AGENT-BASED MODEL OF ECONOMICS: MARKET MECHANISMS, DECISION MAKING, TAXATION

Leonid Hulianytskyi, Diana Omelianchyk

Abstract: *This paper describes the elaborated agent-based model of functioning of economics. We also include different ways of implementation of main structural components of the model (economic agents) and methods of their interaction (market mechanisms). The procedures of making production decisions by agents are being of particular attention to us. The final part of this article presents results of the computational experiment designed to analyze dynamics of tax revenues depending on the tax rate value.*

Keywords: *mathematical modeling; agent-based modeling; intelligent agents; mathematical economics; tax rates.*

ACM Classification Keywords: *H.4.2 Information Systems Applications: Types of Systems: Decision Support, I.2.11 Computing technologies: Artificial intelligence: Distributed artificial intelligence: Multiagent systems, I.6.5 Computing Methodologies: Simulation and modeling: Model development, J.4 Computer applications: Socio and behavioral sciences: Economics.*

Introduction

Traditional economic theory showed its inability to discover preconditions of the global crises in time. Therefore, the need of qualitatively new approaches to the economic system modeling was established. Agent-based modeling is one of the most modern techniques of mathematical modeling. The main idea of this approach is in conception of economics as the complex adaptive system whose behavior is defined by multiple interactions between autonomous heterogeneous economic agents that feature various behavior patterns and ability to learn.

Using mathematical methods, researchers of agent-based models (ABM) are able to create computer simulations of the real world, where they gain an opportunity to explore consequences of different scenarios, forecast evolution of economic systems or analyze cause-effect relations between variables.

The goal of this paper is to present the elaborated ABM of functioning of economics and formalize main stages of its construction, as well as market mechanisms and decision making processes. We illustrate performance of our ABM with computational experiment on analyzing dependence between tax revenues and changes of tax rate values.

General structure of ABM of economics functioning

Agent-based approach to economic system modeling enables creating computational models with numerous degrees of freedom and extraordinary flexibility. In fact, a real tool of the modeler would be a model constructor, i.e. an opportunity to modify a particular implementation of the model without changing its structure and compare results. Proposed ABM of economics functioning [Hulianytskyi, 2014] is also a family of models with homogeneous structure and alternative implementations of the independent components.

Our model contains three types of active agents: raw materials producers, consumption good producers and households. They interact indirectly, via special mediator entities – market of raw materials, consumption good market and labor market.

Each iteration of the model consists of the following structural action blocks, which are performed in the presented order:

- Interaction between producers and households on the labor market;
- Production of raw materials;
- Pricing of raw materials;
- Interaction between raw materials producers and consumption good producers on the raw materials market;
- Production of consumption goods;
- Price formation of consumption goods;
- Determination of consumption budget;
- Interaction between consumption good producers and households on the consumption good market;
- Taxation;
- Decision making by firms.

Each structural block can be implemented in different ways, depending on the goals of the research, and independently from implementation of other blocks. Below, we present the key aspects of implementation of the specified blocks in the proposed ABM and we also briefly note alternative approaches that will be implemented in future.

Market mechanisms in the model

Labor market. Labor market describes labor demand from firms. Firm's request for vacancy filling includes the needed number of workers and suggested wage. Labor market is populated by unemployed households and by workers unsatisfied with their salary. Labor market interaction between firms and households is implemented using modified version of the matching algorithm proposed in [Dawid, 2009]:

- Firms post vacancies at the labor market;
- Unemployed households and households unsatisfied with their current salary consider opened vacancies and apply for interesting jobs;
- Firms receive applications and send invitations to selected households. The number of such invitations cannot exceed the number of lacking workers;
- Households consider received invitations and choose the most profitable option in sense of wage;
- Firms update their information in correspondence with decisions of households.

Matching cycle is performed twice and then is aborted even if firms could not fill all opened vacancies.

Note that in this model, labor market is common for raw materials and consumption good producers.

Consumption good market. Consumption good market interaction takes place exclusively between consumption good producers and households. We assume that firms are able to produce only one kind of consumption good, which is also homogeneous, i.e. goods differ only by their price. Produced consumption good are sent to the consumption good market, where households could buy them. Good offer consists of unit price and available amount of consumption good.

Household's need in consumption good can be measured in conventional monetary or natural units. In our ABM, household's need is estimated via consumption budget that is used to buy goods.

On the consumption good market, household selects one of the offers randomly with probability of selecting an offer being inversely proportional to its price. Therefore, if we denote the value of good to the household by $v(p) = -\ln p$, where p is the price, then a probability $prob_j$ to select good of firm j is given by the formula [Deissenberg, 2008]:

$$prob_j = \frac{Exp\left[v\left(p_j\right)\right]}{\sum_i Exp\left[v\left(p_i\right)\right]}.$$

If there is not enough of selected good on the market, i.e. after purchase household still has some money designated for consumption, then it selects next offer and tries to spend the rest of the budget on the purchase. This process is repeated until the consumption budget is totally spent, or there are no more offers on the market.

Note that the presence or absence of assumption about infinite divisibility of good is extremely important to the model specification. In the latter case, household can find itself in situation when it cannot use non-zero consumption budget on purchase of good because prices are too high. This can significantly affect the profit of firms and correspondingly their decisions.

Raw materials market. The main idea of modeling raw materials market is the same as for consumption good market described above. However, in this case interaction takes place exclusively between consumption good producers and raw materials producers. Consumption budget is represented by assets allocated for raw materials purchase during decision making process.

Production processes

Production activity of firms is defined by its production function, which associates available capital and labor with the quantity of output. In the developed ABM we assume that productive capacities of the firm are not exposed to significant changes during the observable period. Therefore, only labor is relevant to the production process, i.e. the quantity of output Y is expressed by formula

$$Y = \alpha L ,$$

where α – productivity of labor, L – actual number of workers.

Well-known production functions of Cobb-Douglas or CES (with constant elasticity of substitution) could be used as alternative production functions in the proposed ABM.

On the other hand, the quantity of output of consumption good is limited by the availability of appropriate amount of raw materials. Thus, if we denote need in raw materials to produce a unit of consumption good with β and with raw – available amount of raw materials, then the quantity of output of consumer good Y can be defined with expression

$$Y = raw/\beta .$$

Decision making by agents

Pricing. The price of good is one of the main control parameters of firms, i.e. by changing price producer can directly affect the reaction of the environment. In the developed ABM we use the approach from [Nagle, 1987], according to which price is defined via interrelation between the cost of unit production and elasticity of demand. Therefore, raw materials producers can compute the unit price of raw materials p_{raw} using expression

$$p_{raw} = \frac{w_{raw}}{\alpha_{raw}\left(1 + 1/\varepsilon_{raw}\right)},$$

where w_{raw} – proposed wage, α_{raw} – productivity of labor, ε_{raw} – elasticity of demand of firm's good.

In turn, producers of consumption good need to include the cost of raw materials in their price. Appropriate formula for price computation is

$$p_{cons} = \frac{\beta \cdot \left(w_{cons} \cdot L + raw\right)}{raw \cdot \left(1 + 1/\varepsilon_{cons}\right)},$$

where w_{cons} – proposed wage, ε_{cons} – elasticity of demand of firm's good.

Consumption budget. Life cycle of households in the model includes consumption of goods produced by firms. In the existing implementation of the ABM, household's needs are estimated with consumption budget B_{T+1}. Hereby previous income, received by the household in earlier periods, is taken into account implicitly, using average level of income. Therefore, the final formula for consumption budget is expressed with

$$B_{T+1} = \begin{cases} \overline{I},\ if\ I_{T+1} \geq \overline{I}, \\ \overline{I} - I_{T+1}, otherwise, \end{cases}$$

where $\overline{I} = \sum_{i=1}^{T} I_t \Big/ T$ – average income of the household over the observed period, I_{T+1} – household income at the time moment $T+1$, which is determined by wage if the household is employed, or by unemployment benefit otherwise.

Alternatively, household's needs may be measured with utility function.

Decision making by firms. In each iteration, firms-producers should make a decision about proposed wage and desired number of workers. In this version of the ABM we suggest doing this by determining budget (financial) constraints on raw materials purchase and wage payments as shares of profit [Ballot, 1999].

Producers of raw materials spend their financial assets exclusively on provision of the wage fund $money_{salary}$:

$$money_{salary} = \begin{cases} 0,5\pi, if\ (sold > 0) \wedge (\pi > 0), \\ 0,5|\pi|, if\ (sold > 0), \\ 0,3 \cdot money, otherwise, \end{cases}$$

where π – received profit, $money$ – total current financial assets of firm.

Producers of consumption good distribute their funds between wage payments $money_{salary}$ and raw materials purchase $money_{raw}$:

$$money_{salary} = \begin{cases} 0,3\pi, if\ (sold > 0) \wedge (\pi > 0), \\ 0,3|\pi|, if\ (sold > 0), \\ 0,2 \cdot money, otherwise, \end{cases} \qquad money_{raw} = \begin{cases} 0,3\pi, if\ (sold > 0) \wedge (\pi > 0), \\ 0,3|\pi|, if\ (sold > 0), \\ 0,2 \cdot money, otherwise. \end{cases}$$

After evaluation of budget constraints, firms can directly determine values of the main control parameters that are used to interact with the environment. Thus, to determine the number of desired workers, firms estimate the required amount of production via measuring level of demand for good. To do so, we use one of the intuitive approaches to learning, which says that future repeats a trend from the past. Therefore, the desired number of workers L_T can be expressed by

$$L_T = \frac{\xi \cdot \hat{Y}_T + (1-\xi) \cdot \sum_{k=T-t}^{T-1} \hat{Y}_k \Big/ t}{\alpha},$$

where $\hat{Y}_k$ – firm sales at the time moment k, t – interval of smoothing, ξ – smoothing coefficient.

Sales time series is smoothed in order to avoid excessive oscillations of production plan. Therefore, firms could be inert in adjusting sales volumes to real needs of buyers.

Wage w in this case is determined, knowing suggested amount of wage payments:

$$w = \frac{money_{salary}}{L_t}.$$

Computational experiment

In this stage of the ABM development, we consider solely profit tax [Omelianchyk, 2013], which is collected from firms that had received profit in the current period. The general scheme of taxation is presented below:

$$money_{tax} = \begin{cases} tax \cdot \pi, if\ \pi > 0, \\ 0, otherwise, \end{cases}$$

where $money_{tax}$ – collected profit tax, tax – profit tax rate.

Note that in this ABM an individual entity like state budget, where tax revenues are handled and accumulated, is not modeled. The computation of the total amount of tax revenues is conducted on each individual iteration of the model run and is not extended to other periods.

To illustrate capabilities of the developed ABM, we suggest a computational experiment that helps us to find an answer to the following question: How do tax revenues change depending on the tax rate?

Assume that firms need some time to adapt to the conditions of the environment, which include availability of workforce, demand and tax rate. We consider two options: fixed tax rate is either increased or decreased by the same value. Computational experiment is used to analyze observed dynamics of tax revenues in this case.

Initial conditions are defined as follows: $f_{raw} = 2, f_{cons} = 3, h = 500, money_{salary} = 500, money_{raw} = 300,$ $L_0 = 50, \alpha_{raw} = 20, \alpha_{cons} = 5, \beta = 4, \varepsilon = -1.5, money_0^f = money_0^h = 1000,$ where f_{raw} – number of raw materials producers, f_{cons} – number of consumption goods producers, h – number of households, w_0 – initial wage, L_0 – initial desired number of workers, α_{raw} – productivity of labor while producing raw materials, α_{cons} – productivity of labor while producing consumption goods, β – amount of raw materials, required to produce a unit of consumption good, ε – elasticity of demand, $money_0^f$ и $money_0^h$ – initial financial assets of firms and households correspondingly.

Figure1 below shows dynamics of tax revenues when tax rate has been altered from 10% of profit to (1) 5% and (2) 15% correspondingly.

Therefore, we can assert that tax rate increase really causes growth of tax revenues during initial periods. However, decrease of tax rate enables firms to have larger financial assets, which they can use for further investments in production. Thus, in the long-term perspective tax revenues with decreased tax rate begin to exceed the ones with increased tax rate. However, we should take into account that both increase and decrease of tax rates makes sense only to some threshold value. The problem of finding this value, as well as optimal tax rate, remains the subject of our further research.

Conclusion

This article describes the elaborated ABM of economics functioning. On the first stage of creating of this model, the purpose was to establish mechanisms of interaction between main economic agents, firms and households, on the markets of raw materials, consumption goods and labor. The procedures of making production decisions were of great interest to us as well. The second stage was dedicated to the analysis of adequacy of various intelligent agents learning algorithms for appropriate modeling of firm behavior and comparison of their results. In future, we plan to extend capabilities of the model by introducing new types of agents (bank, government, and external market), new mechanisms of interaction (bank loans, openings of deposits) and new subsystems of national economics (region, field).

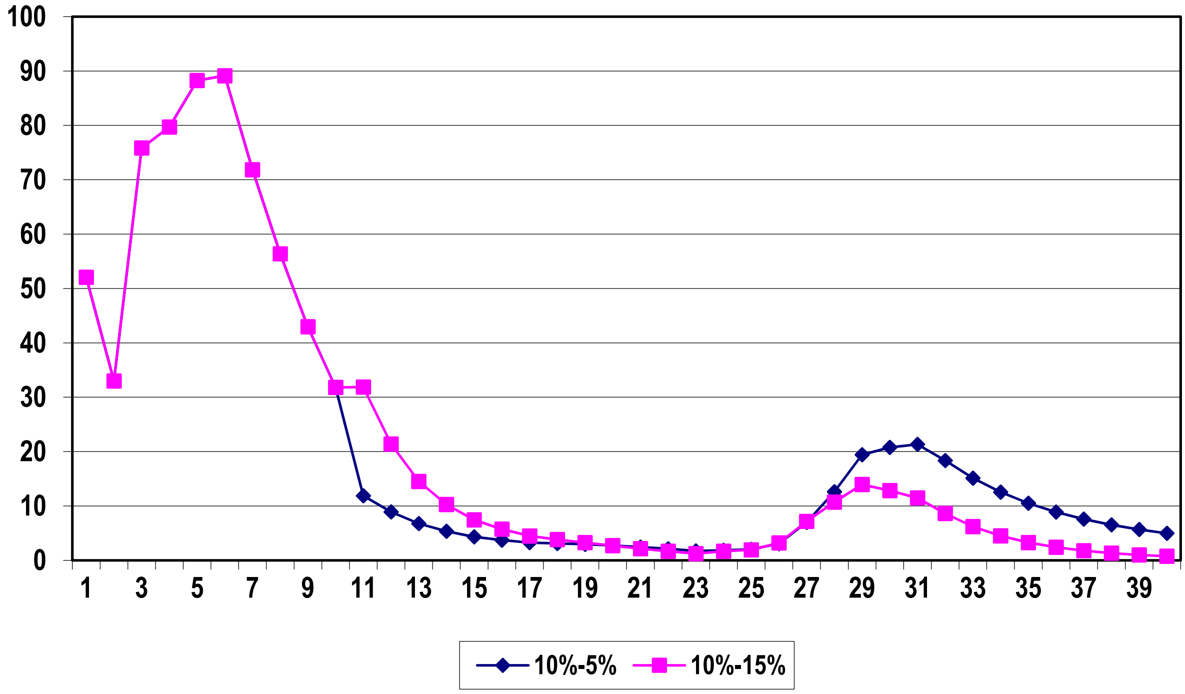

Figure 1. Tax revenues changes in correspondence with tax rate change

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Bibliography

[Ballot, 1999] Ballot G., Taymaz E. Technological change, learning and macro-economic coordination: An evolutionary mode // J. of Artificial Societies and Social Simulation. – 1999. – N 2(2). URL http://jasss.soc.surrey.ac.uk/2/2/3.html

[Dawid, 2009] Dawid H., Gemkow S., Harting P., Neugart M. On the effects of skill upgrading in the presence of spatial labor market frictions: An agent-based analysis of spatial policy design // J. of Artificial Societies and Social Simulation. – 2009. – N 12 (4).

[Deissenberg, 2008] Deissenberg C., van der Hoog S., Dawid H. EURACE: A massively parallel agent-based model of the European economy // Applied Mathematics and Computation. – 2008. – N 204(2). – P.541–552.

[Hulianytskyi, 2014] Гуляницкий Л.Ф., Омельянчик Д.А. Разработка и исследование базовой агентно-ориентированной модели функционирования экономики // Компьютерная математика. – № 1. – 2014. – С. 26-36.

[Nagle, 1987] Nagle T. Strategy and Tactics of Pricing. – Prentice Hall. – 1987.

[Omelianchyk, 2013] Омельянчик Д.А. Дослідження оптимальної податкової ставки в агентно-орієнтованій моделі економіки // Сучасні проблеми моделювання соціально-економічних систем. Матеріали V Міжн. науково-практичної конф. – Х.: ВД «ІНЖЕК», 2013. – С. 314-317.

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