ON RECONSTRUCTION OF IMAGES

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Abstract: *This paper considers the problems related to processing of optically registered non-focused images in frequency domain, by the use of the two principal theorems of the domain: Kotelnikov [Kotelnikov, 2000] an Wiener [Ahmed & Rao, 1980] theorems, concerning signal and image processing.*

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Introduction

Methods and solutions of image processing problems (quality increase, reconstruction, segmentation, etc.) are subdivided into two principal classes – transforms in space domain, based on transformations of image describing matrices and elements (pixels), and frequency transformations, based on image transforms in spectral domain (Fourier transforms).

It is well known that there doesn't exist a general theory to solve the problem of image quality improvement. Thus, it could be that the algorithms for optically registered image improvement don't work to improve image quality when registered in other ways (X-ray or gamma-ray, by radiolocation, etc.).

In this paper the optically registered non-focused images processing problem is considered in frequency domain, based on two principal theorems concerning signal and image processing - by Kotelnikov [Kotelnikov, 2000] and Wiener [Ahmed & Rao, 1980; Pratt, 1982]. There exist large number of different publications and algorithmic realizations [SmartDeblur, 2015] solving these problems but some technique arising during the algorithmic realizations still remains open. For example, even if it is known the dispersion of inducing noise of impulse function, there appear an iterative time consuming process during the Wiener and inverse filtration processes.

One of the reasons of appearing iteration is the requirement on images to be stationary in the Wiener theorem. Also if the mean square value for image is the least, this fact can't be the quality characteristic of being images the best.

Evaluation of the number of the required iterations is a sensitive characteristic of the algorithm and its termination, but this is a separate problem not considered in this paper.

While registered, often there appear distortions of different type on the image, depending on characteristics of registering devices (sensitivity), technical situation or location and also peculiarities of the image area being registered. On images obtained by optical devices there can be violations of focal distance; there can appear diffusions when moving objects are registering, etc.

We consider the image as a function of two variables f(x,y) which is the projection of three dimensional field on to the two dimensional field of view, where (x,y) is a coordinate of any point of plane and f(x,y) is the light intensity in the point (x,y).

We'll consider the problem of optically [Chen & Yong, 2010] registered distorted images reconstruction in spectral area. Because of optical systems focus on the falling light and that can be expressed by the Fourier transformation, then as it is known, the image reconstruction problem reduces to the solving of integral equations of second order.

Image reconstruction

Let $g(x, y)$ be the given/registered image and $f(x, y)$ be the reconstructed image. Then the following equation takes place

$$g(x, y) = \iint f(u, v)h(x, y, u, v)dudv, \quad (1)$$

where the function $h(x, y, u, v)$ is called image registering system's impulse response (output value corresponding to unit impulse) [Chen & Yong, 2010].

To solve this equation we'll give some assumptions.

Definition. The system is called space-invariant, if its impulse function response depends on the difference between input (x, y) and output (x, y) plane coordinates:

$$h(x, y, u, v) = h(x - u, y - v).$$

For such system the equation (1) may be represented as

$$g(x,y) = \iint f(u,v)h(x-u,y-v)dudv, \tag{2}$$

which is usually called the convolution. Equation (2) can also be represented as

$$g(x,y) = f(x,y) * h(x,y). \tag{3}$$

Since $f(x,y)$ is a function of image describing the range of vision, and $g(x,y)$ is a function of the registered image, we can see that $h(x,y)$ is a noise describing function.

In general case linear filtration algorithms are realized by transforms of type (2) having the following discrete representation

$$g_{i,j} = \sum_{k=i-r/2}^{i+r/2} \sum_{l=j-r/2}^{j+r/2} f_{k,l} h_{k-i+{}^{r}/_{2},\, l-j+{}^{r}/_{2}},\ i \in \left[{}^{r}/_{2}, M + {}^{r}/_{2}\right], j \in \left[{}^{r}/_{2}, N + {}^{r}/_{2}\right]. \tag{4}$$

M is number of image rows, N is number of image columns, the sum includes points of rectangle with the center at (i,j) and the edge $2r+1$. Before applying the calculation of the transform (4) all parts of image should be already widened by the rectangular areas of width $r/2$ (using zeros or repeating the boarder values).

In spectral domain the linear filtration algorithm is also based on convolution theorem, so instead of calculating by formula (4) it can be realized by the following formula:

$$G(u,v) = F(u,v)H(u,v), \tag{5}$$

where G, F, H are Fourier transforms of functions g, f, h. Note, that complex multiplication is realized by all u, v frequencies.

Now we'll represent the mathematical model of the system:

$f(x, y)$ - reconstructed image function (undistorted),

$h(x, y)$ - noise causing function,

$n(x, y)$ - total noise,

$g(x, y)$ - distorted registered image (fuzzified, unfocused).

So we have the following representation of the process:

$$g(x, y) = f(x, y) * h(x, y) + n(x, y). \tag{6}$$

It is required to find the impulse characteristic function which will be for the system the best reconstruction function $\hat{f}$ by means of square distortion value

$$\sigma = \sqrt{\frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\left(\hat{f}_{i,j-}f_{i,j}\right)^2} \rightarrow \min$$

The problem solution for linear stationary processes was given by Wiener. The proof and the detail are in given in [Ahmed & Rao, 1980]. The best approximating filter's spectral representation of function $f(x, y)$ represents as [Ahmed & Rao, 1980]

$$\hat{F}(u, v) = \frac{1}{H(u, v)} \cdot \frac{|H(u, v)|^2}{|H(u, v)|^2 + {S_n(u, v)}\big/{S_f(u, v)}} \cdot G(u, v), \tag{7}$$

where $S_n(u, v)$ is the spectral density of additive noise and $S_f(u, v)$ is the spectral density of the function $f(x, y)$. In general case these values are unknown. The ratio $S_n(u. v)/S_f(u. v)$ is the inverse value of the signal-noise value. Its value in time domain is considered acceptable if it is in the interval of 30 - 40 decibels.

The noises induced by focal distance violations on the images registered by the optical devices mainly depend on the light dispersion, a problem described by the following two Gaussian functions:

$$h(x,y)=\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}},$$

$$h(x,y)=\begin{cases}\frac{1}{\pi r^2}, & \text{if } x^2+y^2<r^2\\ 0, & \text{if } x^2+y^2\ge r^2.\end{cases}$$

If the image doesn't include an additive noise then $n(x,y)=0$ and formula (7) represents as

$$\hat{F}(u,v)=\frac{G(u,v)}{H(u,v)}, \tag{8}$$

which is called an inverse filter.

Inverse filters

In the model above, because of some device errors during the image registration under some frequencies the value of denominator $H(u,v)$ of equation (8) becomes equal to 0 causing an indeterminacy situation. In such cases, the spectral value corresponding to this value of image is set equal to zero. As a result, on the filtered image there appear obvious horizontal or vertical (sometimes-curved) phenomena.

To reduce such occurrences we offer to realize in spectral domain the low-frequency interpolation:

$$\hat{F}(u,v)=\sum_{i=u-w}^{u+w}\sum_{j=v-w}^{v+w}s_{i,j}, \tag{9}$$

Where

$$s_{i,j}=\begin{cases}F(i,j)\frac{\sin(2\pi fi)}{i}\frac{\sin(2\pi fj)}{j}, & \text{if } i>0, j>0,\\ 2\pi fF(i,j)\frac{\sin(2\pi fj)}{j}, & \text{if } i=0, j\ne 0,\\ 2\pi fF(i,j)\frac{\sin(2\pi fi)}{i}, & \text{if } i\ne 0, j=0,\\ 4\pi^2f^2F(i,j), & \text{if } i=0, j=0,\\ 0, & \text{in other cases}\end{cases}$$

$f\in(0;\ 0{,}5)$, $2w+1-$ length of kernel.

From Wiener's theorem it follows that the minimum of the mean square distortion value doesn't depend on the type of orthogonal transformation. Therefore for speeding up the calculations other orthogonal transformations also can be used: cosine, Walsh, Haar etc. In this case Wiener's filtration is called suboptimal. Here we bring an example of suboptimal filtration.

Let the *covariance* matrices of the image and noise has the following appearance.

$$C_{\text{image}} = \begin{pmatrix} 1 & \rho & \rho^2 & \cdots \rho^{n-1} \\ \rho & 1 \cdots & & \rho^{n-2} \\ \cdots & \cdots & \cdots & \cdots \cdots \cdots \\ \rho^{n-1} & \rho^{n-2} & \cdots & \rho \; 1 \end{pmatrix}, \; C_{\text{noise}} = k \cdot I$$

where $0 < \rho < 1$ is the correlation coefficient of the first-order Markov process, k is constant number and I is an *identity* matrix. In this case Wiener's filtration is considered suboptimal and instead of interpolation formula (9) could also be applied one-dimensional interpolation formula.

$$\hat{F}(u,v) = \sum_{j=v-w}^{v+w} s_j,$$

$$s_j = \begin{cases} F(u,j)\dfrac{\sin(2\pi f j)}{j}, & \text{if } j > 0, \\ 2\pi f F(u,j), & \text{if } j = 0, \\ 0, & \text{if } j < 0, \end{cases}$$

where $f \in (0; 0{,}5)$.

There are many internet investigations and program realizations of this problem.

We think, the system SmartDeblur-1.27-win is one of the best program realizations, but its mathematical apparatus is not presented in the work.

The program realization of method (5) - (9) presented in this paper has been fulfilled.

The result of the system work is given below.

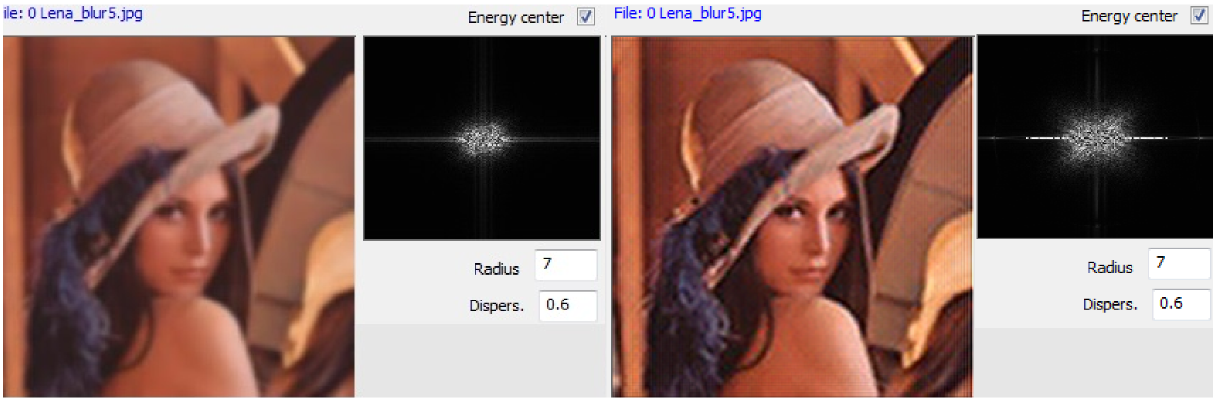

Figure 1. a. input image (blur $\sigma = 5$) b. restore image Inverse filter, iteration = 6

a) Input image, that includes the gauss noise with dispersion $\sigma = 5$ and radius $r = 5$;

b) The result of the developed system, $f = 0.45$ is used.

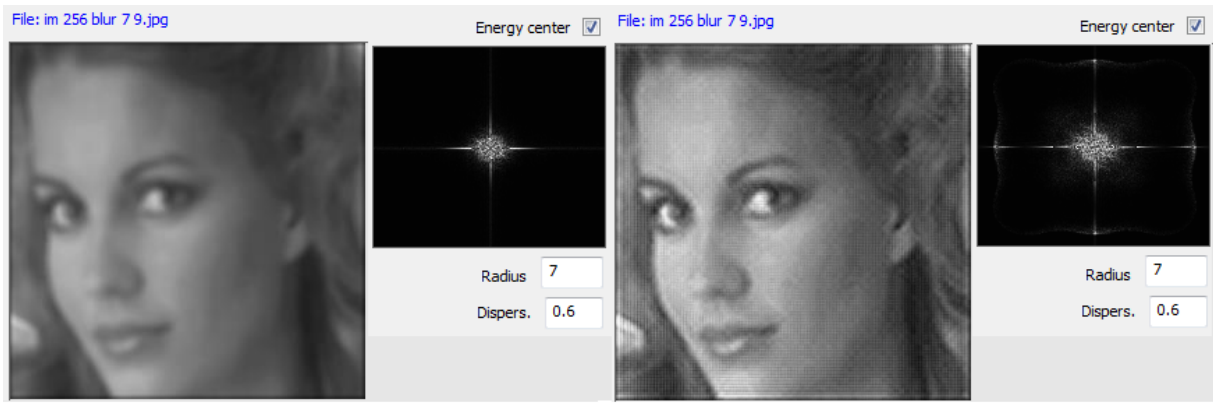

Figure 2. a. input image (blur $\sigma = 9, r = 7$) b. restore image Inverse filter, iteration = 5

a) Input image, that includes the gauss noise with dispersion $\sigma = 9$ and radius $r = 7$;

b) The result of the developed system, $f = 0.45$ is used.

In the above two figures it is implemented a Gaussian blurring on the original images with appropriate mentioned parameters, which gave as a result the images in Figure 1a and Figure 2a. Those images then considered as an input images for inverse filtering. Figures 1b and 2b are the images reconstructed via inverse filter.

Conclusion

In case of decreasing the power spectral density of the image, which is typical for high frequencies, denominator of the inverse filter given by formula (8) tends to zero

$$e_{u,v} = \left(Re\big(H(u,v)\big)\right)^2 + \left(Im\big(H(u,v)\big)\right)^2 \to 0. \tag{10}$$

During the implementation of the filtering it is usually considered $\hat{\mathrm{F}}(\mathrm{u},\mathrm{v}) = 0$ [Ahmed & Rao, 1980] in such cases, which could cause horizontal and vertical smoothing on the reconstructed image.

In the paper for inverse filter's singular (10) values it is obtained a new method for calculation of spectral values of reconstructed image, which is given by interpolation formula (9). The experimental examples show good results, the horizontal and vertical smoothing are disappearing.

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