	0,91833
0,01112	0,02085	0,02391	0,02383	0,92029
0,00333	0,01992	0,02517	0,02476	0,92682
0,0021	0,01579	0,02457	0,02344	0,9341
0,00004	0,00921	0,02423	0,02135	0,94517
0,00224	0,00144	0,01825	0,01095	0,96712
0,00044	0,00682	0,02508	0,02058	0,94708
0,0011	0,00917	0,02448	0,02039	0,94486
0,00294	0,01206	0,02533	0,02154	0,93813

Table 10. Parameters of the optimal portfolio (dual task)

Low bound	Expected profitableness	Upper bound	Risk level	Critical rate of return
0,58944	0,78264	0,97584	0,00025	0,6
0,59846	0,79141	0,98437	0,01468	0,65
0,61478	0,80735	0,99991	0,04973	0,7
0,6229	0,81531	1,00772	0,13347	0,75
0,63606	0,82822	1,02037	0,26399	0,8
0,64945	0,84181	1,03417	0,49937	0,85
0,63712	0,82933	1,02153	0,72631	0,86
0,63382	0,82612	1,01843	0,8333	0,87
0,62559	0,81805	1,01052	0,91214	0,88

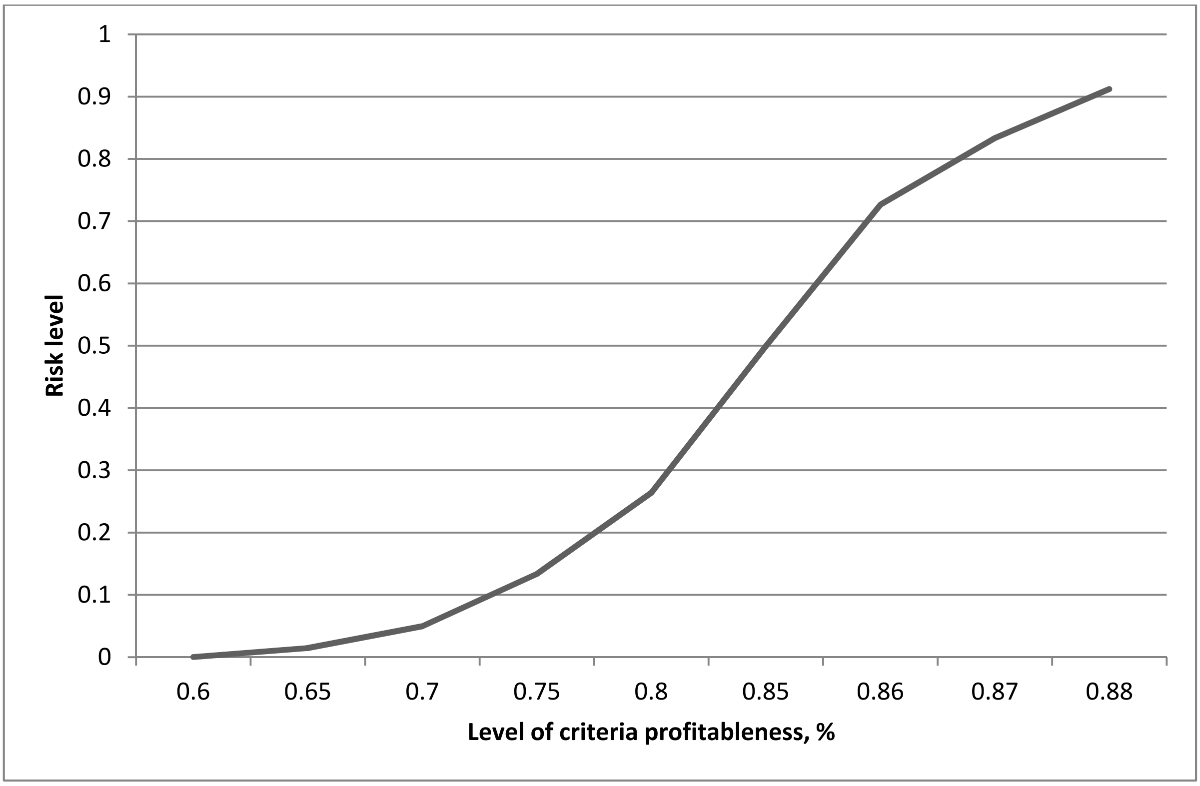

Figure 7. Dependence of the risk level on a given critical return

From these results we can see that the dependence risk - given critical level of profitability takes a growing character, because the growth of the critical profitability increases the probability that the expected return will be lower than a given critical value.

Conclusion

The problem of optimization the investment portfolio under uncertainty is considered in this paper. Particular we use the fuzzy-set approach for solving the direct and dual optimization problem. In the direct problem we used triangular, bell-shaped and Gaussian membership functions. The results of solving the tasks were presented. The optimal portfolio for the five assets was constructed. We got the input to the system by using FGMDH.

As a result of this research we obtained based on fuzzy set-approach mathematical model for the structure of the optimal investment portfolio, devoid of most shortcomings of classical probabilistic models.

- From the results for the direct problem we see that the dependence profitableness - risk has descending type, the greater risk the lesser is profitableness opposite from classical probabilistic methods. It may be explained so that at fuzzy approach by risk is meant the situation when the expected profitableness happens to be less than the given criteria level. When the expected profitableness decreases, the risk grows.
- The profitableness obtained by using Gaussian and bell-shaped MF is higher than the profitableness obtained by using triangular MF. The reason is the looks of used curves.
- The dependence risk - given critical level of profitability takes a growing character, because the growth of the critical profitability increases the probability that the expected return will be lower than a given critical value.

Thus, we create a system that not only automates the search for the optimal portfolio, but also provides a flexible and effective management of portfolio investments.

In future studies planned to consider the problem of optimizing the average return of the portfolio over a given period of time.

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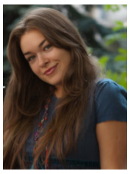

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