ON THE BEHAVIOR OF A CLASS OF INFINITE STOCHASTIC AUTOMATON IN A RANDOM ENVIRONMENT

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Abstract. *It is proposed the behavior algorithm of a wide class of infinite stochastic automaton in a stationary random environment that reacts to the behavior of the automaton with three possible reactions (win, loss, indifference). Explicit analytical formulas and offered numerical algorithm for computing the probability characteristics of the behavior of this automaton. In terms of these characteristics is given complete classification of the possible behavior of infinite stochastic automaton in this environment.*

Keywords: *infinite automaton, random environment, reaction of the environment, the behavior of the* automaton, change of actions, win of automaton.

Introduction

The problem of the behavior of finite automata in a random environment was first formulated Tsetlin [1], in which, as in the works of other authors, the study behavior of the asymptotic sequences of finite automata was based on a study of the final probability (at time $t \to \infty$) Markov chains describing the behavior of finite automata in random environments. However, the disadvantage of this method is what the behavior of finite automata in random environments have been studied insufficiently full , in particular, there was no complete classification of possible asymptotic behavior of finite automata in stationary random environments. Such an analysis proved to possible thanks to the study of the behavior of infinite (with countably many states) stochastic automata, the definition of convergence (in a reasonable sense) sequences of finite automata to their respective infinite automata. With this approach the asymptotic behavior of finite automaton is classified in accordance with behavior of the limit automaton [2].

Studies related to the study of the behavior of automata in stationary random environments have shown that the construction of the automaton, is the best for of some signs in any environment, is unrealistic. Therefore, need to build structure and development analytical and numerical methods for finding the statistical characteristics of the behavior of broad classes of Automata that can be used for solutions a variety of practical problems.

In this paper propose an algorithm of behavior of a wide class of infinite stochastic automaton operating in a stationary random environment in the assumption that all possible reactions of the environment perceived by automaton, as belonging to one of three classes: class favorable reactions (win), class of adverse reactions (loss) and the class of neutral reactions (indifferent). Explicit analytical formulas and offered numerical algorithm for computing the probability characteristics of the behavior of this automaton. In terms of these characteristics is given complete classification of the possible behavior of infinite stochastic automaton in this environment.

Analysis of the behavior of infinite stochastic automaton $T_2^{(x)}(l, 1; \varepsilon, \eta)$ in a ternary stationary random environment $C(a_1, r_1; a_2, r_2)$

Consider the scheme of behavior of automata in a random environment under the assumption that every possible reactions $S\epsilon\{s_1, s_2, \dots, s_g\}$ environment perceived automaton, as opposed to [1.2], as referring to one of three classes: class of favorable reactions (win, $s = +1$), class of adverse reactions (loss, $s = -1$) and the class of neutral reactions (indifference, s = 0).

Let automaton A_k functioning in ternary stationary random environment $C(a_1, r_1; a_2, r_2; \dots; a_k, r_k)$ and if the automaton produces action $f_\alpha(\alpha = \overline{1;k})$, then the environment C generates value of the signal on the input of automaton $s = +1$ with probability $q_\alpha = \frac{1-r_\alpha+a_\alpha}{2}$, value of the signal $s = -1$ with probability $p_\alpha = \frac{1-r_\alpha-a_\alpha}{2}$ and value of the signal $s = 0$ with probability $r_\alpha = 1 - q_\alpha - p_\alpha$ $(\alpha = \overline{1;k})$.

Here value $a_\alpha = q_\alpha - p_\alpha$ ($|a_\alpha|$ < 1-r_α) it makes sense to mathematical expectation of win for action f_α in environment $C(a_1, r_1; a_2, r_2; \dots; a_k, r_k)$. For definiteness we assume that $a_1 > a_2 \geq \dots \geq a_k$, so that the action f_1 automaton A_k with middle win a_1 in environment $C(a_1, r_1; a_2, r_2; \dots; a_k, r_k)$ it is optimal.

In studying the of the possible behavior of the infinite automaton in a stationary random environment $(a_1, r_1; a_2, r_2; \dots; a_k, r_k)$, essential is the calculation such of the statistical characteristics of the behavior of the automaton, how probability σ_α change (when - ever) action f_α and mathematical expectations of random τ_α time before change f_α action at start of the automaton $x \in L_\alpha$ $(\alpha = \overline{1;k})$ [2].

In terms of this set of characteristics behavior of the infinite automaton A_k in a random environment is classified as follows.

Definition. Following [2], we say that the infinite automaton A_k , functioning in the ternary stationary random environment $C(a_1, r_1; a_2, r_2; \dots; a_k, r_k)$, is:

optimal, when $\sigma_{x,1} < 1,\ \sigma_{x,\alpha} = 1,\ \alpha = \overline{2;k},\ \forall x;$

strictly optimal, when $\sigma_{x,1} < 1,\ \sigma_{x,\alpha} = 1,\ \tau_{x,\alpha} < \infty,\ \alpha = \overline{2;k},\ \forall x;$

quasi optimal, when $\sigma_{x,\alpha} = 1,\ \alpha = \overline{1;k},\ \tau_{x,1} = \infty,\ \tau_{x,\alpha} < \infty,\ \alpha = \overline{2;k},\ \forall x;$

retractable, when $\sigma_{x,\alpha} < 1\ ,\ \forall x, \alpha;$

pushed out, when $\sigma_{x,\alpha} = 1,\ \ \tau_{x,\alpha} < \infty\ ,\ \forall x, \alpha;$

anti-optimal, when $\sigma_{x,k} < 1,\ \sigma_{x,\alpha} = 1,\ \alpha = \overline{1;k-1},\ \forall x;$

anti- quasi - optimal, when $\sigma_{x,\alpha} = 1,\ \alpha = \overline{1;k},\ \tau_{x,k} = \infty,\ \forall x.$

Let infinite (with countably many states) stochastic automaton $T_2^{(x)}(l, 1; \varepsilon, \eta)$ ($0 \le \varepsilon, \eta \le 1,\ 0 \le \varepsilon + \eta \le 1,\ l = 1,2, \dots$) with $L = L_1 \cup L_2 = \{0, \pm 1, \pm 2, \dots, \pm n, \dots\}$ internal states and two $F_2 = \{f_1, f_2\}$ actions, functioning in the ternary stationary random environment $C(a_1, r_1; a_2, r_2)$. The automaton in the states of the area L_1 with numbers $x = \{\dots, -n, \dots, -1, 0'\}$ performs action f_1, in the states of the area L_2 with numbers $x = \{0,1,2, \dots, n, \dots\}$ - action f_2. For symmetry automaton the area L_1 and L_2 we have implemented with the number 0. If this will cause misunderstanding, then the state 0 the area L_1 will be called $0'$.

We define algorithm behavior of the infinite stochastic automaton $T_2^{(x)}(l, 1; \varepsilon, \eta)$ in the ternary stationary random environment $C(a_1, r_1; a_2, r_2)$ as follows: when the signal $s = +1$ (win) states with the numbers $x = i$ and $x = -i$ $(i = 0,1,2, \dots)$ respectively, goes into a states with the numbers $x = i + 1$ and $x = -(i + 1)$ $(i = 1,2, \dots)$; when the signal $s = -1$ (loss) state of with the numbers $x = i$ and $x = -i$ $(i = 1,2, \dots)$ are moving to states with the numbers $x = i - 1$ and $x = -(i - 1)$ respectively; the state with number $x = 0$ goes over at any one state with number $x = -i$, $i = 0', 1,2, \dots$, and the state with number $x = 0'$ at any one state with number $x = i$, $i = 0,1,2, \dots$; when the signal $s = 0$ all states $x = i$ $(i = 0, \pm 1, \pm 2, \dots)$ with probability $1 - \varepsilon - \eta$ are mapped into themselves, and with probability ε (with probability η) transitions between states are determined also as a signal when $s = -1$ $(s = +1)$.

Thus, the automaton $T_2^{(x)}(l, 1; \varepsilon, \eta)$ can make jumps on one state of in the direction of the area L_α with a probability $P_\alpha = p_\alpha + \varepsilon r_\alpha$; on l of state $(l = 1,2, \dots)$ deep into the area L_α with probability $Q_\alpha = q_\alpha + \eta r_\alpha$ or stay at the same of states with probability $R_\alpha = (1 - \varepsilon - \eta) r_\alpha,\ \alpha = 1,2$. Easy to see that a change the actions of in one clock cycle of functioning of only possible from one state $x = 0\ (0')$.

In the future we will mainly consider the behavior of the automaton in the area, marked by some action before replace it and the index , for brevity ignore.

Let $u_{x,d}$ the probability that infinite stochastic automaton $T_2^{(x)}(l,1;\varepsilon,\eta)$ at time d for the first time replace the action of f, lifting off from any state with number x area L.

Taking into account the tactics of behavior of the automaton $T_2^{(x)}(l,1;\varepsilon,\eta)$ in a stationary random environment $C(a_1,r_1;a_2,r_2)$, relative to probabilities of $u_{x,d}$ will have the difference equation

$$u_{x,d+1} = Pu_{x-1,d} + Qu_{x+l,d} + Ru_{x,d}, \quad x = 1,2,\dots \quad d = 0,1,2,\dots \quad , \tag{1}$$

where

$$P = p + \varepsilon r, \ \ Q = q + \eta r, \ \ R = (1 - \varepsilon - \eta)r, \ P + Q + R = 1$$

and arising from the of the probabilistic of meaning $u_{x,d}$ boundary conditions

$$u_{-1,0} = 1, \ \ u_{x,0} = 0 \ \ \forall x \geq 0. \tag{2}$$

Then the respect to the derivative function probabilities change action

$$U_x(z) = \sum_{d=0}^{\infty} u_{x,d}\, z^d,$$

from (1), (2) obtain the boundary value problem

$$U_x(z) = \frac{Pz}{1-Rz} U_{x-1}(z) + \frac{Qz}{1-Rz} U_{x+l}(z), \quad x = 0,1,2, \tag{3}$$

$$U_{-1}(z) = 1.$$

A solution of equation (3) is

$$U_x(z) = \lambda^{x+1}(z), \tag{4}$$

where $\lambda(z)$ the roots of the characteristic equation

$$Qz\lambda^{l+1}(z) - (1 - Rz)\lambda(z) + Pz = 0. \tag{5}$$

On roots of the equation (5) we have the following lemma, evidence which is based on the Rouche theorem and take place in the same way as in [3].

Lemma. 1. For $|z| < 1$ $(z \neq 0)$all the roots of equation (5) is a simple; one root $\lambda_1(z)$ is situated in unit circle k_1 complex λ plane, others roots l $\lambda_j(z)$, $j = \overline{2, l+1}$ - outside.

2. For $P > lQ$: $\lambda_1(1) = 1$, $|\lambda_{j+1}(1)| > 1$, $j = \overline{1, l}$;

For $P = lQ$: $\lambda_1(1) = \lambda_2(1) = 1$, $|\lambda_{j+1}(1)| > 1$, $j = \overline{2, l}$;

For $P < lQ$: $\lambda_1(1) < 1$, $\lambda_2(1) = 1$, $|\lambda_{j+1}(1)| > 1$, $j = \overline{2, l}$.

From the limited function $|U_x(z)| \leq 1$, based on the lemma and the boundary condition (3), have

$$U_x(z) = \lambda_1^{x+1}(z). \tag{6}$$

Since when $P \geq lQ$ $\lambda_1(1) = 1$, while $P < lQ$ $|\lambda_1(1)| < 1$, then

$$\sigma_x = U_x(1) = \begin{cases} 1, & at\ \ P \geq lQ \\ \lambda_1^{x+1}(1) < 1, & at\ \ P < lQ \end{cases}.$$

To calculate the of the mathematical expectation of time τ_x , which automaton spends in the area L before change the action f, multiply the (1) on $d + 1$ and sum over all $d = 0,1,2, ...$ As a result for of define the τ_x obtain the equation

$$\tau_x = \frac{P}{Q+P}\tau_{x-1} + \frac{Q}{Q+P}\tau_{x+l} + \frac{1}{Q+P}, \ \ \tau_{-1} = 0, \ x = \{0,1,2, \dots\}. \tag{7}$$

When $P > lQ$ equation (7) has a solution

$$\tau_x = \frac{x+1}{P-lQ}. \tag{8}$$

Be noted that when $P = lQ$ $\tau_x = \infty$. In the case of $P < lQ$ $\sigma_x < 1$ and $\tau_x = \infty$ [4].

Thus, the following theorem holds.

Theorem1. Statistical characteristics for infinite stochastic automaton $T_2^{(x)}(l, 1; \varepsilon, \eta)$ in the ternary stationary random environment $C(a_1, r_1; a_2, r_2)$ defined by the formulas:

at $P_\alpha > lQ_\alpha$: $\sigma_{x,\alpha} = 1$, $\tau_{x,\alpha} = \frac{|x|+1}{P_\alpha - lQ_\alpha}$;

at $P_\alpha = lQ_\alpha$: $\sigma_{x,\alpha} = 1$, $\tau_{x,\alpha} = \infty$;

at $P_\alpha < lQ_\alpha$: $\sigma_{x,\alpha} = \lambda_1^{|x|+1}(1)$, $\tau_{x,\alpha} = \infty$ $(\alpha = 1,2)$.

Note that the condition $P_\alpha < lQ_\alpha$ equivalent to the condition

$$a_\alpha > \frac{1 - l + [(1-2\eta)l - (1-2\varepsilon)]r_\alpha}{l+1}, \qquad \alpha = 1,2$$

and the results obtained makes it possible to make the following statement for classification depending on the values of the quantity $[(1-2\eta)l-(1-2\varepsilon)](r_1-r_2)$.

Theorem. The behavior of infinite stochastic automaton $T_2^{(x)}(l,1;\varepsilon,\eta)$ in the ternary stationary random environment $C(a_1,r_1;a_2,r_2)$ is:

1) if $[(1-2\eta)l-(1-2\varepsilon)](r_1-r_2)<0$, then

at $a_1 \le \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_1}{l+1}, a_2 > \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_2}{l+1}-$ optimal:

$$\sigma_{x,1}=1,\ \sigma_{x,2}<1,\quad \tau_{x,1}=\frac{|x|+1}{P_1-lQ_1},\ \tau_{x,2}=\infty;$$

at $a_1 < \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_1}{l+1},\ a_2 > \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_2}{l+1}-$ strictly optimal:

$$\sigma_{x,1}=1,\quad \sigma_{x,2}<1,\quad \tau_{x,1}=\frac{|x|+1}{P_1-lQ_1}<\infty,\ \tau_{x,2}=\infty;$$

at $a_1 > \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_1}{l+1}, a_2 \le \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_2}{l+1}-$ optimal or anti optimal:

$$\sigma_{x,1}=\lambda_1^{|x|+1}(1)<1,\ \sigma_{x,2}=1,\ \tau_{x,1}=\infty,\quad \tau_{x,2}=\frac{x+1}{P_2-lQ_2};$$

at $a_1 = \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_1}{l+1},\ a_2 < \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_2}{l+1}-$ quasi optimal or anti quasi-optimal: $\sigma_{x,\alpha}=1,\ \alpha=1,2,\ \tau_{x,1}=\infty,\quad \tau_{x,2}=\frac{x+1}{P_2-lQ_2}<\infty;$

at $a_\alpha > \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_\alpha}{l+1}-$ retractable:

$$\sigma_{x,\alpha}=\left(\frac{P_\alpha}{Q_\alpha}\right)^{|x|+1}<1,\ \tau_{x,\alpha}=\infty, \alpha=1,2;$$

at $a_\alpha < \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_\alpha}{l+1}-$ pushed out: $\sigma_{x,\alpha}=1,\ \tau_{x,\alpha}=\frac{|x|+1}{P_\alpha-lQ_\alpha}<\infty,\ \alpha=1,2;$

2) if $[(1-2\eta)l-(1-2\varepsilon)](r_1-r_2)\geq 0$, then

at $a_1 > \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_1}{l+1},\ a_2 \leq \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_2}{l+1}$ – optimal:

$$\sigma_{x,1}=\lambda_1^{|x|+1}(1)<1,\ \ \sigma_{x,2}=1, \tau_{x,1}=\infty,\ \tau_{x,2}=\frac{x+1}{P_2-lQ_2};$$

at $a_1 > \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_1}{l+1},\ a_2 < \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_2}{l+1}$ – strictly optimal:

$$\sigma_{x,1}=\lambda_1^{|x|+1}(1)<1,\ \ \sigma_{x,2}=1,\ \tau_{x,1}=\infty,\ \tau_{x,2}=\frac{x+1}{P_2-lQ_2}<\infty;$$

at $a_1 = \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_1}{l+1},\ a_2 < \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_2}{l+1}$ – quasi optimal:

$$\sigma_{x,\alpha}=1,\qquad \alpha=1,2,\ \ \tau_{x,1}=\infty,\ \ \tau_{x,2}=\frac{x+1}{P_2-lQ_2}<\infty;$$

at $a_\alpha > \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_\alpha}{l+1}$ – retractable:

$$\sigma_{x,\alpha}=\lambda_1^{|x|+1}(1)<1,\ \tau_{x,\alpha}=\infty,\ \alpha=1,2;$$

at $a_\alpha < \frac{1-l+[(1-2\eta)l-(1-2\varepsilon)]r_\alpha}{l+1}$ – pushed out:

$$\sigma_{x,\alpha}=1,\ \tau_{x,\alpha}=\frac{|x|+1}{P_\alpha-lQ_\alpha}<\infty,\ \ \alpha=1,2.$$

From these formulas see that in the case of $P_\alpha < lQ_\alpha$ probability $\sigma_{x,\alpha}$ action change f_α is a function of the smallest root of the characteristic equation (5) for $z=1$.

The decomposition of the generating function obtained by the method of the Newton-Puiseux diagrams (at starting of the automaton $x=0$ and at $P_\alpha < lQ_\alpha$) allows approximately calculate the probability of $\sigma_{0,\alpha}$ action change [5].

This expansion in this case is as follows:

$$U_0(z)=\frac{P_\alpha z}{1-R_\alpha z}+\frac{Q_\alpha z}{1-R_\alpha z}\left(\frac{P_\alpha z}{1-R_\alpha z}\right)^{l+1}+(l+1)\left(\frac{Q_\alpha z}{1-R_\alpha z}\right)^2\left(\frac{P_\alpha z}{1-R_\alpha z}\right)^{2l+1}+ \\ +\frac{(l+1)(3l+2)}{2}\left(\frac{Q_\alpha z}{1-R_\alpha z}\right)^3\left(\frac{P_\alpha z}{1-R_\alpha z}\right)^{3l+1}+\cdots \tag{9}$$

and $\sigma_{0,\alpha}=U_0(1)$.

With the help of this expansion can be carried out approximate computations of the $\sigma_{0,\alpha}$ at $l \geq 3$.

In the particular case, at $l = 1$: $\sigma_{0,\alpha} = \frac{P_\alpha}{Q_\alpha}$, and when $l = 2$: $\sigma_{0,\alpha} = \frac{\sqrt{1+4\frac{P_\alpha}{Q_\alpha}}-1}{2}$.

Observe, that the expansion (9) allows obtain an approximate value of the probability $T_2^{(x)}(l, 1; \varepsilon, \eta)$ action changing of the automaton in case of when the initial state automata is $x = 0$.

In the general case, for an approximate calculation $\sigma_{x,\alpha}$ and $\tau_{x,\alpha}$, for any starting state of $x \geq 0$ of the automaton, can be used a numerical algorithm, built on the basis of the expansion of the function generating $U_0(z)$ [5].

On the basis of (9) we introduce the following parameters

$$\xi = \sqrt[l+1]{QP^l}, \quad \theta = \sqrt[l+1]{\frac{Q}{P}}$$

and consider the function

$$W_x(z) = \theta^{x+1} U_x(z).$$

Multiplying (3) by θ^{x+1} with respect to $W_x(z)$ we obtain the following boundary value problem:

$$\begin{gathered} W_x(z) = \frac{\xi z}{1 - Rz} W_{x-1}(z) + \frac{\xi z}{1 - Rz} W_{x+l}(z), \\ W_{-1}(z) = 1. \end{gathered} \tag{10}$$

From (10) it follows that

$$W_x(z) = \sum_{d=0}^{\infty} \sum_{j=1}^{d} \xi^j \, R^{d-j} \, A_x^{(d-j)}(d) \; z^d,$$

where the magnitudes $A_x^{(d-j)}(d)$ require the determination. Then

$$U_x(z) = \theta^{-(x+1)} \sum_{d=0}^{\infty} \sum_{j=1}^{d} \xi^j \, R^{d-j} \, A_x^{(d-j)}(d) \; z^d,$$

$$u_{x,d} = \theta^{-(x+1)} \sum_{j=1}^{d} \xi^j \, R^{d-j} \, A_x^{(d-j)}(d) \, z^d. \tag{11}$$

From (1), taking into account (11), with respect to the quantities $A_x^{(d-j)}(d)$ obtain the equations:

$$A_x^{(d-j)}(d) = A_{x-1}^{(d-j)}(d-1) + A_{x+l}^{(d-j)}(d-1) + A_x^{(d-j-1)}(d-1),$$

$$x \geq 0, \qquad j = 0,1,2,..d,$$

$$A_x^{(-1)}(d-1) = A_{x-1}^{(d)}(d) = A_{x+l}^{(d)}(d) = 0,$$

$$A_{-1}^{(0)}(0) = 1,\ A_{-1}^{(j)}(0) = 0,\ j = 1,..,d,\ A_x^{(j)}(0) = 0\ \ \forall x \geq 0, \qquad j = 0,1,..,d.$$

To find quantities $A_x^{(d-j)}(d)$ is necessary to determine the number of paths when the automaton with a starting state $x \geq 0$ change action (Fig. 1).

If for some d $(d = 1,2,\dots)$ $x - d \geq 0$ condition is not satisfied, then this means that the automaton can at the d-th action change operation. Since automaton $T_2^{(x)}(l,1;\varepsilon,\eta)$ makes one state transition in the direction of the area, then such a state is only one state $x = -1$.

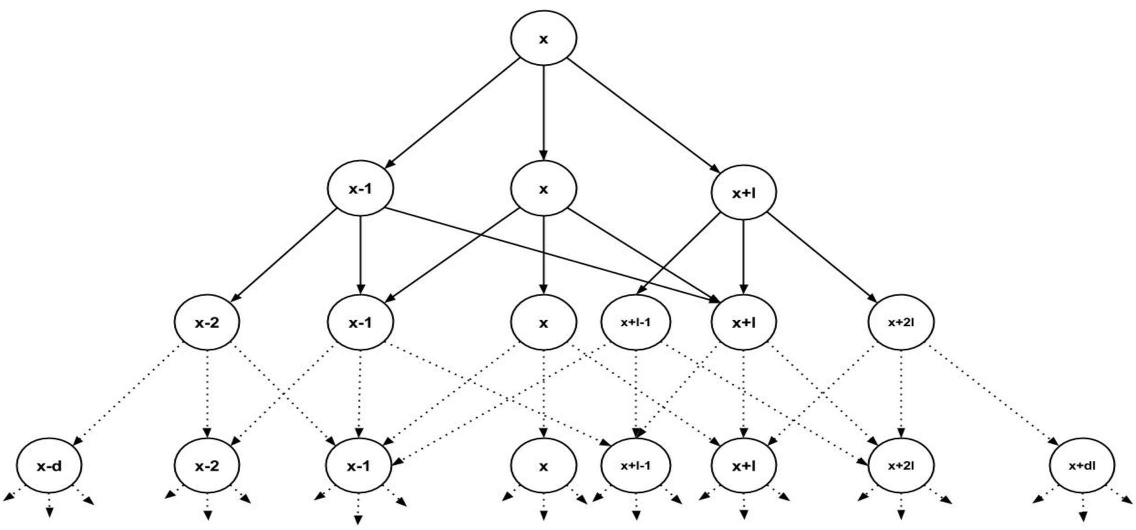

Fig.1. Graf of transition of the automaton $T_2^{(x)}(l,1;\varepsilon,\eta)$ in the area of the states provided $x - d \geq 0$.

Let $N(d)$ the maximum number of possible states of d-tier, and by $H^{(d-j)}(x,i,d)$- the number of paths that lead from state automaton with the number x in the i- th state of the d- tier at $d-j$ stopovers.

Easy notice that the $N(d)=x+ld$ and for determining $H^{(d-j)}(x,i,d)$ have the following recursive relation:

$$H^{(d-j)}(x,N(d)-i,d)=$$

$$=H^{(d-j)}(x,N(d-1)+l+1-i,d-1)\,\mu(N(d-1)+l+1-i)+$$

$$+H^{(d-j)}(x,N(d-1)-i,d-1)\,\mu(N(d-1)-i)+$$

$$+H^{(d-j)}(x,N(d-1)+l-i,d-1)\ \ \mu(N(d-1)+l-i),$$

$$H^{(d-j)}(x,N(d),d)=\begin{cases}1, & j=d\\ 0, & j\neq d\end{cases},\qquad H^{(d-j)}(x,x-1,1)=\begin{cases}1, & j=d\\ 0, & j\neq d\end{cases},$$

$$H^{(d-j)}(x,x,1)=\begin{cases}1, & j=d-1\\ 0, & j\neq d-1\end{cases},\ H^{(d-j)}(x,i,1)=0\ \ ,\quad \forall\ i\neq x,\ x-1,x+l,$$

$$H^{(-1)}(x,i,d)=0\ ,\ \forall\ i,\ d,\ N(d)=x+ld.$$

$$j=1,2,\dots,d,\ \ i=1,2,\dots,x+dl,\ \ \mu(x)=\begin{cases}1, & x\geq 0\\ 0, & x<0\end{cases};$$

Then $A_x^{(d-j)}(d)=H^{(d-j)}(x,-1,d)$ and finally will have:

$$\sigma_x=\theta^{-(x+1)}\sum_{d=0}^{\infty}\sum_{j=1}^{d}\xi^j\,R^{d-j}\ \ H^{(d-j)}(x,-1,d),$$

$$\tau_x=\theta^{-(x+1)}\sum_{d=0}^{\infty}\sum_{j=1}^{d}d\ \ \xi^j\,R^{d-j}\,H^{(d-j)}(x,-1,d).$$

It should be noted that on the fruitfulness of the of the method of diagrams Newton-Puisaye, both for the constructing numerical algorithms, and for the direct calculation of the statistical characteristics of the behavior of infinite automata, was first pointed out in [3].

Conclusion

Thus, the possible behavior of infinite stochastic automaton $T_2^{(x)}(l,1;\varepsilon,\eta)$ in the ternary stationary random environment $C(a_1,r_1;a_2,r_2)$, is completely determined by the statistical characteristics of behavior $\sigma_{x,\alpha}$ of the automaton and $\tau_{x,\alpha}$, $\alpha = 1,2$.

The decomposition of the generating function, obtained by the method of the Newton - Puiseux diagrams (when the starting state of the automaton $x = 0$) allows calculate the probability of approximately by $\sigma_{0,\alpha}$ change action. The number of terms in the expansion (9) depends not only on the environmental parameters, but also a on a parameter l, and of the degree of accuracy. In the general case, numerical algorithm allows with sufficient accuracy to calculate $\sigma_{x,\alpha}$ and $\tau_{x,\alpha}$ for any starting state the automaton $x \in \{0,1,2,\dots\}$.

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